Simulation and Scientific Computing 2
Assignment 2 - Beam Waveguide

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But first of all..

New Assignment!

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- physical structure that guides electromagnetic waves
- used for transmitting light over long distance (e.g. telecommunication systems) and maintaining high optical intensities over considerable lengths

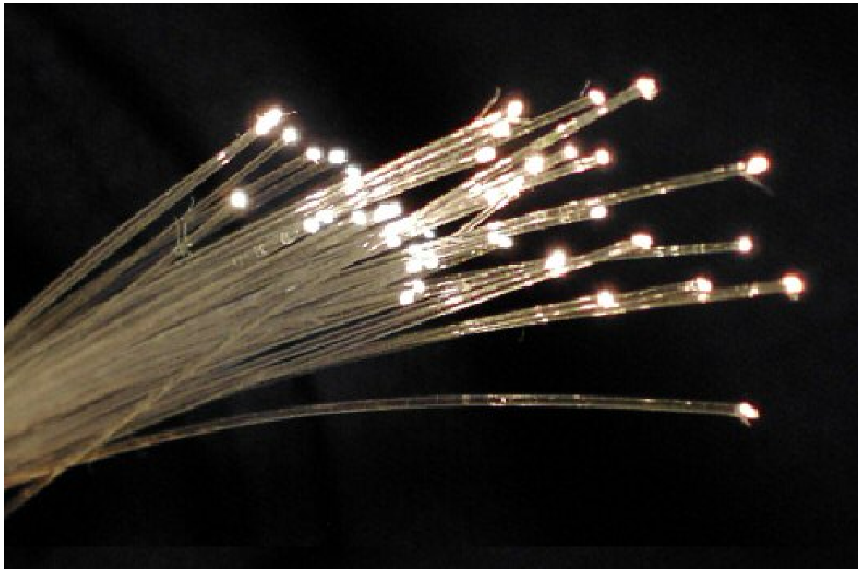

Figure : Bundle of optical fibers[1]

[1] `http://en.wikipedia.org/wiki/Optical_fiber#/media/File:Fibreoptic.jpg`

- spatially inhomogeneous structure for guiding light, i.e. for restricting the spatial region in which light can propagate inside the waveguide
- Usually, waveguides contain a region (*core*) of **increased** refractive index compared to the surrounding medium (*cladding*)

$\rightarrow$ total internal reflection at the interface between core and cladding

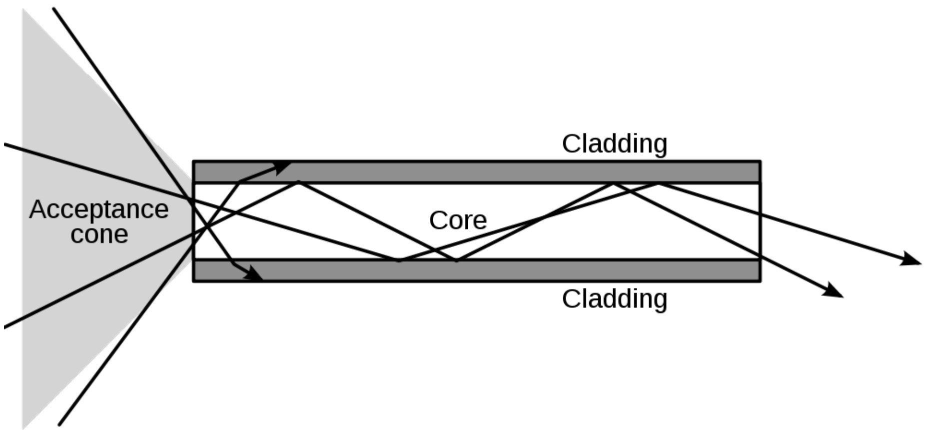

Figure : Guidance of light due to total reflection in an optical fiber[2]

refractive index:

$$n_M = c_0 / c_M$$

with:

- c_0: speed of light in vacuum
- c_M: speed of light in material M

[2] http://de.wikipedia.org/wiki/Datei:Optical-fibre.svg

- One can distinguish different kinds of fibers, based on
  - refractive index profile: gradient-index vs. step-index
  - number of supported propagation modes: Single-mode vs. Multi-mode

