Report

Topics: Sparsity (PCA and Compressive Sensing)

Assigned: Wednesday May 23

Due: Sunday June 10 by midnight

Part I: PCA

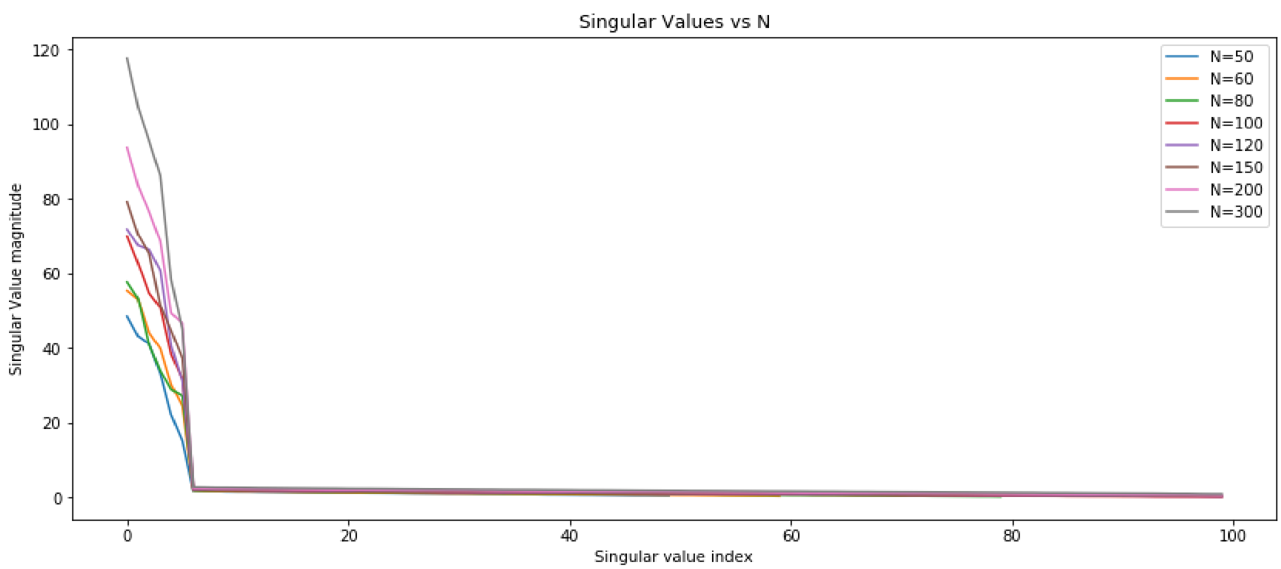
1. (a) d0 = 6 are the dominant singular values. After 6 the values are extremely low and effectively do not contribute a lot to the ddataset.

When we run a comparison for the standard deviation values. We see that when considering the 6 dominant values there is a minor error in

Reduced this Matrix from = (2000 × 100) to = (2000, 6)

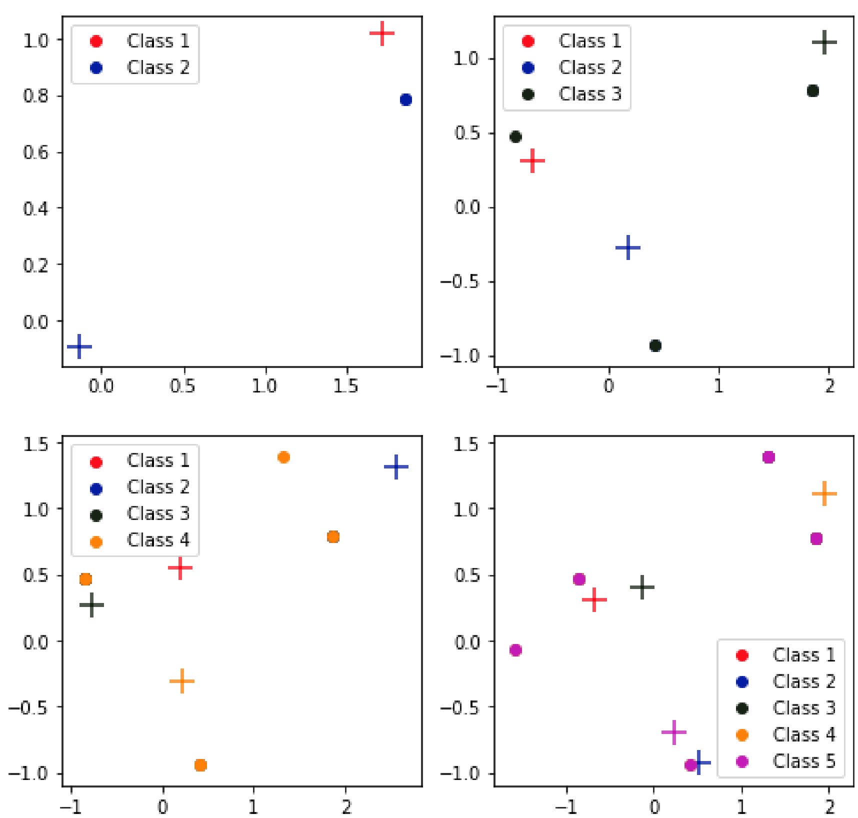
- SD-inp: 1.2395057807174528
- SD-out: 1.235722754594632
- SD-Err: 0.09679696927444671

6 dominant singular values from Matrix (2000 × 100) : [282.76080712 266.71486558 242.47285716 229.01120717 161.32588682 133.6358575 5.38798619]



1. (b) Performed PCA and implemented KMeans for the dominant d0 components. We can see the probabilities of various components based on the cluster center plot.

K= 2
comp= 1 -> Prob=0.00 -> Prob=1.00
comp= 2 -> Prob=0.80 -> Prob=0.20
comp= 3 -> Prob=0.00 -> Prob=1.00
K= 3
comp= 1 -> Prob=0.00 -> Prob=1.00 -> Prob=0.00
comp= 2 -> Prob=0.30 -> Prob=0.00 -> Prob=0.70
comp= 3 -> Prob=0.49 -> Prob=0.51 -> Prob=0.00
K=4
comp= 1 -> Prob=0.00 -> Prob=0.00 -> Prob=0.00 -> Prob=1.00
comp= 2 -> Prob=0.51 -> Prob=0.45 -> Prob=0.04 -> Prob=0.00
comp= 3 -> Prob=0.00 -> Prob=0.00 -> Prob=0.56 -> Prob=0.44
K=5
comp= 1 -> Prob=0.00 -> Prob=0.50 -> Prob=0.13 -> Prob=0.00 -> Prob=0.37
comp= 2 -> Prob=0.30 -> Prob=0.00 -> Prob=0.00 -> Prob=0.70 -> Prob=0.00
comp= 3 -> Prob=0.46 -> Prob=0.00 -> Prob=0.54 -> Prob=0.00 -> Prob=0.00



Observing the Cluster Centers

2. Insights into how the cluster centers found by K-means relate to the d0-dimensional projections of the vectors {uj} in the model

