Homework #1a: MIMO Radar - Direction of Arrival (DoA) estimation

Prof. Andrea Virgilio Monti Guarnieri
Dr. Marco Manzoni

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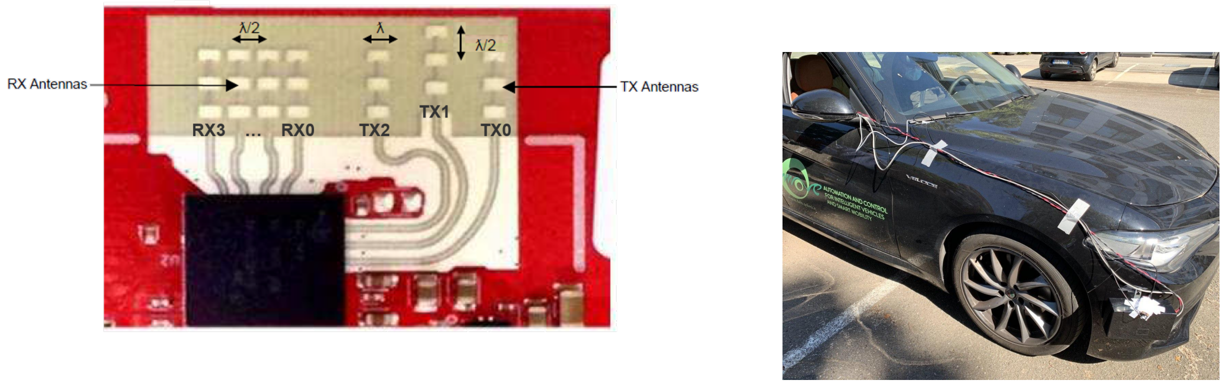
Instructions

- The output of the homework is a set of MATLAB files and a report (Word, LaTeX or PowerPoint are allowed) containing the required figures and a brief explanation of them. The MATLAB files are demo so they should be error-free and they should not require any user input.
- The text defines requirements and parameters. Missing information -if any- are a free choice of the engineer.
- Compress all files in a ZIP package and give the name H1XXXXXXXX.zip where XXXXXXXX is the ID number of the student.
- The ZIP must be sent by email at marco.manzoni@polimi.it
- If you have any question or doubts, send a request at marco.manzoni@polimi.it
- The oral discussion of the homework will be scheduled later on.
- Partial solving of the following points is allowed.

1 Introduction

Multiple-Input Multiple-output (MIMO) radar is a recent method that exploits multiple antennas both in transmission (TX) and reception (RX). One example of MIMO radar is depicted in Figure 1a where we can see 3 TX antennas and 4 RX antennas. Each antenna is, in reality, an array itself composed by 3 elements.
In Figure 1b a MIMO radar (code-name ScanBrick) is mounted on a car: the idea is to generate a synthetic aperture.

One of the main advantages of MIMO radar is that the degrees of freedom can be greatly increased by the concept of virtual array. In order to properly understand this concept, the knowledge of the standard monostatic array used for the Direction of Arrival estimation (DoA) is mandatory. The theory about this topic is presented in Appendix A.



(a)

(b)

Figure 1: The core of the ScanBrick and the ScanBrick itself mounted on a car.

1.1 Phase Center Approximation (PCA)

Let T be the position of the transmitter and R the position of the receiver as in Figure 2. This configuration is called bistatic since transmitter and receiver are located in different positions in space.
The idea of the PCA is to replace the bistatic configuration with an equivalent monostatic one. It is possible to prove that the transmission and reception occurs from a point V placed in the middle between the two sensors ($V = (T + R)/2$). Can you prove it using geometrical considerations?

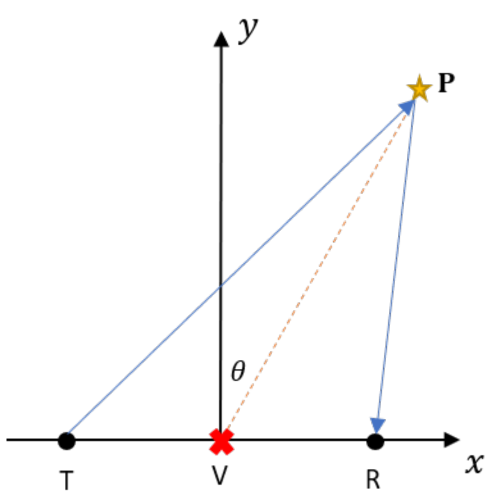


Figure 2: Geometry of the PCA.