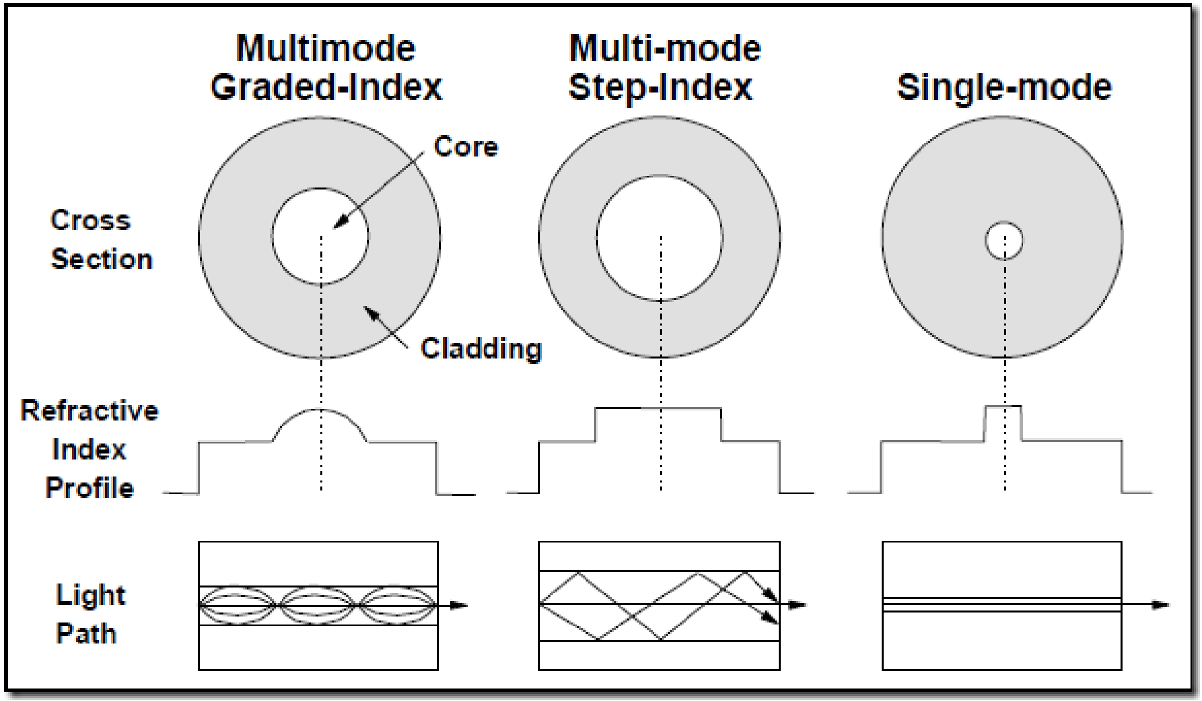

Figure : Different fiber types[3]

[3] http://www.fiberoptics4sale.com/wordpress/wp-content/uploads/2009/06/opticalfiberrefractiveindexprofile-thumb.gif

Modes in a Waveguide

- Depending on the geometry and structure of the waveguide, only a certain number of guided propagation modes are supported.
- Intensity profile of a mode does not change during propagation through the waveguide.