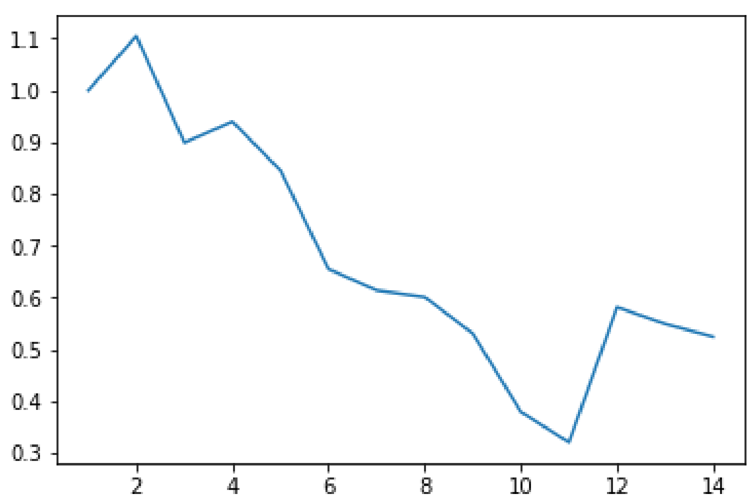
The mean of data points from component 1 is the projection of u1 into m = 6 dimension, and the mean data points from component u2 is the projection of 2 × u4 to the m = 6 dimension, and for the data points from component three, it is the projection of (2)u6 to the m = 6 dimension. Thus, for every cluster, the centers found by K-means is the weighted average of the means of each component in lower dimension, based on how many points of that component are in the cluster.

Part II: Random Projections & Compressed Sensing

3. Generated the dataset with the following variable shapes & values

```
N = 2000,
M = 20,
Y_xp   : (2000, 20),
Labels: (2000, 3),
Phi    : (20, 100),
B      : (100, 7)
```

4. Find a sparse reconstruction of s based on y using Lasso. The minimum M is 11



MSE vs M values

5. normalized MSE over many draws, with reconstruction performance.

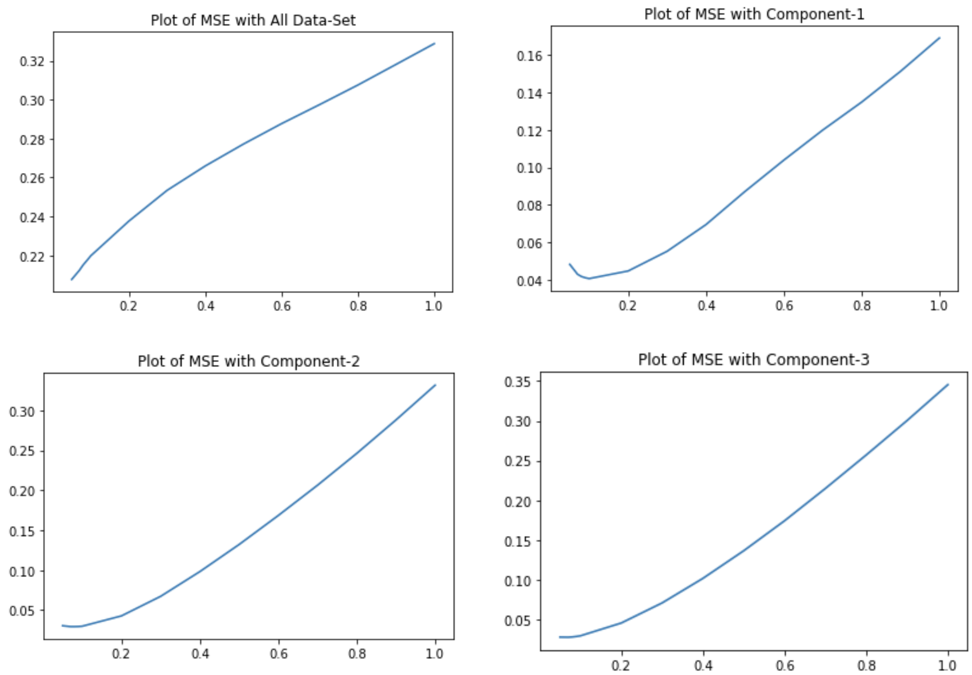


Fig: We can see the mean square error increases as the lambda value increase from 0 to 1

There was another case in which it did not perform any good irrespective of the lambda value and the plot of that is available in the code logs.

6. Down projected the data to 6 dimensions with m =11

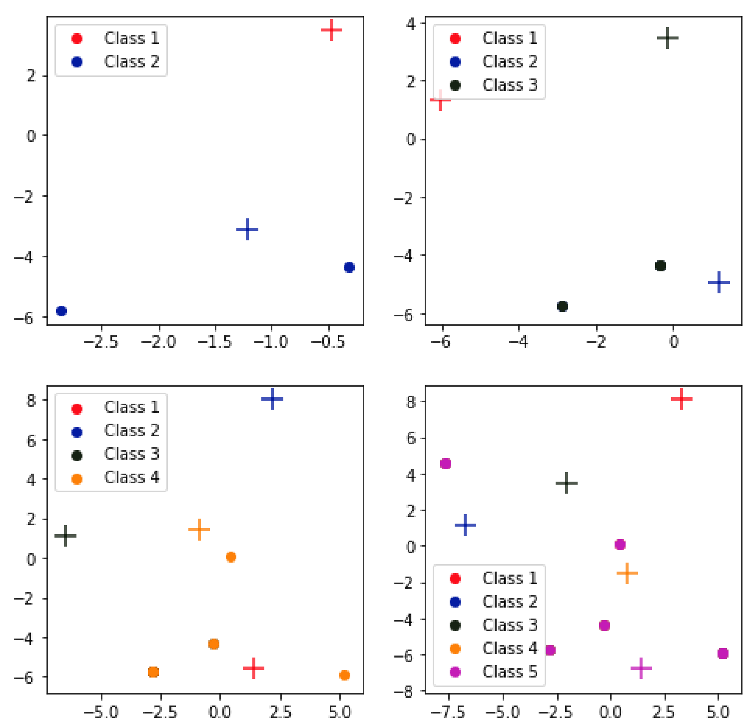
The projected distance are mentioned below in both the spaces. We can see that it did maintain some consistency across the values however not accurate.

```
projected_dist_matrix
        [[  0.     37.45  56.36  73.45  23.64  46.82]
        [ 37.45   0.     66.91  66.55  34.91  87.55]
        [ 56.36  66.91   0.    131.27  59.64 114.45]
        [ 73.45  66.55 131.27   0.     70.18  67.18]
        [ 23.64  34.91  59.64  70.18   0.     43.18]
        [ 46.82  87.55 114.45  67.18  43.18   0.  ]]
dist_matrix
        [[ 0.  2.  2.  7.  3.  4.]
        [ 2.  0.  2.  3.  3.  2.]
        [ 2.  2.  0.  7.  3.  4.]
        [ 7.  3.  7.  0. 10.  7.]
        [ 3.  3.  3. 10.  0.  7.]
        [ 4.  2.  4.  7.  7.  0.]]
```

