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 CSCI E-89c Deep Reinforcement Learning
 Part I of Assignment 10

Suppose each state $s \in \mathcal{S}$ of the Markov Decision Process can be represented by a vector of 2 real-valued features: $\mathbf{x}(s) = (x_1(s), x_2(s))^T$.

Given some policy π , suppose we model the state value function $v_\pi(s)$ with a *fully connected feedforward neural network* (please see the table below) which has two inputs ($x_1(s)$ and $x_2(s)$), one hidden layer that consists of two neurons (u_1 and u_2) with ReLU activation functions, and one output ($\hat{v}(s, \mathbf{w})$) with the ReLU activation function.

The explicit representation of this network is

input layer	hidden layer	output layer
x_1	$u_1 = f(w_{01}^{(1)} + w_{11}^{(1)}x_1 + w_{21}^{(1)}x_2)$	$\hat{v} = f(w_0^{(2)} + w_1^{(2)}u_1 + w_2^{(2)}u_2)$
x_2	$u_2 = f(w_{02}^{(1)} + w_{12}^{(1)}x_1 + w_{22}^{(1)}x_2)$	

Here, $f(x)$ denotes the rectified linear unit (ReLU) defined as follows:

$$f(x) = \begin{cases} x, & \text{if } x \geq 0, \\ 0, & \text{if } x < 0. \end{cases}$$

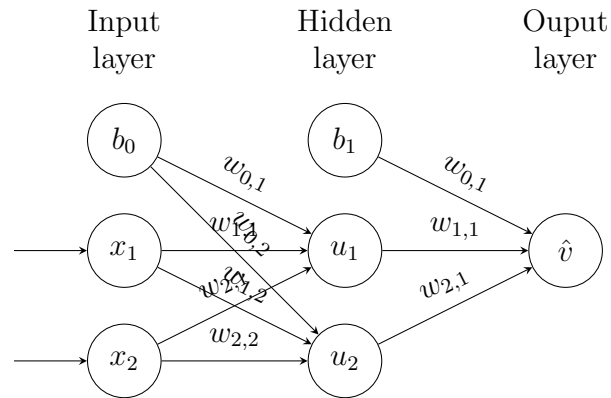
- (a) Sketch the graphical representation of this neural network. Indicate values \hat{v} , x_1 , x_2 and u_1 , u_2 next to the corresponding neurons in the graph. Also, indicate weights next to corresponding connections.
- (b) Assume we trained the network (by minimizing the loss function) and came up with the following weights \mathbf{w} :

hidden layer	output layer
$w_{01}^{(1)} = -1.2, w_{11}^{(1)} = 0.1, w_{21}^{(1)} = 0.5$ $w_{02}^{(1)} = 0.9, w_{12}^{(1)} = 0.8, w_{22}^{(1)} = -0.3$	$w_0^{(2)} = 0.2, w_1^{(2)} = -0.8, w_2^{(2)} = 1.2$

If the features of some state s are $x_1(s) = 1.3$ and $x_2(s) = 0.7$, what is the approximation $\hat{v}_\pi(s, \mathbf{w})$ of the state value $v_\pi(s)$ obtained with this neural network?

SOLUTION:

a)



b)

Of the form: $\text{Relu}(xW + b)$

Hidden layer:

$$\text{relu} \left(\begin{bmatrix} 1.3 \\ 0.7 \end{bmatrix} * \begin{bmatrix} 0.1 & 0.5 \\ 0.8 & -0.3 \end{bmatrix} + \begin{bmatrix} -1.2 \\ 0.9 \end{bmatrix} \right) = \begin{bmatrix} 0 \\ 1.34 \end{bmatrix}$$

Output layer:

$$\text{relu} \left(\begin{bmatrix} 0 \\ 1.34 \end{bmatrix} * \begin{bmatrix} -0.8 & 1.2 \end{bmatrix} + \begin{bmatrix} 0.2 \end{bmatrix} \right) = [1.808]$$

$$\hat{v}_{\pi}(s, w) = 1.808$$