

# Markerless LiDAR-Camera Calibration

## Initial Formulation

$$r_i = I_2(x_i) - I_1(\pi(g(\delta, \pi^{-1}(x_i, d(g(G(\xi), X_i))))));$$

where,

$d$  is the Z-coordinate selection operator, defined as

$$\begin{aligned} d : \mathbb{R}^3 &\rightarrow \mathbb{R} \\ d(X, Y, Z) &\mapsto Z \end{aligned}$$

$r_i$  is the difference in intensity between the second image  $I_2$  and the first image  $I_1$  after warping it to the point of view of the second image, at the pixel  $x_i$ .

$\delta$  is the known transformation between the two cameras.

$\xi$  is the transformation between the first camera and the LiDAR

$x_i$  is the 2D image coordinates of the 3D point in velodyne's coordinates,  $X_i$

$$J = J_I \cdot J_\pi \cdot J_g \cdot J_{\pi^{-1}} \cdot J_d \cdot J_g \cdot J_G$$

$$J_I = [\nabla I_x \nabla I_y]$$

$$J_\pi = \begin{pmatrix} f_x \frac{1}{z''} & 0 & -f_x \frac{x''}{z''^2} \\ 0 & f_y \frac{1}{z''} & -f_y \frac{y''}{z''^2} \end{pmatrix}$$

$$J_g = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix}$$

$$J_{\pi^{-1}} = \begin{pmatrix} \frac{(x_i - c_x)}{f_x} \\ \frac{(y_i - c_y)}{f_y} \\ 1 \end{pmatrix}$$

$$J_d = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}$$

$$J_g = \begin{pmatrix} x & 0 & 0 & y & 0 & 0 & z & 0 & 0 & 1 & 0 & 0 \\ 0 & x & 0 & 0 & y & 0 & 0 & z & 0 & 0 & 1 & 0 \\ 0 & 0 & x & 0 & 0 & y & 0 & 0 & z & 0 & 0 & 1 \end{pmatrix}$$

$$J_G = \begin{pmatrix} 0 & 0 & 0 & 0 & R_{31} & -R_{21} \\ 0 & 0 & 0 & -R_{31} & 0 & R_{11} \\ 0 & 0 & 0 & R_{21} & -R_{11} & 0 \\ 0 & 0 & 0 & 0 & R_{32} & -R_{22} \\ 0 & 0 & 0 & -R_{32} & 0 & R_{12} \\ 0 & 0 & 0 & R_{22} & -R_{12} & 0 \\ 0 & 0 & 0 & 0 & R_{33} & -R_{23} \\ 0 & 0 & 0 & -R_{33} & 0 & R_{13} \\ 0 & 0 & 0 & R_{23} & -R_{13} & 0 \\ 1 & 0 & 0 & 0 & T_z & -T_y \\ 0 & 1 & 0 & -T_z & 0 & T_x \\ 0 & 0 & 1 & T_y & -T_x & 0 \end{pmatrix}$$

The product of all the jacobian terms,  $J$  is

$$J = \begin{pmatrix} 0 & 0 & A & B & C & 0 \end{pmatrix}$$

where,

$$\begin{aligned} A = & \frac{(x_i - c_x) \left( r_{31} \left( -\frac{f_x g_x x''}{(z'')^2} - \frac{f_y g_y y''}{(z'')^2} \right) + \frac{r_{11} f_x g_x}{z''} + \frac{r_{21} f_y g_y}{z''} \right)}{f_x} \\ & + \frac{(y_i - c_y) \left( r_{32} \left( -\frac{f_x g_x x''}{(z'')^2} - \frac{f_y g_y y''}{(z'')^2} \right) + \frac{r_{12} f_x g_x}{z''} + \frac{r_{22} f_y g_y}{z''} \right)}{f_y} \\ & + r_{33} \left( -\frac{f_x g_x x''}{(z'')^2} - \frac{f_y g_y y''}{(z'')^2} \right) + \frac{r_{13} f_x g_x}{z''} + \frac{r_{23} f_y g_y}{z''} \end{aligned}$$