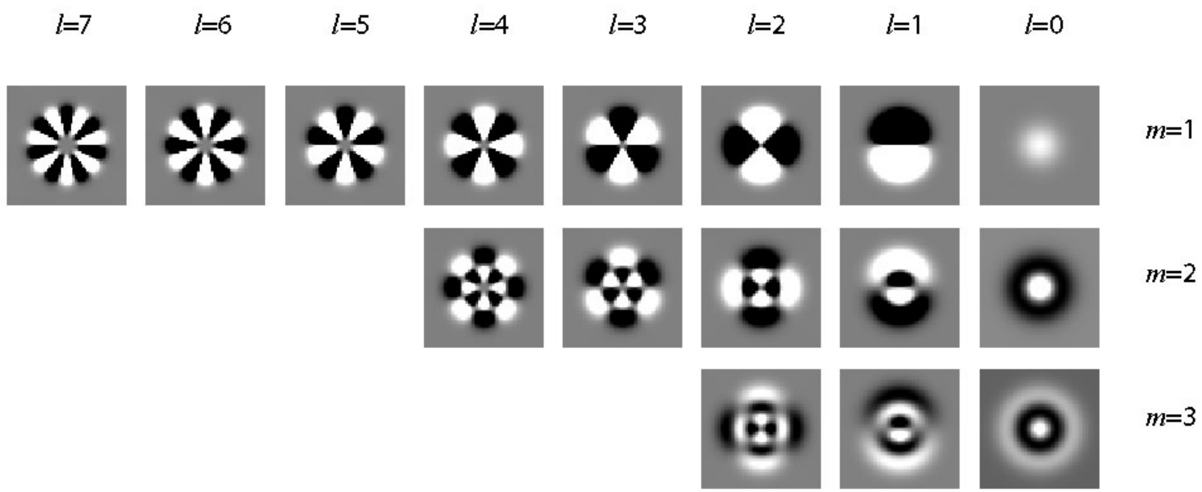

Figure : Examples for different LP l,m-modes in an optical fiber[4].

[4] http://de.wikipedia.org/wiki/Lichtwellenleiter#/media/File:Optical_fibre_modes.jpg

- Behavior of light in the waveguide can be explained by total reflection (at interface) and ray optics.
- However, ray optics are invalid when diffraction occurs, as in small waveguide structures.

$\rightarrow$ Description of light as electromagnetic wave is needed.

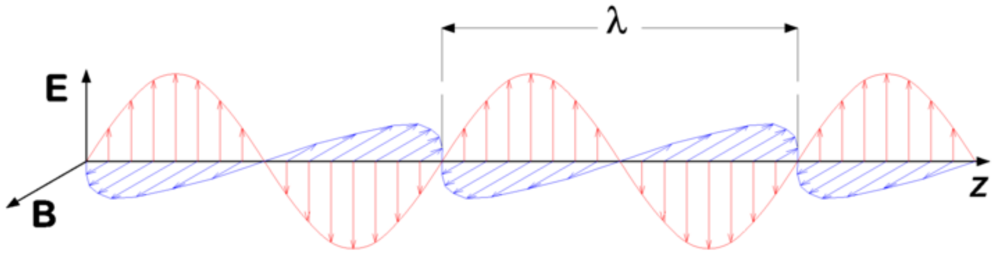

Figure : Electromagnetic wave[5]

$\rightarrow$ Start with Maxwell's equations and make approximate assumptions.

$\rightarrow$ Finally arrive at eigenvalue problem based on the Helmholtz equation.

[5] http://upload.wikimedia.org/wikipedia/commons/0/0a/Electromagnetic_wave.png

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Overview

- four partial differential equations that describe the phenomena of electromagnetism
- state the generation and interaction of electric and magnetic fields

Figure : James Clerk Maxwell[6]

- named after Scottish physicist and mathematician James Clerk Maxwell

$\rightarrow$ combined and extended known phenomena in a physically meaningful way around 1860

[6] http://de.wikipedia.org/wiki/Maxwell-Gleichungen#/media/File:James_Clerk_Maxwell_big.jpg

Faraday's law of induction

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

with

- $\vec{\nabla}\times$: curl operator
- $\vec{E}$: electric field
- $\vec{B}$: magnetic field

Generalisation of: induction of a current due to change in magnetic flux

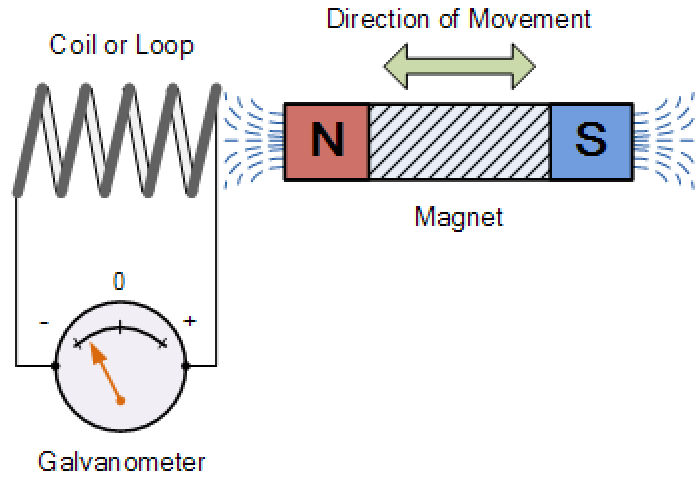

Figure : Electromagnetic induction by moving magnet[7]

