IBSS Heuristic Notes

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Contents

"Justification" for running IBSS with general likelihood

With $q = \prod q_l$

$$\begin{aligned}
\mathcal{L}_{\text{SuSiE}}(q_l; q_{-l}) &= \mathbb{E}_q[\log p(y, \beta_l, \beta_{-l}|X) - \log q] && (1)\\
&\approx \mathbb{E}_{q_l}\left[\log p(y, \beta_l, \bar{\beta}_{-l}) - \log q_l\right] + C && (2)\\
&= \mathcal{L}_{\text{SER}}(q_l, \bar{\beta}_{-l}) + H(q) && (3)
\end{aligned}$$

In the second line we "approximate" by simply pushing the expectation over q_{-l} into the log likelihood term. It turns out this holds exactly when $\log p$ is quadratic in $\beta = \beta_l + \beta_{-l}$ (e.g. Gaussian likelihood case). Noting that $\mathbb{E}_q\left[\beta^2\right] = (\beta_l + \bar{\beta}_{-l})^2 + V(\beta_{-l})$, we can see that $\mathbb{E}_q[Q(\beta)] = \mathbb{E}_{q_l}\left[Q(\beta_l + \bar{\beta}_{-l})\right] + C$ where C is a constant with respect to q_l, and Q is a quadratic function.

The approximation should be good when $\log p(y, \beta_l, \beta_{-l})$ is well approximated by a quadratic function in β, that is, in the asymptotic regime (large n small $|\beta|$). Then, we might hope this approximation works well when $\log p(y, \beta_l, \bar{\beta}_{-l})$:

$$\begin{aligned}
\mathbb{E}_{q_l}\left[\mathbb{E}_{q_{-l}}[\log p(y, \beta_l, \beta_{-l})]\right] &\approx \mathbb{E}_{q_l}\left[\mathbb{E}_{q_{-l}}[Q(\beta_l + \beta_{-l})]\right]\\
&= \mathbb{E}_{q_l}\left[Q(\beta_l + \bar{\beta}_{-l}) + C\right]\\
&\approx \mathbb{E}_{q_l}\left[\log p(y, \beta_l, \bar{\beta}_{-l})\right] + C
\end{aligned}$$