7. K-means algorithm post-projection

Dataset Dimension: (2000, 11)

K= 2
comp= 1 -> Prob=0.69 -> Prob=0.31
comp= 2 -> Prob=0.63 -> Prob=0.37
comp= 3 -> Prob=0.27 -> Prob=0.73
K= 3
comp= 1 -> Prob=0.00 -> Prob=0.31 -> Prob=0.69
comp= 2 -> Prob=0.43 -> Prob=0.02 -> Prob=0.55
comp= 3 -> Prob=0.08 -> Prob=0.66 -> Prob=0.26
K= 4
comp= 1 -> Prob=0.23 -> Prob=0.03 -> Prob=0.00 -> Prob=0.74
comp= 2 -> Prob=0.01 -> Prob=0.30 -> Prob=0.39 -> Prob=0.30
comp= 3 -> Prob=0.57 -> Prob=0.01 -> Prob=0.06 -> Prob=0.36
K= 5
comp= 1 -> Prob=0.00 -> Prob=0.00 -> Prob=0.47 -> Prob=0.43 -> Prob=0.10
comp= 2 -> Prob=0.25 -> Prob=0.37 -> Prob=0.28 -> Prob=0.10 -> Prob=0.00
comp= 3 -> Prob=0.01 -> Prob=0.05 -> Prob=0.13 -> Prob=0.41 -> Prob=0.40



Observing the Cluster Centers

8. geometric insight. cluster centers found by K-means relate to the m=11 dimensional projections of the vectors

K-means seeks to represent all n data vectors via small number of cluster centroids so K-means can be seen as a highly-sparse PCA.

The mean of data points from component 1 is the projection of u1 into m = 11 dimension, and the mean data points from component u2 is the projection of 2 × u4 to the m = 11 dimension, and for the data points from component three, it is the projection of (2)u6 to the m = 11 dimension. Thus, for every cluster, the centers found by K-means is the weighted average of the means of each component in lower dimension, based on how many points of that component are in the cluster.

h5-pca-cs

June 11, 2018

Topics: Sparsity (PCA and Compressive Sensing)

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```
In [1]: # -*- coding: utf-8 -*-
        import numpy as np
        from math import *
        import matplotlib.pyplot as plt
        from matplotlib import cm
        from mpl_toolkits.mplot3d import Axes3D
        from scipy.stats import norm


        # Params
        N = 2000
        dims = 100
        ulength = 7
```

1 Code & Logs

1.1 Part I PCA

Generate a random vector u in d dimensions as follows: The components of u are i.i.d., with - P [u[i] = 0] = 2=3; P [u[i] = +1] = 1=6; P [u[i] = 1] = 1=6

```
In [2]: # Generate the IID
        def generateMultiDimGaussian(d, ulength):
            u = np.ndarray((ulength,d))
            angles = np.zeros((ulength,ulength))
            deviation = 10
            while np.amax(angles+90*np.eye(ulength)) > (90+deviation) or np.amin(angles+90*np.e
                for i in range(ulength):
                    for j in range(d):
                        r = np.random.rand()
                        if(r<4/6):
```

```
                    u[i,j] = 0
                elif(r<5/6):
                    u[i,j] = 1
                else:
                    u[i,j] = -1
        if(u.any(axis=1).all()):
            for i in range(ulength):
                for j in range(i):
                    angles[i,j] = np.arccos(  np.clip( np.dot(u[i,:]/np.linalg.norm(u[i
                    angles[j,i] = angles[i,j]
    #print(angles)
    return u

# Uj be i.i.d
Uj = generateMultiDimGaussian(dims, ulength)
print(Uj.shape)
```

```
(7, 100)
```

Generate d-dimensional data samples for a Gaussian mixture distribution with 3 equiprobable components 1. Zm : Standard Gaussian (N(0, 1)) distribution 2. N : noise vector" N N(0, 2Id) (default value 2 = 0:01) 3. Component 1: Generate X = u1 + Z1u2 + Z2u3 + N. 4. Component 2: Generate X = 2u4 + sqrt(2)Z1u5 + Z2u6 + N. 5. Component 3: Generate X = sqrt(2)u6 + Z1(u1 + u2) + (1/sqrt(2))Z2u5 + N

```
In [3]: """
        Generate the higher dimension dataset and sample equiprobable from components
        """
        def generateDataset(u, num_data = 50, d = 30):
            sigma_sq = 0.01
            #print('\nX(Nxd):',num_data,"x",d,',\tUj:',Uj[0].shape)
            dataset = np.ndarray((num_data,d))
            labels  = np.zeros((num_data,3))  # will be containing [0,1,0] one hot value
            # Assign the values based on the three component function
            for i in range(0, num_data):
                # Random numbers Zm {Z1, Z2} and N are drawn afresh
                Z1 = np.random.normal()
                Z2 = np.random.normal()
                noise = np.random.multivariate_normal(np.zeros(d), (sigma_sq)*np.eye(d))
                # choose which comnponent to pick from
                idx_comp = np.random.choice([0, 1, 2],1,p=[0.333, 0.333, 0.334])
                if(idx_comp == 0):   # Sample from component 1
                    dataset[i,:] = Uj[1,:] + Z1*Uj[2,:] + Z2*Uj[3,:] + noise
                elif(idx_comp == 1): # Sample from component 3
                    dataset[i,:] = 2*Uj[4,:] + np.sqrt(2)*Z1*Uj[5,:] + Z2*Uj[6,:] + noise
                elif(idx_comp == 2): # Sample from component 3
                    dataset[i,:] = np.sqrt(2)*Uj[6,:] + Z1*(Uj[1,:] + Uj[2,:]) + 1/np.sqrt(2) *
```

```
            # Assign a label
            labels[i,idx_comp]=1
        return dataset, labels

