

Please consider a Markov Decision Process (MDP) with $\mathcal{S} = \{s^A, s^B, s^C\}$.

Given a particular state $s \in \mathcal{S}$, the agent is allowed to either try staying there or switching to any of the other states. Let's denote an intention to move to state s^A by a^A , to state s^B by a^B , and to state s^C by a^C . The agent does not know transition probabilities, including the distributions of rewards. There is, however, some evidence that the agent gets rewards only at the entrance to s^C ; and transition MDP probabilities to/from s^A appear to be same (or nearly same) as to/from s^B .

Suppose the agent chooses policy $\pi(a^A|s) = 0.05$, $\pi(a^B|s) = 0.05$, $\pi(a^C|s) = 0.90$ for all $s \in \{s^A, s^B, s^C\}$. Because of the apparent symmetry between s^A and s^B , it makes sense to assume that $v_\pi(s^A) \approx v_\pi(s^B)$ and approximate the state-values as follows:

$$v_\pi(s) \approx \hat{v}(s, \mathbf{w}) = w_1 \cdot \mathbb{1}_{(s=s_A)} + w_1 \cdot \mathbb{1}_{(s=s_B)} + w_2 \cdot \mathbb{1}_{(s=s_C)}.$$

Please notice that $\hat{v}(s^A, \mathbf{w}) = \hat{v}(s^B, \mathbf{w})$ for any choice of weights.

Assume the agent runs the Semi-gradient 1-step Temporal-Difference (TD) prediction algorithm with $\alpha = 0.1$ for estimating v_π :

$$\mathbf{w}_{t+1} \doteq \mathbf{w}_t + \alpha \left[\underbrace{R_{t+1} + \gamma \hat{v}(S_{t+1}, \mathbf{w}_t)}_{\approx v_\pi(S_t)} - \hat{v}(S_t, \mathbf{w}_t) \right] \nabla \hat{v}(S_t, \mathbf{w}_t)$$

with $\gamma = 0.9$ and zero weights at time $t = 0$.

If the agent observes the following sequence of states, actions, and rewards:

$$\begin{aligned} S_0 &= s^A, A_0 = a^C, R_1 = 20, \\ S_1 &= s^C, A_1 = a^B, R_2 = 0, \\ S_2 &= s^B, A_2 = a^C, R_3 = 20, \\ S_3 &= s^C, A_3 = a^C, R_4 = 20, \\ S_4 &= s^C, A_4 = a^B, R_5 = 0, \\ S_5 &= s^B, \end{aligned}$$

find (a) weights \mathbf{w}_t and (b) corresponding approximations $\hat{v}(s, \mathbf{w}_t)$ for $t = 1, 2, \dots, 5$. Specifically, please fill the tables in below:

SOLUTION:

Gradient is: $\nabla \hat{v}(S_t, \mathbf{w}_t) = (\mathbb{1}_{(s=s_A)} + \mathbb{1}_{(s=s_B)}, \mathbb{1}_{(s=s_C)})^T$,

(a) weights $\mathbf{w}_t = (w_{1,t}, w_{2,t})^T$:

	$t = 0$	$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$
$w_{1,t}$	0	2	2	3.816	3.816	3.816
$w_{2,t}$	0	0	0.18	0.18	2.178	2.304

(b) approximations $\hat{v}(s, \mathbf{w}_t)$:

	$t = 0$	$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$
$\hat{v}(s^A, \mathbf{w}_t)$	0	2	2	3.816	3.816	3.816
$\hat{v}(s^B, \mathbf{w}_t)$	0	2	2	3.816	3.816	3.816
$\hat{v}(s^C, \mathbf{w}_t)$	0	0	0.18	0.18	2.178	2.304