CS6364 – Artificial Intelligence
Project – Nine Men Morris Variant

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Problem Statement:

To obtain the next best move for the maximizer playing a variant of Nine Men Morris game with a given input board position and ply using algorithms such as Minimax and Alpha-Beta pruning.

Modules:

The following are the list of files delivered.

	File name	**Purpose**
Support	MorrisVariant.py	Contains all support functions
Part I	MiniMaxOpening.py	Program for opening using minimax
	MiniMaxGame.py	Program for game using minimax
Part II	ABOpening.py	Program for opening using alpha-beta
	ABGame.py	Program for game using alpha-beta
Part III	MiniMaxOpeningBlack.py	Program for opening using minimax for black player as maximizer
	MiniMaxGameBlack.py	Program for game using minimax for black player as maximizer
Part IV	MiniMaxOpeningImproved.py	Program for opening using minimax with improved static estimation
	MiniMaxGameImproved.py	Program for game using minimax with improved static estimation
Support	MINIMAX	Support file for minimax algorithm
	ALPHABETA	Support file for alpha-beta pruning algorithm
Test cases	S1.txt, S2.txt, S3.txt, S4.txt, S5.txt	Input text file for 5 different test cases
	Project-test-cases.xlsx	Results for the above test cases
Readme	readme	Instructions to run the program
Report	Report_kxs200001.pdf	Document showing examples and results
Extras for tournament	ABOpeningImproved.py	Program for opening using alpha-beta with improved static estimation
	ABGameImproved.py	Program for game using alpha-beta with improved static estimation

Test cases and results:

The following image shows different test cases applied to different parts of the project. This can also be found in Project-test-cases.xlsx for better view.