#X_data, Y_hot_labels = generateDataset(Uj, 50, 30)
#print("> X: Data Set:",X_data.shape,", Y: One hot :",Y_hot_labels.shape)
```

1.1.1 1. SVD of the A(N Œ d) data matrix

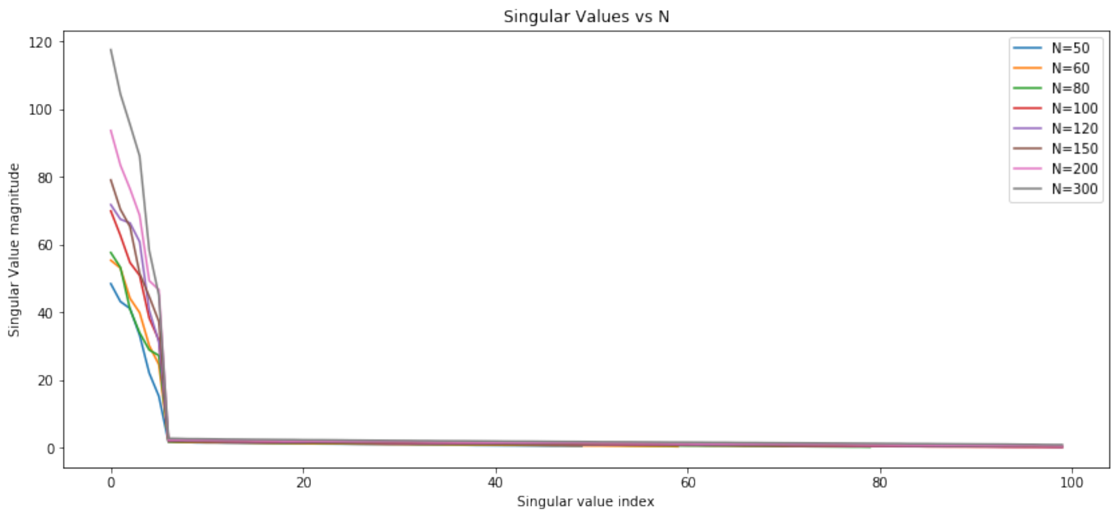
A = U SIGMA VT

1. V must diagonalize ATA and vi are eigenvectors of ATA.
2. SIGMA where SIGMAii are singular values of A.
3. U must diagonalize AAT and ui are eigenvectors of AAT

```
In [4]: # input sample N=? from components
        def getSingularValues(N = 50, d=30):
            X_data, Y_hot_labels = generateDataset(Uj, N, d)
            print("> X: Data Set:",X_data.shape,", Y: One hot :",Y_hot_labels.shape)
            #Xt = X_data.T
            U, S, V = np.linalg.svd(X_data, full_matrices=False)
            X_a = np.dot(np.dot(U, np.diag(S)), V)
            print("> SD-inp:", np.std(X_data), "\nSD-out:", np.std(X_a), "\nSD-Err:",np.std(X_d
            print("Singular values ",N,"Œ",d," : ",S[0:7])
            return S

        plt.figure(figsize=(14,6))
        plt.title('Singular Values vs N')
        plt.xlabel('Singular value index')
        plt.ylabel('Singular Value magnitude')

        for Num in [50, 60, 80, 100, 120, 150, 200, 300]:
            S = getSingularValues(Num, dims)
            plt.plot(range(len(S)), S,label = str('N='+str(Num)))
        plt.legend()
        plt.show()
```

```
> X: Data Set: (50, 100) , Y: One hot : (50, 3)
> SD-inp: 1.2425699236269871
SD-out: 1.242569923626987
SD-Err: 3.048454173224238e-15
Singular values  50 Œ 100  :  [48.39048927 43.11330642 41.06337937 33.16084801 21.96589821 15.
  1.5994595 ]
> X: Data Set: (60, 100) , Y: One hot : (60, 3)
> SD-inp: 1.3498319267986385
SD-out: 1.3498319267986376
SD-Err: 3.4819962065686176e-15
```

```
Singular values  60 Œ 100  :  [55.31684119 53.0200592  44.0650705  39.84964441 30.09112566 24.5
  1.69841237]
> X: Data Set: (80, 100) , Y: One hot : (80, 3)
> SD-inp: 1.1498704791356968
SD-out: 1.1498704791356964
SD-Err: 1.806028648889445e-15
Singular values  80 Œ 100  :  [57.5893642  53.216893   40.83616548 33.89971405 28.80635373 27.
  1.76064902]
> X: Data Set: (100, 100) , Y: One hot : (100, 3)
> SD-inp: 1.2997668301927499
SD-out: 1.2997668301927496
SD-Err: 4.440394990401163e-15
Singular values  100 Œ 100  :  [69.90863454 62.67001101 54.6252193  50.84244765 38.22080618 31
  1.90438301]
> X: Data Set: (120, 100) , Y: One hot : (120, 3)
> SD-inp: 1.3060641823925117
SD-out: 1.3060641823925123
SD-Err: 1.6985712333628717e-15
Singular values  120 Œ 100  :  [71.74773111 67.46007854 66.34224228 60.79853815 40.5416491  31
  1.98174145]
> X: Data Set: (150, 100) , Y: One hot : (150, 3)
> SD-inp: 1.1982438899074066
SD-out: 1.198243889907406
SD-Err: 2.0649813402642227e-15
Singular values  150 Œ 100  :  [79.06779613 70.36741234 65.26769035 51.45642124 44.6072953  37
  2.11513307]
> X: Data Set: (200, 100) , Y: One hot : (200, 3)
> SD-inp: 1.2448438513004056
SD-out: 1.2448438513004045
SD-Err: 1.9863919347571614e-15
Singular values  200 Œ 100  :  [93.67871151 83.432265   76.45619056 68.60885691 49.28696661 46
  2.31861979]
> X: Data Set: (300, 100) , Y: One hot : (300, 3)
> SD-inp: 1.24972911012126
SD-out: 1.2497291101212598
SD-Err: 1.6247784093964919e-15
Singular values  300 Œ 100  :  [117.56893054 104.47921109  95.45651027  86.14543701  58.243263
  45.01554453   2.66531009]
```



1.1.2 1.(a) d0 = 6 are the dominant singular values

we can see this based on the variation for N = [50, 60, 80, 100, 120, 150, 200, 300]

1.1.3 Now, project the data down to the dominant d0 components to obtain an N Œ d0 data matrix.

```
In [5]: X_data, Y_hot_labels = generateDataset(Uj, N, dims)
        print("> X: Data Set:",X_data.shape,", Y: One hot :",Y_hot_labels.shape)
        U, S, V = np.linalg.svd(X_data, full_matrices=False)
        X_a = np.dot(np.dot(U, np.diag(S)), V)
        print(">\nSD-inp:", np.std(X_data), "\nSD-out:", np.std(X_a), "\nSD-Err:",np.std(X_data
        print("Singular values ",N,"Œ",dims," : ",S[0:7])


        d0 = 6 # Dominant vectors d0 = 6
        print("Singular: ", np.diag(S[:d0]).shape)
        reconst_matrix = np.dot(U[:,:d0],np.dot(np.diag(S[:d0]),V[:d0,:]))
        print(">\nSD-inp:", np.std(X_data), "\nSD-out:", np.std(reconst_matrix), "\nSD-Err:",np


        # Using eigen vector V as the basis for projecting the data
        evecs = V[:, :d0]
        X_reduced_matrix = np.dot(X_data, evecs)
        print("Reduced Matrix[From =",N,"Œ",dims,"]:" ," To =",X_reduced_matrix.shape)
        #print(X_reduced_matrix)
```

```
> X: Data Set: (2000, 100) , Y: One hot : (2000, 3)
>
SD-inp: 1.2395057807174528
```