$$\begin{aligned}
B = & R_{21}x \left( \frac{(x_i - c_x) \left( r_{31} \left( -\frac{f_x g_x x''}{(z'')^2} - \frac{f_y g_y y''}{(z'')^2} \right) + \frac{r_{11} f_x g_x}{z''} + \frac{r_{21} f_y g_y}{z''} \right)}{f_x} \right) \\
& + R_{21}x \left( \frac{(y_i - c_y) \left( r_{32} \left( -\frac{f_x g_x x''}{(z'')^2} - \frac{f_y g_y y''}{(z'')^2} \right) + \frac{r_{12} f_x g_x}{z''} + \frac{r_{22} f_y g_y}{z''} \right)}{f_y} + r_{33} \left( -\frac{f_x g_x x''}{(z'')^2} - \frac{f_y g_y y''}{(z'')^2} \right) + \frac{r_{13} f_x g_x}{z''} + \frac{r_{23} f_y g_y}{z''} \right) \\
& + R_{22}y \left( \frac{(x_i - c_x) \left( r_{31} \left( -\frac{f_x g_x x''}{(z'')^2} - \frac{f_y g_y y''}{(z'')^2} \right) + \frac{r_{11} f_x g_x}{z''} + \frac{r_{21} f_y g_y}{z''} \right)}{f_x} \right) \\
& + R_{22}y \left( \frac{(y_i - c_y) \left( r_{32} \left( -\frac{f_x g_x x''}{(z'')^2} - \frac{f_y g_y y''}{(z'')^2} \right) + \frac{r_{12} f_x g_x}{z''} + \frac{r_{22} f_y g_y}{z''} \right)}{f_y} + r_{33} \left( -\frac{f_x g_x x''}{(z'')^2} - \frac{f_y g_y y''}{(z'')^2} \right) + \frac{r_{13} f_x g_x}{z''} + \frac{r_{23} f_y g_y}{z''} \right) \\
& + R_{23}z \left( \frac{(x_i - c_x) \left( r_{31} \left( -\frac{f_x g_x x''}{(z'')^2} - \frac{f_y g_y y''}{(z'')^2} \right) + \frac{r_{11} f_x g_x}{z''} + \frac{r_{21} f_y g_y}{z''} \right)}{f_x} \right) \\
& + R_{23}z \left( \frac{(y_i - c_y) \left( r_{32} \left( -\frac{f_x g_x x''}{(z'')^2} - \frac{f_y g_y y''}{(z'')^2} \right) + \frac{r_{12} f_x g_x}{z''} + \frac{r_{22} f_y g_y}{z''} \right)}{f_y} + r_{33} \left( -\frac{f_x g_x x''}{(z'')^2} - \frac{f_y g_y y''}{(z'')^2} \right) + \frac{r_{13} f_x g_x}{z''} + \frac{r_{23} f_y g_y}{z''} \right) \\
& + T_y \left( \frac{(x_i - c_x) \left( r_{31} \left( -\frac{f_x g_x x''}{(z'')^2} - \frac{f_y g_y y''}{(z'')^2} \right) + \frac{r_{11} f_x g_x}{z''} + \frac{r_{21} f_y g_y}{z''} \right)}{f_x} \right) \\
& + T_y \left( \frac{(y_i - c_y) \left( r_{32} \left( -\frac{f_x g_x x''}{(z'')^2} - \frac{f_y g_y y''}{(z'')^2} \right) + \frac{r_{12} f_x g_x}{z''} + \frac{r_{22} f_y g_y}{z''} \right)}{f_y} + r_{33} \left( -\frac{f_x g_x x''}{(z'')^2} - \frac{f_y g_y y''}{(z'')^2} \right) + \frac{r_{13} f_x g_x}{z''} + \frac{r_{23} f_y g_y}{z''} \right)
\end{aligned}$$