Opening Scenario (S1): xxxxxxWxxxxxxBxxxx					
	depth1 (D1)	**depth2 (D2)**	**depth3 (D3)**	**depth4 (D4)**	**depth5 (D5)**
MiniMax (M)	16, 1, WxxxxxWxxxxxxBxxxx	240, 0, WxxxxxWxxxxxxBxxxx	3416, 1, WxxxxxWxxxxxxBxxxx	45944, 0, WxxxxxWxxxxxxBxxxx	603920, 1, WxxxxxWxxxxxxBxxxx
AlphaBeta (A)	16, 1, WxxxxxWxxxxxxBxxxx	30, 0, WxxxxxWxxxxxxBxxxx	450, 1, WxxxxxWxxxxxxBxxxx	1396, 0, WxxxxxWxxxxxxBxxxx	21961, 1, WxxxxxWxxxxxxBxxxx
Improved (I)	16, 5, xxxxxxWxxxxWxBxxxx	240, 1, xxxxxxWxxxxWxBxxxx	3416, 4, xxxxxxWxxxxWxBxxxx	45944, 0, xxxxxxWxxxxWxBxxxx	603920, 2, xxxxxxWxWxxxxBxxxx
Midgame/Endgame Scenario (S2): BWWxxxWxWBBWWBWxWB					
	depth1 (D1)	**depth2 (D2)**	**depth3 (D3)**	**depth4 (D4)**	**depth5 (D5)**
MiniMax (M)	13, 3997, xWWxxxWWWBBWWBxxWB	48, 3995, xWWxxxWWWBBWWBxxWB	738, 4979, BWWxxxWWWBBWWBxxWx	3469, 4977, BWWxxxWWWBBWWBxxWx	53420, 10000, BxWWxxWxWBBWWBWxWB
AlphaBeta (A)	13, 3997, xWWxxxWWWBBWWBxxWB	23, 3995, xWWxxxWWWBBWWBxxWB	320, 4979, BWWxxxWWWBBWWBxxWx	353, 4977, BWWxxxWWWBBWWBxxWx	3693, 10000, BxWWxxWxWBBWWBWxWB
Improved (I)	13, 4007, BWWxxxWWWBBWWBxxWx	48, 4002, BWWxxxWWWBBWWBxxWx	738, 4990, BWWxxxWWWBBWWBxxWx	3469, 4985, BWWxxxWWWBBWWBxxWx	53420, 10000, BxWWxxWxWBBWWBWxWB
Opening Scenario (S3): xWxxxWxxBxxxxxBxxx					
	depth1 (D1)	**depth2 (D2)**	**depth3 (D3)**	**depth4 (D4)**	**depth5 (D5)**
MiniMax (M)	15, 2, xWxWxWxxxxxxxxBxxx	197, 1, xWxWxWxxxxxxxxBxxx	2752, 2, xWxWxWxxxxxxxxBxxx	33924, 1, xWxWxWxxxxxxxxBxxx	431734, 2, xWxWxWxxxxxxxxBxxx
AlphaBeta (A)	15, 2, xWxWxWxxxxxxxxBxxx	40, 1, xWxWxWxxxxxxxxBxxx	260, 2, xWxWxWxxxxxxxxBxxx	1036, 1, xWxWxWxxxxxxxxBxxx	4897, 2, xWxWxWxxxxxxxxBxxx
MiniMaxBlack (ends in B)	14, 1, BWxxxWxxBxxxxxBxxx	208, 0, xWxBxWxxBxxxxxBxxx	2715, 1, xWxBxWxxBxxxxxBxxx	36360, 0 xWxBxWxxBxxxxxBxxx	444990, 1, xWxBxWxxBxxxxxBxxx
Improved (I)	15, 6, xWxWxWxxxxxxxxBxxx	197, 3, xWxWxWxxxxxxxxBxxx	2752, 5, xWxWxWxxxxxxxxBxxx	33924, 1, xWxWxWxxxxxxxxBxxx	431734, 3, xWxWxWxxxxxxxxBxxx
Midgame/Endgame Scenario (S4): xBBxxxWxxWxxBxBxxW					
	depth1 (D1)	**depth2 (D2)**	**depth3 (D3)**	**depth4 (D4)**	**depth5 (D5)**
MiniMax (M)	33, -1009, WBBxxxxxxWxxBxBxxW	352, -1012, WBBxxxxxxWxxBxBxxW	12150, -36, xBBxxxxxxWxWBxBxxW	143704, -36, xBBxxxxxxWxWBxBxxW	4922535, -36, WBBxxxxxxWxxBxBxxW
AlphaBeta (A)	33, -1009, WBBxxxxxxWxxBxBxxW	192, -1012, WBBxxxxxxWxxBxBxxW	2130, -36, xBBxxxxxxWxWBxBxxW	3894, -36, xBBxxxxxxWxWBxBxxW	48829, -36, WBBxxxxxxWxxBxBxxW
MiniMaxBlack (ends in B)	11, 967, BxBxxxWxxWxxBxBxxW	363, 964, xBBxxxWxxWxBBxxxxW	3801, 964, xxBBxxWxxWxxBxBxxW	128112, -36, BxBxxxWxxWxxBxBxxW	1485986, -36, BxBxxxWxxWxxBxBxxW
Improved (I)	33, -1007, xBBxxxxxxWxWBxBxxW	352, -1010, xBBxxxWxxxxWBxBxxW	12150, -33, xBBxxxxxxWxWBxBxxW	143704, -35, xBBxxxxxxWxWBxBxxW	4922535, -36, WBBxxxxxxWxxBxBxxW
Midgame/Endgame Scenario (S5): WWxxWxxxBxBxxBxxxx					
	depth1 (D1)	**depth2 (D2)**	**depth3 (D3)**	**depth4 (D4)**	**depth5 (D5)**
MiniMax (M)	38, 10000, WxWxWxxxxxBxxBxxxx	1327, 10000, WxWxWxxxxxBxxBxxxx	45441, 10000, WxWxWxxxxxBxxBxxxx	1627830, 10000, WxWxWxxxxxBxxBxxxx	
AlphaBeta (A)	38, 10000, WxWxWxxxxxBxxBxxxx	200, 10000, WxWxWxxxxxBxxBxxxx	2771, 10000, WxWxWxxxxxBxxBxxxx	23317, 10000, WxWxWxxxxxBxxBxxxx	207314, 10000, WxWxWxxxxxBxxBxxxx
Improved (I)	38, 10000, WxWxWxxxxxBxxBxxxx	1327, 10000, WxWxWxxxxxBxxBxxxx	45441, 10000, WxWxWxxxxxBxxBxxxx	1627830, 10000, WxWxWxxxxxBxxBxxxx	
NOTE: The output listed here are in the format => number of positions visited by static estimation, static estimate value, output board position					
Some of the test cases are developed based on the discussion with peers from the discussion board in elearning					

Figure 1 Test Cases and Results

Part I MiniMaxOpening output:

From the Figure 1, Scenarios 1 and 3 are tested for opening game.