```
SD-out: 1.2395057807174528
SD-Err: 1.7785550318342964e-15
Singular values  2000 Œ 100  :  [282.76080712 266.71486558 242.47285716 229.01120717 161.325886
 133.6358575    5.38798619]
Singular:  (6, 6)
>
SD-inp: 1.2395057807174528
SD-out: 1.235722754594632
SD-Err: 0.09679696927444671
Reduced Matrix[From = 2000 Œ 100 ]:  To = (2000, 6)
```

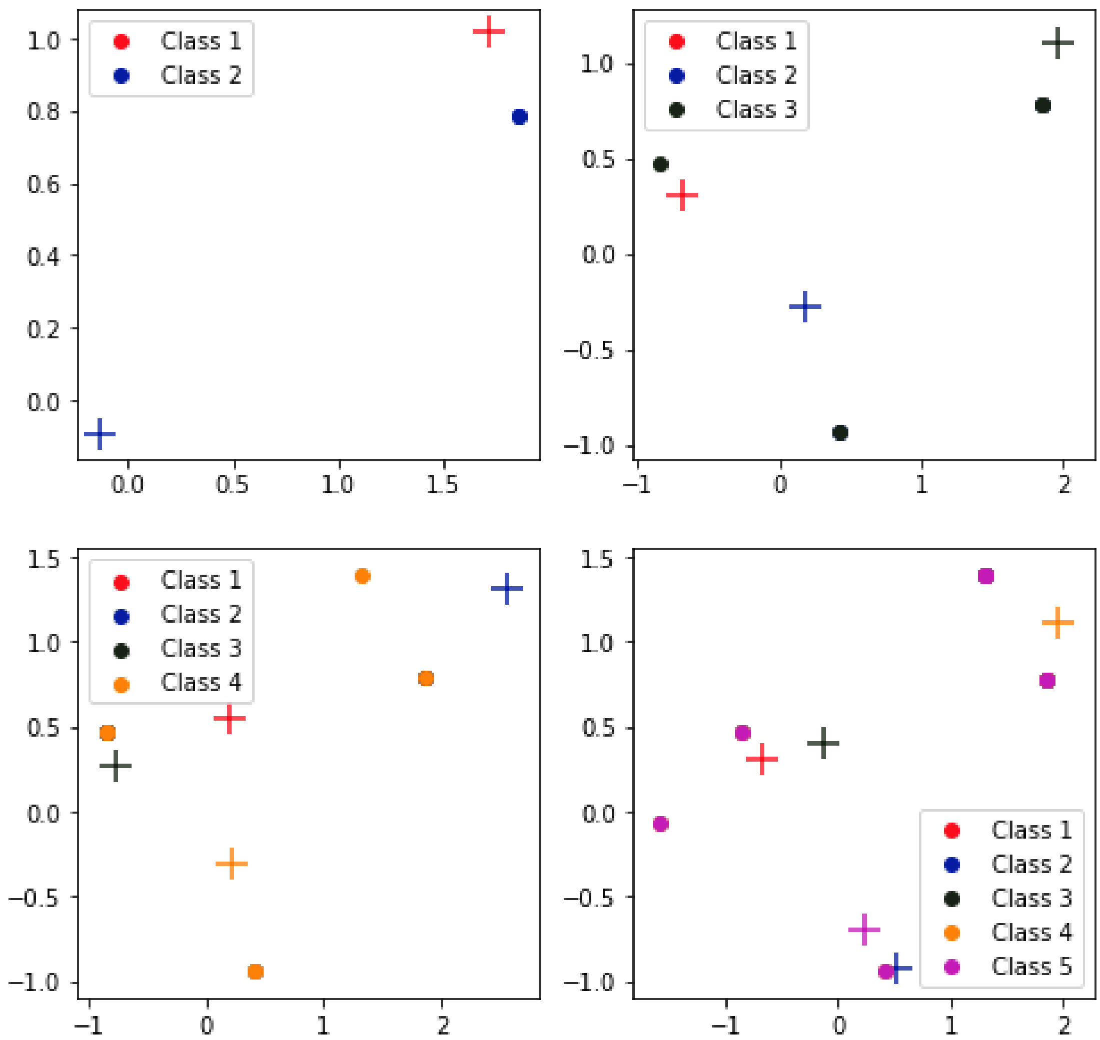
```
In [47]: def runKmeansCluster(data, labels, d):
             class_means_dict=dict()
             preds_dict=dict()
             fig = plt.figure(figsize=(8,8))
             subplot_id = 221
             print("Data Dims: ",N,"x",d)
             print("Dataset  : ", data.shape)
             # Kmeans trial
             for K in range(2,6):
                 pred = np.zeros((N, K))
                 class_means = np.ndarray((K, d))
                 initial_indices = np.random.choice(N,K)

                 for k in range(K):
                     class_means[k,:] = data[initial_indices[k],:]
                 old_class_means = np.zeros((K,d))

                 print("\n-------- K=",K,"----------")

                 while np.linalg.norm(old_class_means-class_means)/np.linalg.norm(class_means)
                     norm_mse = np.linalg.norm(old_class_means-class_means)
                     norm_mu = np.linalg.norm(class_means)
                     #print("> ",norm_mse,"/",norm_mu," \t=\t ",norm_mse/norm_mu)
                     old_class_means = np.array(class_means)
                     for i in range(N):
                         distance_to_means = np.zeros(K)
                         for k in range(K):
                             distance_to_means[k] = np.linalg.norm(class_means[k] - data[i])
                         #print("Distance: ",distance_to_means)
                         nearest_mean = np.argmin(distance_to_means)
                         #print("Nearest : ",nearest_mean)
                         # labels as the min dist
                         pred[i,:] = np.zeros(K)
                         pred[i, nearest_mean] = 1 # one hot encoding

                     # new mean
```

```
                for k in range(K):
                    class_means[k] = np.mean(data[np.where(pred[:,k]==1)], axis=0)
                #print("Class MU : ",class_means)
            ax = fig.add_subplot(subplot_id)
            subplot_id = subplot_id +1


            for k in range(K):
                colors = ('#fc0d1b','#041ca2','#162214','#fd8008','#c41bb6')
                ax.scatter(data[np.where(pred[:k] == 1),0], data[np.where(pred[:k] == 1),
                ax.scatter(class_means[k,0], class_means[k,1], s=200, marker='+', color =
            ax.legend()
            #plt.savefig('dim_%d_q1_%d_means.png' % (d,K), dpi=600)
            #plt.show

            # save means
            class_means_dict[K] = class_means
            preds_dict[K] = pred
            probabilities = np.ndarray((3,K))

            for predicted in range(K):
                print('______________________ \n')
                for true_label in range(3):
                    interssect = np.intersect1d(np.where(labels[:, true_label] == 1), np.
                    tots = np.where(labels[:, true_label] == 1)[0]
                    #print('K=',predicted,", idx=",true_label,"\nP -> ",interssect,"\nT -
                    probabilities[true_label, predicted] = len(interssect) / len(tots)
                    print('K=',predicted,", idx=",true_label,'  -> Prob=%.2f' % probabili
        fig.show
    print("Run on the reduced dimensions data with d0 = ", d0)
    runKmeansCluster(X_reduced_matrix, Y_hot_labels, d0)
```

