Computer Graphics Assignment 2

Francesco Costa, Arnaud Fauconnet

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Exercise 2

Task 1

To compute the position on the ground plane in which the viewer observes the peak of the highlight we have to find the point such that, if we take the reflection vector $\vec{r}$ of the light source with respect to the normal vector $\vec{n}$ of the plane $y = 0$, it passes by the viewer/camera position $\vec{c}$. Now given

$$\vec{c} = \begin{pmatrix} 4 \\ 6 \\ 7 \end{pmatrix}, \quad \vec{l} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \quad \text{and} \quad \vec{n} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

To find $\vec{r}$ we use the given formula (Lecture #3, slide 13)

$$\begin{aligned}
\vec{r} &= 2n \cdot \langle n, l \rangle - l \\
&= 2 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \cdot \left[\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \right] - \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \\
&= \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} \cdot (0 + 2 + 0) - \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \\
&= \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix}
\end{aligned}$$

We can now construct the line passing through $\vec{c}$ and with direction $\vec{r}$, which is

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \\ 7 \end{pmatrix} + t \cdot \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix}$$

and since we know that the point is on the plane x, z we can set $y = 0$ and solve the system of equations to find the point

$$\begin{cases} x = 4 - t \\ 0 = 6 + 2t \\ z = 7 - 2t \end{cases} \implies \begin{cases} x = 4 - t \\ t = -3 \\ z = 7 - 2t \end{cases} \implies \begin{cases} x = 4 - (-3) \\ t = -3 \\ z = 7 - 2(-3) \end{cases} \implies$$

$$\implies \begin{cases} x = 7 \\ y = 0 \\ z = 13 \end{cases}$$

So the point where the highlight is at its peak is $(7, 0, 13)$.

Task 2

To find the intensity of the light at the peak of the highlight we simply apply the Phong model formula without taking into account the ambient component, so

$$I = \rho_d \cdot \cos(\phi) + \rho_s \cdot \cos^k(\alpha) \cdot I_l$$

where $\rho_d = \rho_s = 0.5,\ \alpha = 0,\ I_l = 1$, and

$$\cos(\phi) = \frac{\langle \vec{n}, \vec{l} \rangle}{\|\vec{n}\| \cdot \|\vec{l}\|} = \frac{2}{1 \cdot 3} = \frac{2}{3}$$

hence

$$I = \frac{1}{2} \cdot \frac{2}{3} + \frac{1}{2} \cdot 1 \cdot 1 = \frac{5}{6}$$

In this case we have assumed a distance of the light to the plane of $r = 1$, so that the distance attenuation $att(r)$ given by

$$att(r) = \frac{1}{r^2} = \frac{1}{1} = 1$$

which means that there is no change in light intensity.

Bonus Exercise 3

Since the sun is extremely far from the moon, which is proportionally small compared to the distance between them, the light source can't be modeled as a single point source, rather a directional light source with parallel rays. Also since the moon is really far from the earth we cannot perceive it as a spherical object but only as a disk. The combination of these two factors give the final result we observe.