[7] http://www.electronics-tutorials.ws/electromagnetism/

Maxwell-Ampere’s law (for linear material)

$$\vec{\nabla} \times (\frac{1}{\mu}\vec{B}) = \vec{\jmath} + \frac{\partial(\varepsilon\vec{E})}{\partial t}$$

with

- $\vec{B}$: magnetic field
- $\vec{E}$: electric field
- $\vec{\jmath}$: electric current density
- μ: permeability
- ε: permittivity

Generalisation of: electric current yields a magnetic field

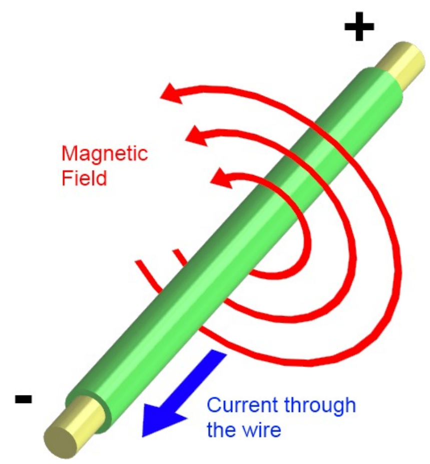

Figure : Electromagnetic induction by moving magnet[8]

[8] https://www.teachengineering.org/collection/van_/lessons/van_cleanupmess_less/less3_header.jpg

Gauss's law (for linear material)

$$\vec{\nabla} \cdot (\varepsilon \vec{E}) = \rho$$

with
- $\vec{E}$: electric field
- ρ: free electric charge density
- ε: permittivity

Statement: charges are sources of the electric field

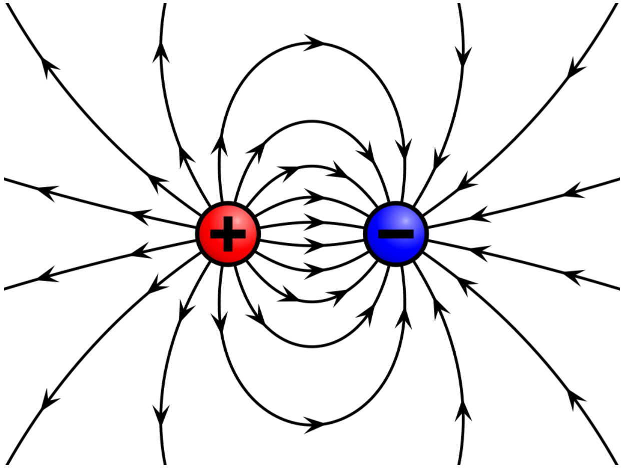

Figure : Electric field around positive and negative charges[9]

[9] http://en.wikipedia.org/wiki/Electric_field

Gauss's law for magnetism

$$\vec{\nabla} \cdot \vec{B} = 0$$

with

- $\vec{B}$: magnetic field

Statement: There are no magnetic monopoles.

Complete set of Maxwell's equations

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times (\frac{1}{\mu}\vec{B}) = \vec{\jmath} + \frac{\partial(\varepsilon \vec{E})}{\partial t}$$

$$\vec{\nabla} \cdot (\varepsilon \vec{E}) = \rho$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

with

- $\vec{B}$: magnetic field
- $\vec{E}$: electric field
- $\vec{\jmath}$: electric current density
- ρ: free electric charge density
- μ: permeability
- ε: permittivity

- They are the foundation of classical electrodynamics, classical optics and electric circuits.

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Electromagnetic wave equation

Assumptions:

- μ & ε approx. constant
- $\rho = 0$, i.e. no free charges
- $\vec{j} = 0$, i.e. no current

$$\Delta\vec{E} = \mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2}$$

- $c_M = 1/\sqrt{\mu\varepsilon} = c_0/n_M$: speed of light in the medium
- magnetic and electric field are perpendicular to each other

Electromagnetic wave equation

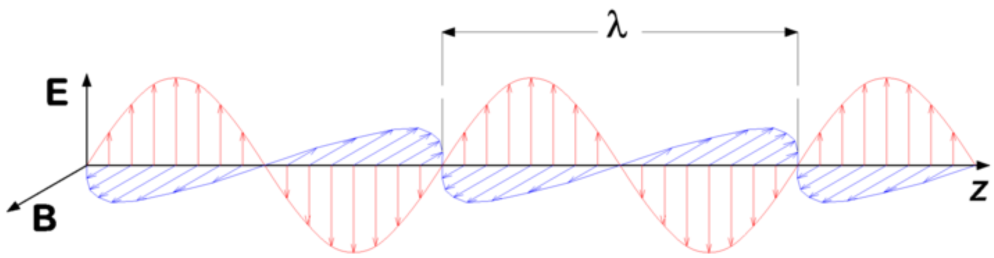

Figure : Electromagnetic wave

- light is a transverse wave: oscillations are perpendicular to direction of propagation
- linear polarization: oscillation only in a plane perpendicular to propagation direction

$\rightarrow$ Description of the fields with a scalar!