```
Run on the reduced dimensions data with d0 =  6
Data Dims:  2000 x 6
Dataset  :  (2000, 6)

-------- K= 2 ----------
______________________

K= 0 , idx= 0   -> Prob=0.00
K= 0 , idx= 1   -> Prob=0.80
K= 0 , idx= 2   -> Prob=0.00
______________________

K= 1 , idx= 0   -> Prob=1.00
K= 1 , idx= 1   -> Prob=0.20
K= 1 , idx= 2   -> Prob=1.00
```

```
-------- K= 3 ----------
----------------------
K= 0 , idx= 0   -> Prob=0.00
K= 0 , idx= 1   -> Prob=0.30
K= 0 , idx= 2   -> Prob=0.49
----------------------
K= 1 , idx= 0   -> Prob=1.00
K= 1 , idx= 1   -> Prob=0.00
K= 1 , idx= 2   -> Prob=0.51
----------------------
K= 2 , idx= 0   -> Prob=0.00
K= 2 , idx= 1   -> Prob=0.70
K= 2 , idx= 2   -> Prob=0.00

-------- K= 4 ----------
----------------------
K= 0 , idx= 0   -> Prob=0.00
K= 0 , idx= 1   -> Prob=0.51
K= 0 , idx= 2   -> Prob=0.00
----------------------
K= 1 , idx= 0   -> Prob=0.00
K= 1 , idx= 1   -> Prob=0.45
K= 1 , idx= 2   -> Prob=0.00
----------------------
K= 2 , idx= 0   -> Prob=0.00
K= 2 , idx= 1   -> Prob=0.04
K= 2 , idx= 2   -> Prob=0.56
----------------------
K= 3 , idx= 0   -> Prob=1.00
K= 3 , idx= 1   -> Prob=0.00
K= 3 , idx= 2   -> Prob=0.44

-------- K= 5 ----------
----------------------
K= 0 , idx= 0   -> Prob=0.00
K= 0 , idx= 1   -> Prob=0.30
K= 0 , idx= 2   -> Prob=0.46
----------------------
K= 1 , idx= 0   -> Prob=0.50
```

```
K= 1 , idx= 1   -> Prob=0.00
K= 1 , idx= 2   -> Prob=0.00
----------------------

K= 2 , idx= 0   -> Prob=0.13
K= 2 , idx= 1   -> Prob=0.00
K= 2 , idx= 2   -> Prob=0.54
----------------------

K= 3 , idx= 0   -> Prob=0.00
K= 3 , idx= 1   -> Prob=0.70
K= 3 , idx= 2   -> Prob=0.00
----------------------

K= 4 , idx= 0   -> Prob=0.37
K= 4 , idx= 1   -> Prob=0.00
K= 4 , idx= 2   -> Prob=0.00
```



1.2 Part II : Random Projections and Compressed Sensing

1.3 3.

(a.) Generate m x d matrix , IID drawn such as P [ij = +1] = 1/2; P [ij = 1] = 1/2

(b.) Compressive Projection, y = 1/sqrt(m) * (x) with sparse reconstruction of s based on y

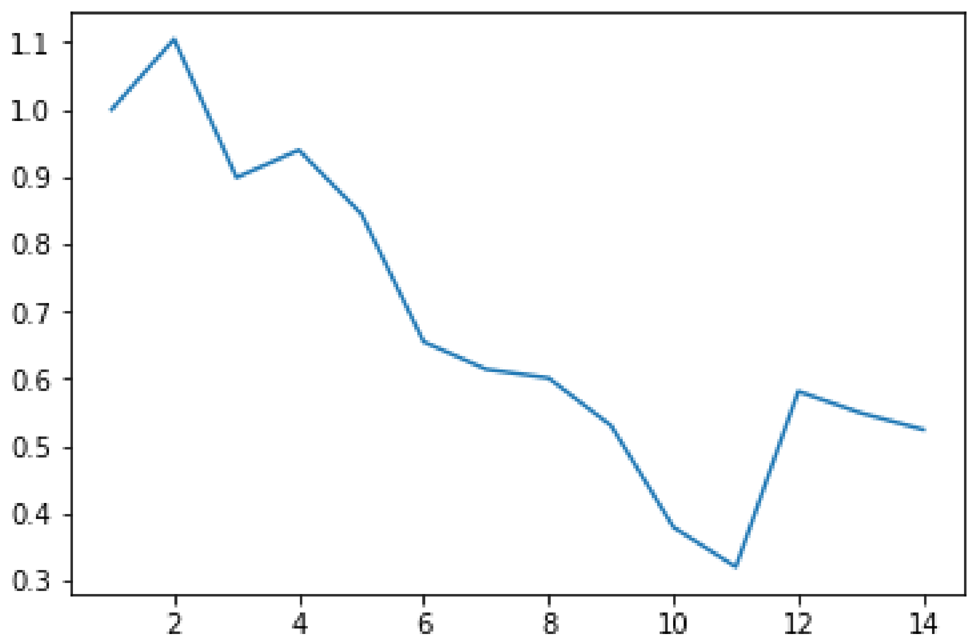
```
In [7]: def generatePhiMatrix(m, d):
            u = np.ndarray((m,d))
            for i in range(m):
                for j in range(d):
                    r = np.random.rand()
                    if(r<1/2):
                        u[i,j] = 1
                    else:
                        u[i,j] = -1
            #print("(mxd):",u.shape)
            return u

        def getCompressedProjection(m, d, log=False):
            # Keeping dimensions as dimensions d = 30 and number of data N = 50
            Xp_data, Zp_hot_labels = generateDataset(Uj, N, d)
            # Generate Phi
            phi = generatePhiMatrix(m,d)
            # Compressive Projection (m-dim projection of the d-dim matrix x)
            Y_xp =  (1/np.sqrt(m)) * np.matmul(Xp_data, phi.T)
            B = np.transpose(Uj)
            if(log == True):
                print("> N =",N,", M =",m,", Y_xp:", Y_xp.shape, ", Labels:",Zp_hot_labels.shap
            return Xp_data, Y_xp, Zp_hot_labels, phi, B

        M = 20
        X, Y_xp, labels, phi, B_evecs = getCompressedProjection(M, dims, True)
```

```
> N = 2000 , M = 20 , Y_xp: (2000, 20) , Labels: (2000, 3) , Phi : (20, 100) , B : (100, 7)
```

1.3.1 4. Lasso problem (using sklearn.linear_model.Lasso)

- (phi) = projection matrix also called A in examples
- x = latent data variables with gaussian noise
- y = observed results (projection matrix is applied on latent data variables and the result has been reduced in dimensions)