Example1: Scenario 1 for depth 2
INPUT:
Board Position: xxxxxxWxxxxxxBxxxx
OUTPUT:
Board Position: WxxxxxWxxxxxxBxxxx
Positions evaluated by static estimation: 240
MINIMAX estimate: 0

Example2: Scenario 3 for depth 4
INPUT:
Board Position: xWxxxWxxBxxxxxBxxx
OUTPUT:
Board Position: xWxWxWxxxxxxxxBxxx
Positions evaluated by static estimation: 33924
MINIMAX estimate: 1

Part I MiniMaxGame output:

From the Figure 1, Scenarios 2, 4, and 5 are tested for midgame/endgame.

Example1: Scenario 2 for depth 2
INPUT:
Board Position: BWWxxxWxWBBWWBWxWB
OUTPUT:
Board Position: xWWxxxWWWBBWWBxxWB
Positions evaluated by static estimation: 48
MINIMAX estimate: 3995

Example2: Scenario 4 for depth 3
INPUT:
Board Position: xBBxxxWxxWxxBxBxxW
OUTPUT:
Board Position: xBBxxxxxxWxWBxBxxW
Positions evaluated by static estimation: 12150
MINIMAX estimate: -36

Part II ABOpening output:

From the Figure 1, Scenarios 1 and 3 are tested for opening game. Same examples are used from the MiniMaxOpening, we can see that the ouput position and static estimate values are same, but number of positions evaluated is less.

Example1: Scenario 1 for depth 2
INPUT:
Board Position: xxxxxxWxxxxxxBxxxx
OUTPUT:
Board Position: WxxxxxWxxxxxxBxxxx
Positions evaluated by static estimation: 30
AlphaBeta estimate: 0

Example2: Scenario 3 for depth 4
INPUT:
Board Position: xWxxxWxxBxxxxxBxxx
OUTPUT:
Board Position: xWxWxWxxxxxxxxBxxx
Positions evaluated by static estimation: 1036
AlphaBeta estimate: 1

Part II ABGame output:

From the Figure 1, Scenarios 2, 4, and 5 are tested for midgame/endgame.

Example1: Scenario 2 for depth 2
INPUT:
Board Position: BWWxxxWxWBBWWBWxWB

OUTPUT:
Board Position: xWWxxxWWWBBWWBxxWB
Positions evaluated by static estimation: 23
AlphaBeta estimate: 3995

Example2: Scenario 4 for depth 3
INPUT:
Board Position: xBBxxxWxxWxxBxBxxW
OUTPUT:
Board Position: xBBxxxxxxWxWBxBxxW
Positions evaluated by static estimation: 2130
AlphaBeta estimate: -36

Alpha-Beta vs Minimax:

From the Figure 1, when we compare the results of MiniMax (M) and AlphaBeta (A), the number of positions evaluated is same at depth 1 and it significantly reduces for AlphaBeta as depth increases.

In scenario (S1), for depth5 (D5) MiniMax has evaluated 603920 nodes whereas AlphaBeta evaluated only 21961, we can see 96% reduction in the number, which is a huge savings in time. The same holds for other scenarios at depth5 (D5). In fact, the last scenario (S5) with a lot of empty spots in board was able to produce results at depth5 (D5) only using AlphaBeta pruning, as MiniMax was taking large amount of time.

Part III MiniMaxOpeningBlack output:

From the Figure 1, Scenarios 1 and 3 are tested for opening game. Here, the black coin is the maximizer. The board is inverted (whites become black and blacks become white) before and after executing the functions defined for white as maximizer.

