Home Work 1

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1

If A and b are constants, then E[Ax + b] = AE[x] +b

To show this:

Sum $\sum((Ax+b)P(x)) = E[Ax+b]$

$A \cdot (\sum(xP(x)))$ + $b \cdot (\sum(P(x)))$= E[Ax + b]

Therefore, E[y] = E[Ax + b] = AE[x] + b

We know that cov[x] = $(E) \cdot [(x - E[x])(x - E[x]^T)]$

By definition, cov[y] = cov[Ax + b]

$= E \cdot [(Ax + b - E[Ax + b])(Ax + b - E[Ax + b])^T]$

$= E \cdot [(Ax + b - AE[x] - b)(Ax + b - AE[x] - b)^T]$

$= E \cdot [A(x - E[x])(x - E[x]^T A^T]$

$= AE \cdot [(x - E[x])(x - E[x])^T]A^T$

$= A \cdot [cov[x]A^T] = A \cdot [\sum A^T]$

2

D = (x,y) = (0,1), (2,3), (3,6), (4,8)

(a).

$$X = \begin{bmatrix} 1 & 0 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \end{bmatrix} \tag{1}$$

$$y = \begin{bmatrix} 1 \\ 3 \\ 6 \\ 8 \end{bmatrix} \tag{2}$$

from which we can derive:

$$(X^T)X = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 4 & 9 \\ 9 & 29 \end{bmatrix} \tag{3}$$

and we also get:

$$(X^T)y = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 6 \\ 8 \end{bmatrix} = \begin{bmatrix} 18 \\ 56 \end{bmatrix} \tag{4}$$

By setting the partial derivatives of the least squares equal to zero, we can see that $(\mathrm{X}^T)X\theta^*$. By using Cramer's rule, we get:

$$\frac{\begin{vmatrix} 18 & 9 \\ 56 & 29 \end{vmatrix}}{\begin{vmatrix} 4 & 9 \\ 9 & 29 \end{vmatrix}} = \frac{18}{35} \tag{5}$$

(5) $= \theta_0^*$ and

$$\frac{\begin{vmatrix} 4 & 18 \\ 9 & 56 \end{vmatrix}}{\begin{vmatrix} 4 & 9 \\ 9 & 29 \end{vmatrix}} = \frac{62}{35} \tag{6}$$

(6) $= \theta_1^*$. From the above, we can see that y $= \theta_0$ - θ_0x.

(b). Using normal equation, we get: $\theta^* = ((\mathrm{X}^T X)^{-1}) \cdot (X^T y)$

$$= \begin{bmatrix} 4 & 9 \\ 9 & 29 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 6 \\ 8 \end{bmatrix} \tag{7}$$

$$= \frac{1}{35} \begin{bmatrix} 29 & -9 \\ -9 & 4 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 6 \\ 8 \end{bmatrix} \tag{8}$$

$$= \frac{1}{35}\begin{bmatrix} 29 & 11 & 2 & -7 \\ -9 & 1 & 3 & 7 \end{bmatrix}\begin{bmatrix} 1 \\ 3 \\ 6 \\ 8 \end{bmatrix} \tag{9}$$

$$= \frac{1}{35}\begin{bmatrix} 18 \\ 62 \end{bmatrix} = \begin{bmatrix} \frac{18}{35} \\ \frac{62}{35} \end{bmatrix} \tag{10}$$

thus we can see that solutions in (a) and (b) are the same

(c) and (d). Refer to files in GitHub repository