```
In [11]: from sklearn import linear_model

         def normalizedME(s, s_hat):
              return np.linalg.norm(s_hat - s)/np.linalg.norm(s)

         def findMinimumM():
             mses = []
             # Keeping dimension fixed and varying M for this
             rng = range(1,15)
             for m in rng:
                 s, Y_xp, labels, phi, B = getCompressedProjection(m, dims, False)
                 matrix = (1/np.sqrt(m)) * np.matmul(phi, B)
                 clf = linear_model.Lasso(alpha = 1.0) # Set lambda ( called alpha  here )
                 clf.fit(matrix, Y_xp.T) # Fit the reduced Y to the one hot
                 a_hat = clf.coef_ # Get a_hat
                 s_hat = np.matmul(a_hat, B.T) # this is the output
                 #print("Mat:",matrix.shape, ",\ts_hat:",s_hat.shape, ",\ts:",s.shape)
                 mses.append(normalizedME(s, s_hat))
             plt.plot(list(rng), mses)
             plt.show()
             return np.argmin(mses)+1

         minM = findMinimumM()
         print("The minimum M is ", minM)
```



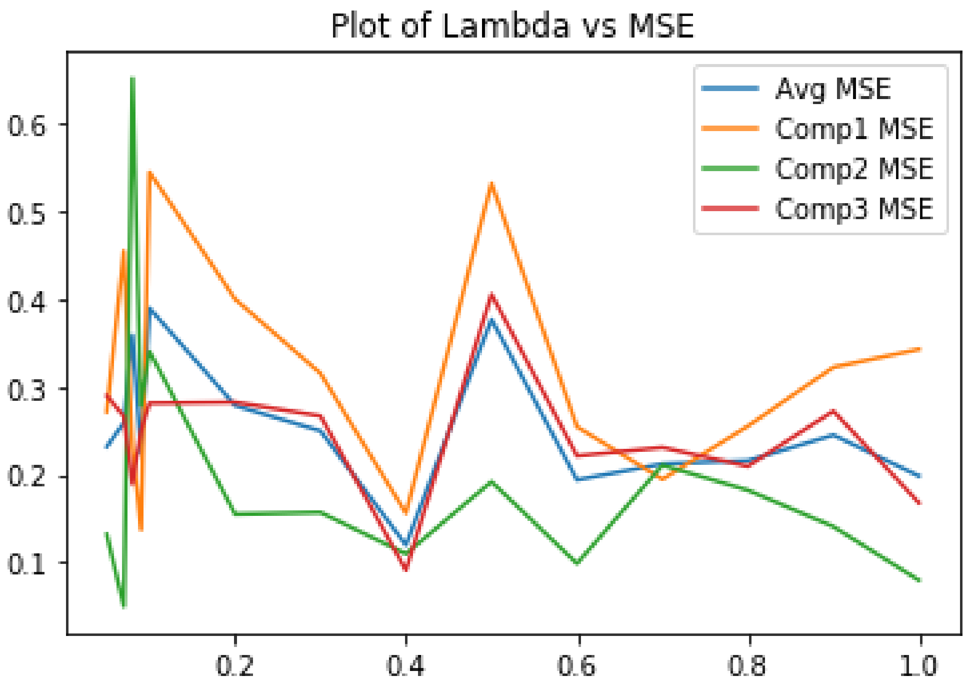

```
The minimum M is  11
```

1.3.2 5. Normalized MSE vs Lambda

In [37]:
```
def normalize(lst):
    s = sum(lst)
    return map(lambda x: float(x)/s, lst)

def mean(lst):
    return sum(lst)/float(len(lst))

def normalizedMSE(s, s_hat):
     return (np.linalg.norm(s_hat - s)/np.linalg.norm(s))**2

def computeMSELambda(N, m, d):
    mse_ = []
    mse_comp1_ = []
    mse_comp2_ = []
    mse_comp3_ = []
    s, Y_xp, labels, phi, B = getCompressedProjection(m, d, False)
    matrix = (1/np.sqrt(m)) * np.matmul(phi, B)
    clf = linear_model.Lasso(alpha = 1.0) # Set lambda ( called alpha  here )
    clf.fit(matrix, Y_xp.T) # Fit the reduced Y to the one hot
    a_hat = clf.coef_ # Get a_hat
    s_hat = np.matmul(a_hat, B.T) # this is the output
    for i in range(N):
        mse = normalizedMSE(s[i],s_hat[i])
        index = np.where(labels[i]==1)[0]
        if index == 0: #component 1
            mse_comp1_.append(mse)
        if index == 1:
            mse_comp2_.append(mse)
        if index == 2:
            mse_comp3_.append(mse)
        mse_.append(mse)
    return mean(mse_), mean(mse_comp1_),mean(mse_comp2_),mean(mse_comp3_)


mse_list=[]
mse_comp1_list = []
mse_comp2_list = []
mse_comp3_list = []

lambda_list = [0.05,0.07,0.08,0.09,0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9,1.0]
for l in lambda_list:
    me, me1, me2, me3 = computeMSELambda(N, minM, dims)
    mse_list.append(me)
    mse_comp1_list.append(me1)
```

```
        mse_comp2_list.append(me2)
        mse_comp3_list.append(me3)
```

```
In [38]: plt.title('Plot of Lambda vs MSE')
         plt.plot(lambda_list, mse_list, label="Avg MSE")
         plt.plot(lambda_list, mse_comp1_list, label="Comp1 MSE")
         plt.plot(lambda_list, mse_comp2_list, label="Comp2 MSE")
         plt.plot(lambda_list, mse_comp3_list, label="Comp3 MSE")
         plt.legend()
         plt.show()
```



1.3.3 6. Compare the Euclidean distances squared vs the corresponding quantity in projected space

```
In [39]: m = minM
         J = 6
         B = np.transpose(Uj)
         projected_u = (1/np.sqrt(m)) * np.matmul(phi, B)
         projected_dist_matrix = np.zeros(shape=(J,J))
         for i in range(J):
             for j in range(J):
                 dist = np.linalg.norm(projected_u[:,i]-projected_u[:,j])**2
                 projected_dist_matrix[i][j] =  dist
         print("projected_dist_matrix\n",projected_dist_matrix)
```

```
dist_matrix = np.zeros(shape=(J,J))
for i in range(J):
    for j in range(J):
        dist = np.linalg.norm(Uj[:,i]-Uj[:,j])**2
        dist_matrix[i][j] =  dist
print("dist_matrix\n",dist_matrix)
```