Helmholtz equation

- assume time periodicity with a given frequency ω:

$$E(x, y, z, t) = E(x, y, z)e^{i\omega t}$$

- inserting this ansatz into the wave equation yields the scalar Helmholtz equation:

$$\Delta E + \mu\varepsilon\omega^2 E = 0$$

- introducing the wave number $k = \omega / c_0$:

$$\Delta E + k^2 n_M^2 E = 0$$

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Slowly varying envelope approximation

- separate the field into slowly-varying envelope and a fast-oscillating phase term (aka paraxial approximation):

$$E(x, y, z) = u(x, y, z)e^{-ikn_0 z}$$

with

$$\partial^2 u/\partial z^2 \approx 0$$

- inserting into Helmholtz equation:

$$\mathbf{P}u = i2kn_0 \frac{\partial u}{\partial z}$$

with propagation operator

$$\mathbf{P} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \tilde{k}^2, \quad \tilde{k}^2 = k^2 n_M^2 - k^2 n_0^2$$

Eigenvalue problem for beam waveguide

- We are interested in the intensity profiles that do not change inside the beam waveguide, i.e. the (eigen)modes

$$u = u(x, y)$$

- Then:

$$\mathbf{P}u = 0$$

- Mathematically speaking, we are looking for the eigenvalues λ and eigenvectors of the propagation operator $\mathbf{P}$:

$$\mathbf{P}u = \lambda u$$

Final problem formulation

- Beam propagation is mainly described by lowest order eigenmode

$\rightarrow$ Search smallest eigenvalue and corresponding eigenfunction for

$$\mathbf{P}u = \lambda u$$

with

$$\mathbf{P} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \tilde{k}^2$$

- This is your task in the new assignment!

- Side note: For more information, see the lecture Computational Optics from Prof. Pflaum

Eigenvalue problems

Eigenvalue problems are of interest in many technical application fields.

E.g. structural mechanics:

- Collapse of the Broughton Suspension bridge in 1831 due to mechanical resonance induced by troops marching in step
- Unwanted resonances at London's Millenium Bridge in 2000 $\rightarrow$ "Wobbly Bridge"

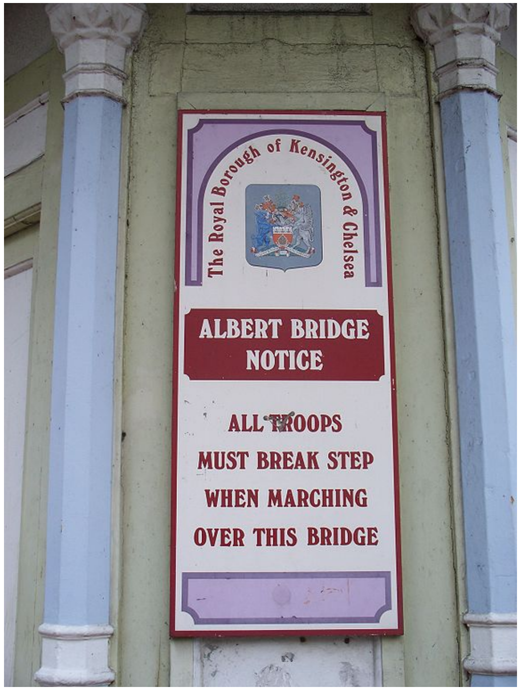

Figure : Sign on Albert Bridge, London[10]

[10] http://en.wikipedia.org/wiki/Broughton_Suspension_Bridge

Outlook

In upcoming exercise classes we will

- learn how to discretize partial differential equations with FEM and assemble the system of equations.
- discuss methods to obtain eigenvalues and -vectors of a system.

Note: No board exercises next week. Computer exercises only on Wednesday.

Enjoy the fifth season of the year: the Berg!