Example1: Scenario 1 for depth 2
INPUT:
Board Position: xxxxxxWxxxxxxBxxxx
OUTPUT:
Board Position: BxxxxxWxxxxxxBxxxx
Positions evaluated by static estimation: 240
MINIMAX estimate: 0

Example2: Scenario 3 for depth 4
INPUT:
Board Position: xWxxxWxxBxxxxxBxxx
OUTPUT:
Board Position: xWxBxWxxBxxxxxBxxx
Positions evaluated by static estimation: 36360

MINIMAX estimate: 0

Part III MiniMaxGameBlack output:

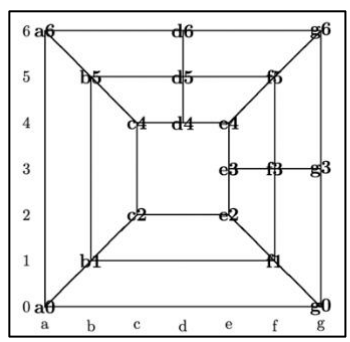
From the Figure 1, Scenarios 2, 4, and 5 are tested for midgame/endgame.

Example1: Scenario 2 for depth 2
INPUT:
Board Position: BWWxxxWxWBBWWBWxWB
OUTPUT:
Board Position: xWWxxxWxWBBWWBWBWB
Positions evaluated by static estimation: 26
MINIMAX estimate: -4014

Example2: Scenario 4 for depth 3
INPUT:
Board Position: xBBxxxWxxWxxBxBxxW
OUTPUT:
Board Position: xxBBxxWxxWxxBxBxxW
Positions evaluated by static estimation: 3801
MINIMAX estimate: 964

Part IV: MiniMaxOpeningImproved and MiniMaxGameImproved:

The original static estimate function doesn't given importance to few special cases which might improve the game. The current function would always try to give the left most node as the best node. The function is improved by the following two ways.



1. The original static estimation function doesn't given importance to special spots in the board such as e4, f5, and g6 where the number of possible mills is 3 for each. Therefore, the number of possible mills (if the corresponding spots forming mill are either empty or occupied by the maximizer) are calculated and added into the static estimate value giving such nodes higher weightage than the others. This is very important in opening move.
2. The tree that is being generated through these algorithms for the Morris game is not a complete tree, that is, the leaf nodes (winning position, either the number of minimizer coins are less than or equal to 2 or the minimizer is out of moves) don't always fall on the same depth. The leaf nodes can be located at any depths and therefore should carry different weights. Closer the leaf node to the root higher should be its weight as the maximizer can win soon if it reaches the closest leaf node. Therefore, the depth at which the leaf node is found is being subtracted from the static estimate value. Higher the depth higher is the penalty and lower will be the static estimate value.

Example 1:

In Figure 1 Scenario 1 (S1) depth 1 (D1), which is an opening position the original static estimation is estimating the same value (that is 1) for all the positions in next move and places the White coin in the left mode empty spot (a0) which is intuitively the worst possible opening spot. Whereas the improved static function places the White coin at spot (e4) which is a good opening move as the player can try to form more mills in upcoming moves.

MiniMaxOpening.py Output for Scenario 1 and depth 1:
INPUT:
Board Position: xxxxxxWxxxxxxBxxxx
OUTPUT:
Board Position: WxxxxxWxxxxxxBxxxx
Positions evaluated by static estimation: 16
MINIMAX estimate: 1

MiniMaxOpeningImproved.py Output for Scenario 1 and depth 1:
INPUT:
Board Position: xxxxxxWxxxxxxBxxxx
OUTPUT:
Board Position: xxxxxxWxxxxWxBxxxx
Positions evaluated by static estimation: 16
MINIMAX estimate: 5

Example 2:

In Figure 1 Scenario 2 (S2) depth 1 (D1), which is a midgame/endgame position, the output result for static estimate value and board position are different for original and improved versions, but the number of positions evaluated by static estimation is same.

MiniMaxOpening Output for Scenario 2 and depth 1:
INPUT:
Board Position: BWWxxxWxWBBWWBWxWB
OUTPUT:
Board Position: xWWxxxWWWBBWWBxxWB
Positions evaluated by static estimation: 13
MINIMAX estimate: 3997

MiniMaxOpeningImproved Output for Scenario 2 and depth 1:
INPUT:
Board Position: BWWxxxWxWBBWWBWxWB
OUTPUT:
Board Position: BWWxxxWWWBBWWBxxWx
Positions evaluated by static estimation: 13
MINIMAX estimate: 4007