```
projected_dist_matrix
 [[  0.    37.45  56.36  73.45  23.64  46.82]
 [ 37.45   0.    66.91  66.55  34.91  87.55]
 [ 56.36  66.91   0.   131.27  59.64 114.45]
 [ 73.45  66.55 131.27   0.    70.18  67.18]
 [ 23.64  34.91  59.64  70.18   0.    43.18]
 [ 46.82  87.55 114.45  67.18  43.18   0.  ]]
dist_matrix
 [[ 0.  2.  2.  7.  3.  4.]
 [ 2.  0.  2.  3.  3.  2.]
 [ 2.  2.  0.  7.  3.  4.]
 [ 7.  3.  7.  0. 10.  7.]
 [ 3.  3.  3. 10.  0.  7.]
 [ 4.  2.  4.  7.  7.  0.]]
```

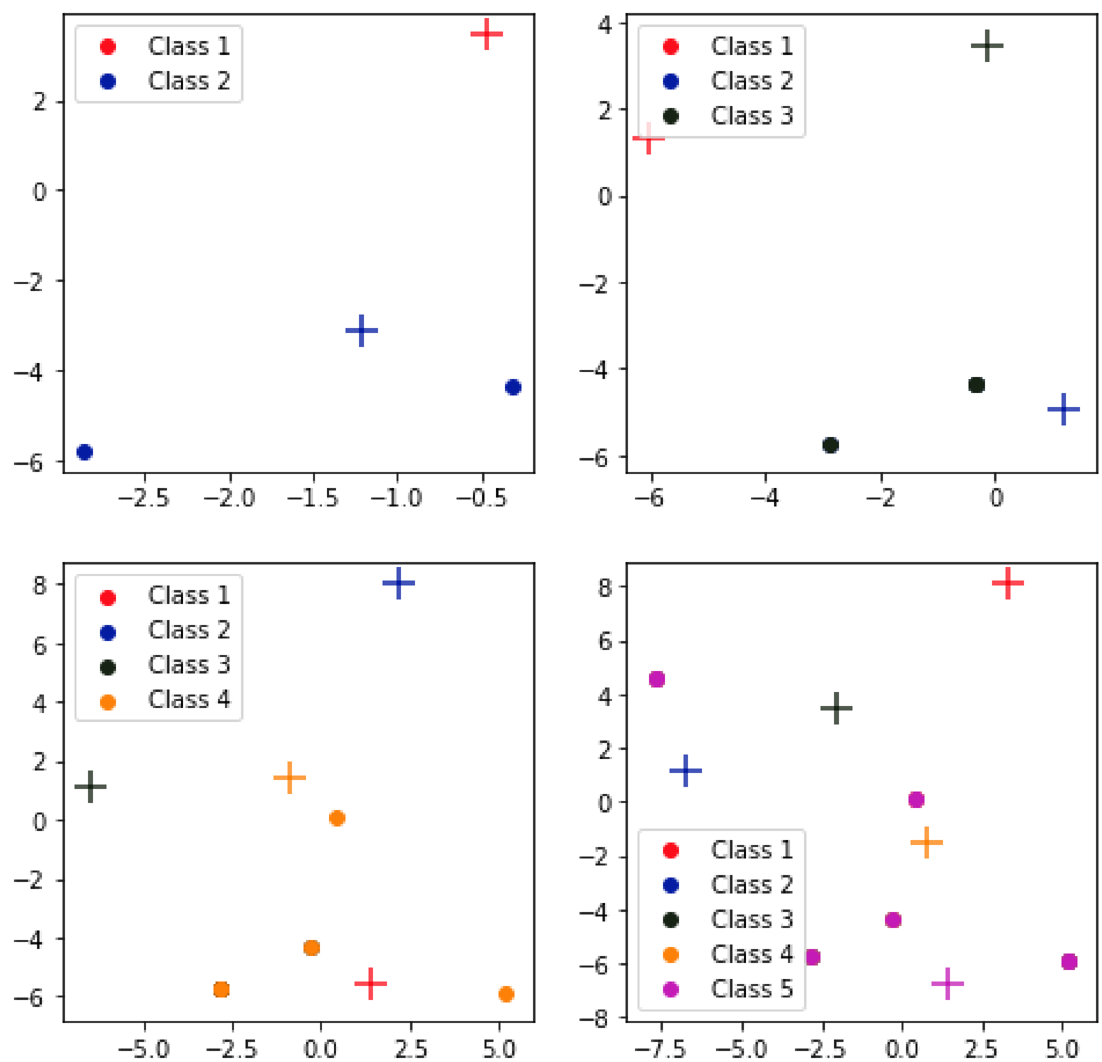
1.3.4 7. K-means algorithm post-projection

In [48]:
```
x, labels = generateDataset(Uj, N, dims)
phi = generatePhiMatrix(minM,dims)
Y = np.matmul(phi,np.transpose(x))/(np.sqrt(minM))
Z = np.transpose(Y)
runKmeansCluster(Z,labels, minM)
```

```
Data Dims:  2000 x 11
Dataset  :  (2000, 11)

-------- K= 2 ----------

----------------------

K= 0 , idx= 0   -> Prob=0.69
K= 0 , idx= 1   -> Prob=0.63
K= 0 , idx= 2   -> Prob=0.27
----------------------

K= 1 , idx= 0   -> Prob=0.31
K= 1 , idx= 1   -> Prob=0.37
K= 1 , idx= 2   -> Prob=0.73

-------- K= 3 ----------

----------------------
```

```
K= 0 , idx= 0   -> Prob=0.00
K= 0 , idx= 1   -> Prob=0.43
K= 0 , idx= 2   -> Prob=0.08
----------------------
K= 1 , idx= 0   -> Prob=0.31
K= 1 , idx= 1   -> Prob=0.02
K= 1 , idx= 2   -> Prob=0.66
----------------------
K= 2 , idx= 0   -> Prob=0.69
K= 2 , idx= 1   -> Prob=0.55
K= 2 , idx= 2   -> Prob=0.26
-------- K= 4 ----------
----------------------
K= 0 , idx= 0   -> Prob=0.23
K= 0 , idx= 1   -> Prob=0.01
K= 0 , idx= 2   -> Prob=0.57
----------------------
K= 1 , idx= 0   -> Prob=0.03
K= 1 , idx= 1   -> Prob=0.30
K= 1 , idx= 2   -> Prob=0.01
----------------------
K= 2 , idx= 0   -> Prob=0.00
K= 2 , idx= 1   -> Prob=0.39
K= 2 , idx= 2   -> Prob=0.06
----------------------
K= 3 , idx= 0   -> Prob=0.74
K= 3 , idx= 1   -> Prob=0.30
K= 3 , idx= 2   -> Prob=0.36
-------- K= 5 ----------
----------------------
K= 0 , idx= 0   -> Prob=0.00
K= 0 , idx= 1   -> Prob=0.25
K= 0 , idx= 2   -> Prob=0.01
----------------------
K= 1 , idx= 0   -> Prob=0.00
K= 1 , idx= 1   -> Prob=0.37
K= 1 , idx= 2   -> Prob=0.05
```

```
----------------------

K= 2 , idx= 0   -> Prob=0.47
K= 2 , idx= 1   -> Prob=0.28
K= 2 , idx= 2   -> Prob=0.13
----------------------

K= 3 , idx= 0   -> Prob=0.43
K= 3 , idx= 1   -> Prob=0.10
K= 3 , idx= 2   -> Prob=0.41
----------------------

K= 4 , idx= 0   -> Prob=0.10
K= 4 , idx= 1   -> Prob=0.00
K= 4 , idx= 2   -> Prob=0.40
```



1.3.5 8. Geometric insights

1.3.6 END