Assignment-3 Computational Physics.

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7) Discrete Fourier transform of w_p is,

$$\tilde{w}_q = \frac{1}{\sqrt{n}} \sum_{p=0}^{n-1} w_p \cdot \exp\left(\frac{-i 2\pi pq}{n}\right) \quad \cdots (i)$$

In (i) there is a sum over n terms.

$$= \frac{1}{\sqrt{n}} \sum_{p=0}^{\frac{n}{2}-1} w_{2p} \cdot \exp\left(\frac{-2\pi (2p) q}{n}\right)$$

$$+ \frac{1}{\sqrt{2}} \sum_{p=0}^{(\frac{n}{2}-1)} w_{2p+1} \cdot \exp\left(\frac{-i 2\pi q (2p+1)}{n}\right)$$

$$= F_q^e + W^q F_q^o \quad \cdots (ii)$$

where,
$$F_q^e = \frac{1}{\sqrt{n}} \sum_{p=0}^{\frac{n}{2}-1} w_{2p} \cdot \exp\left(\frac{-i 2\pi pq}{n/2}\right)$$

$$F_q^o = \frac{1}{\sqrt{n}} \sum_{p=0}^{\frac{n}{2}-1} w_{2p+1} \exp\left(-\frac{i 2\pi pq}{n/2}\right)$$

$$W^q = \exp\left(-i \frac{2\pi q}{n}\right)$$

Now, F_q^e & F_q^o are sums over $\frac{n}{2}$ terms, i.e they are Fourier transform of $\frac{n}{2}$ numbers.

We can break F_q^e & F_q^o further into F_q^{ee}, F_q^{eo} and F_q^{oe}, F_q^{oo} which are Fourier transforms of $\frac{n}{4}$ term. We can keep breaking terms in this way until we ~~ch~~ reach when we need to sum over only one term. But ~~b~~ now, Fourier transform of a single number is the number itself.

So to get $\tilde{w}_q$, we need to start at the last step where we get n different sums over single terms and keep back substituting on the steps above.

This is FFT.

We have to do this for m steps.

$m \rightarrow$ total number of steps.

$n \rightarrow$ total number of terms at the last step.

So, $2^m = n$

$\Rightarrow m = \log_2(n)$.

Now ~~a~~ we have n different values of q.

So to get the total fourier transform ~~for~~ we nee $n \times \log_2(n)$ operations.

$\Rightarrow$ fast fourier transform is $O(n \log_2(n))$