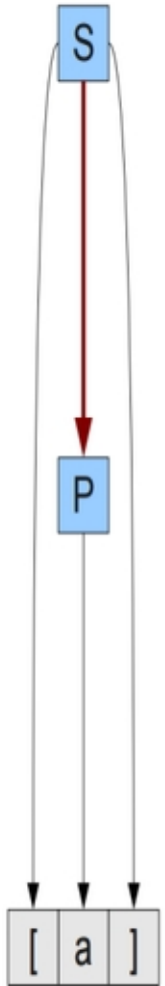
$S \rightarrow [P]$

$P \rightarrow RR \mid a$

$R \rightarrow (P) \mid b$



$S \rightarrow [P]$

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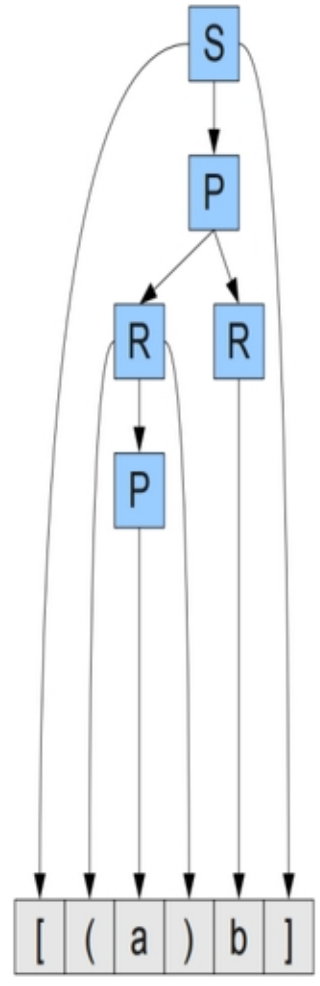
$R \rightarrow (P) \mid b$



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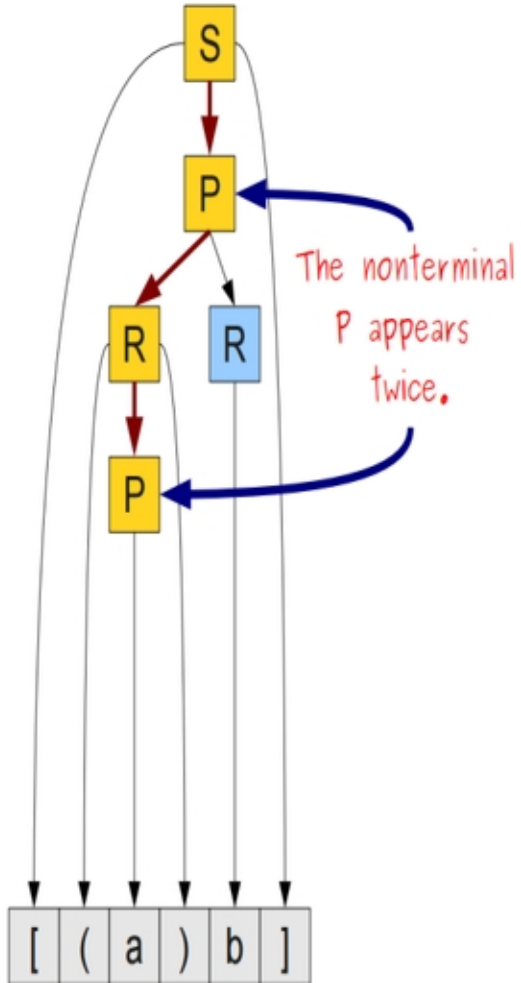