$$\begin{aligned}
C = & -R_{11}x \left( \frac{(x_i - c_x) \left( r_{31} \left( -\frac{f_x g_x x''}{(z'')^2} - \frac{f_y g_y y''}{(z'')^2} \right) + \frac{r_{11} f_x g_x}{z''} + \frac{r_{21} f_y g_y}{z''} \right)}{f_x} \right) \\
& -R_{11}x \left( \frac{(y_i - c_y) \left( r_{32} \left( -\frac{f_x g_x x''}{(z'')^2} - \frac{f_y g_y y''}{(z'')^2} \right) + \frac{r_{12} f_x g_x}{z''} + \frac{r_{22} f_y g_y}{z''} \right)}{f_y} + r_{33} \left( -\frac{f_x g_x x''}{(z'')^2} - \frac{f_y g_y y''}{(z'')^2} \right) + \frac{r_{13} f_x g_x}{z''} + \frac{r_{23} f_y g_y}{z''} \right) \\
& -R_{12}y \left( \frac{(x_i - c_x) \left( r_{31} \left( -\frac{f_x g_x x''}{(z'')^2} - \frac{f_y g_y y''}{(z'')^2} \right) + \frac{r_{11} f_x g_x}{z''} + \frac{r_{21} f_y g_y}{z''} \right)}{f_x} \right) \\
& -R_{12}y \left( \frac{(y_i - c_y) \left( r_{32} \left( -\frac{f_x g_x x''}{(z'')^2} - \frac{f_y g_y y''}{(z'')^2} \right) + \frac{r_{12} f_x g_x}{z''} + \frac{r_{22} f_y g_y}{z''} \right)}{f_y} + r_{33} \left( -\frac{f_x g_x x''}{(z'')^2} - \frac{f_y g_y y''}{(z'')^2} \right) + \frac{r_{13} f_x g_x}{z''} + \frac{r_{23} f_y g_y}{z''} \right) \\
& -R_{13}z \left( \frac{(x_i - c_x) \left( r_{31} \left( -\frac{f_x g_x x''}{(z'')^2} - \frac{f_y g_y y''}{(z'')^2} \right) + \frac{r_{11} f_x g_x}{z''} + \frac{r_{21} f_y g_y}{z''} \right)}{f_x} \right) \\
& -R_{13}z \left( \frac{(y_i - c_y) \left( r_{32} \left( -\frac{f_x g_x x''}{(z'')^2} - \frac{f_y g_y y''}{(z'')^2} \right) + \frac{r_{12} f_x g_x}{z''} + \frac{r_{22} f_y g_y}{z''} \right)}{f_y} + r_{33} \left( -\frac{f_x g_x x''}{(z'')^2} - \frac{f_y g_y y''}{(z'')^2} \right) + \frac{r_{13} f_x g_x}{z''} + \frac{r_{23} f_y g_y}{z''} \right) \\
& -T_x \left( \frac{(x_i - c_x) \left( r_{31} \left( -\frac{f_x g_x x''}{(z'')^2} - \frac{f_y g_y y''}{(z'')^2} \right) + \frac{r_{11} f_x g_x}{z''} + \frac{r_{21} f_y g_y}{z''} \right)}{f_x} \right) \\
& -T_x \left( \frac{(y_i - c_y) \left( r_{32} \left( -\frac{f_x g_x x''}{(z'')^2} - \frac{f_y g_y y''}{(z'')^2} \right) + \frac{r_{12} f_x g_x}{z''} + \frac{r_{22} f_y g_y}{z''} \right)}{f_y} + r_{33} \left( -\frac{f_x g_x x''}{(z'')^2} - \frac{f_y g_y y''}{(z'')^2} \right) + \frac{r_{13} f_x g_x}{z''} + \frac{r_{23} f_y g_y}{z''} \right)
\end{aligned}$$