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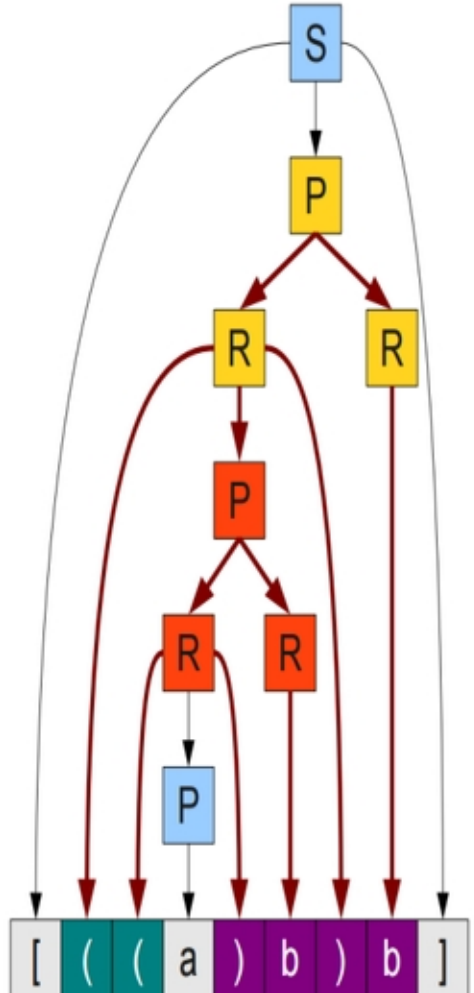

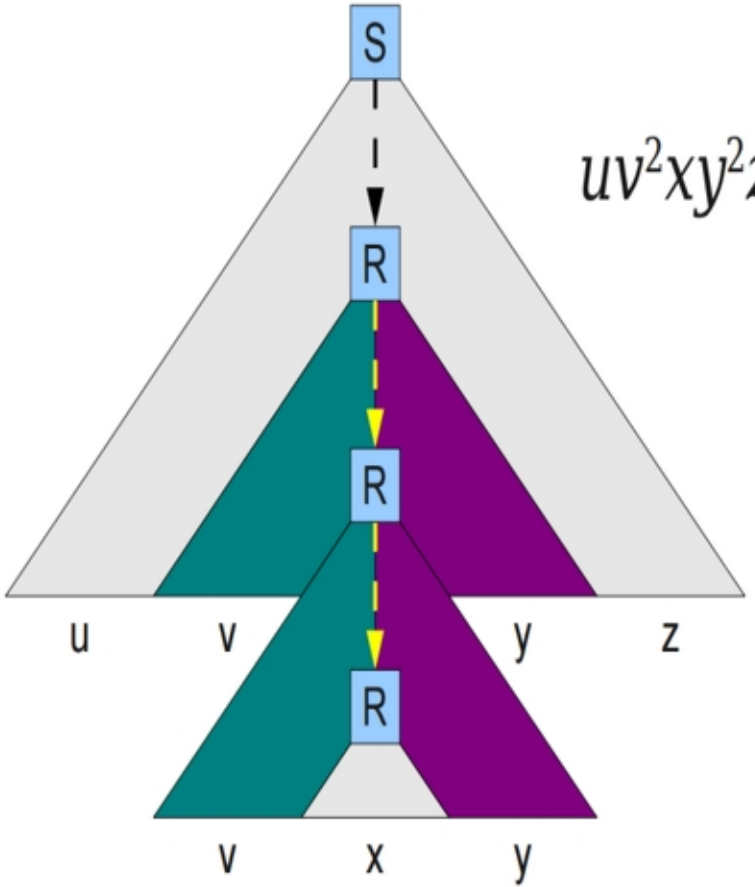

$uv^2xy^2z \in L$

The Pumping Lemma for CFLs

For any context-free language L,

There exists a positive natural number n such that

For any $w \in L$ with $|w| \geq n$,

There exists strings u, v, x, y, z such that

For any natural number i,

$w = uvxyz$, w can be broken into five pieces,

$|vxy| \leq n$, where the middle three pieces aren't too long,

$|vy| > 0$ where the 2nd and 4th pieces aren't both empty, and

$uv^ixy^iz \in L$ where the 2nd and 4th pieces can be replicated 0 or more times

Note that we pump both v and y at the same time, not just one or the other.

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The two strings to pump, collectively, cannot be too long.

They also must be close to one another.

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The pumping length is not simple; see Sipser for details.

The Pumping Lemma Game

$L = \{w \in \{0,1,2\}^* \mid w \text{ has the same number of } 0\text{s}, 1\text{s}, 2\text{s}\}$

ADVERSARY	YOU
Maliciously choose pumping length n.	Cleverly choose a string $w \in L$, $\lvert w \rvert \geq n$
Maliciously split $w = uvxyz$, with $\lvert vy \rvert > 0$ and $\lvert vxy \rvert \leq n$	Cleverly choose k such that $uv^kxy^kz \notin L$
Grrr! Aaaargh!	

Try Your Best!