1.2 The virtual array

Let's now suppose to have N transmitting antennas spaced d_{tx} and M receiving antennas spaced d_{rx} as if Figure 3. The first antenna transmit a signal and all the RX antennas

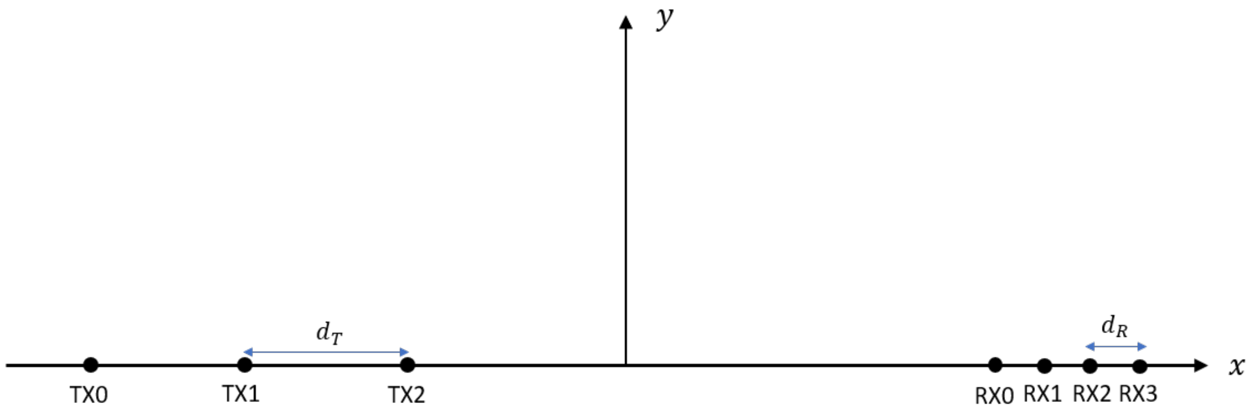


Figure 3: Geometry of MIMO ULA System with $M = 3$ and $N = 4$

receives the echo. Then the second antenna transmit and again all the others receive the echo and so on.
Following the PCA considerations it is easy to see that we can form an equivalent array with MN virtual elements using just $M + N$ physical antennas!
All the considerations of Appendix A about the DoA are still valid when we consider a virtual array instead of a physical one.

2 MIMO radar design

In this section you will design a MIMO radar system for detecting the Direction of Arrival of a target placed in the FOV.

2.1 Design of the equivalent virtual array

Let's suppose to have a 2D field of view (FOV) scanned by a MIMO radar. The operational wavelength is 9.6 GHz and the field of view (the region where the target could be) is delimited by the coordinates

$$x_{min} = -35m$$
$$x_{max} = 35m$$
$$y_{min} = 100m$$
$$y_{max} = 150m$$

The desired minimum resolution in the FOV is $2m$.
Provide the specifications of the ULA needed to meet the requested resolution. What is the minimum number of antenna to place in order to properly sample the signal without aliasing? Remember that you FOV is space-limited...

The output of this task is a set of specification needed to build the actual array.

2.2 Design of the MIMO radar

Transform the design of the virtual array section 2.1 into a physical MIMO design.

1. What is the required spacing in transmission and reception (d_{tx} and d_{rx}) to have a uniform virtual array that properly sample the signal?

2. Where do you place the transmitting elements and receiving elements?

The output of this task is a set of specification needed to place the physical elements in the space (spacing between antennas, number of antennas in TX and RX, etc.).

3 DoA estimation using FFT

In this section we will take the design of Section 2.1 and make a simulation out of it.

3.1 Single target without noise

In Appendix A we learned that the angle of arrival is directly related to the (spatial) frequency excited on the array. The easiest way to estimate the DoA is, therefore, a simple Fourier Transform.

1. Place a single target in a random position in the FOV defined in Section 2.1. (Use the MATLAB function `rand()` to generate the x and y position of the target).

2. Place the TX and RX elements of the array in the 2D space by defining their x and y coordinates.

3. Compute the true angle of arrival using simple geometry (The angle θ in Figure 2). This number will be useful for the validation of the DoA procedure.

4. Propagate the signal from the first TX element to all the RX elements.

$$s_m^n = \frac{\rho_p}{R_0 + R_1} e^{-j\frac{2\pi}{\lambda}(R_0+R_1)} \tag{1}$$

   Where R_0 and R_1 are the distances from the transmitting antenna to the target and from the target to the receiving antenna respectively. Note that we added the path loss due to free space. You can choose ρ_p to be a normally distributed complex random number.
   Do the same thing for all the M transmitters. Save all the received signals in a vector of size MN. This is your signal vector.

5. Compute the Fourier transform of the signal using the `fft()` function in MATLAB. Remember to interpolate the frequency domain signal (zero padding in the space domain...) to achieve better accuracy in the DoA estimation. Plot the power spectrum of the received signal.

6. Convert the frequency in angles and print the estimated angle of arrival. Compare the estimated angle of arrival with nominal one (i.e. the ground truth)

The outputs of this task are:

- A figure of the simulated geometry with the elements of the array and the target as in Figures 2 and 3;
- A figure of the power spectrum of the signal received;
- A screenshot with the true direction of arrival and the estimated direction of arrival in degrees.

3.2 Validation of the nominal resolution - Multiple targets

Repeat section 3.1 using two targets instead of one. The two target should be separated by the nominal resolution of the system. Can you distinguish them? Try again, but this time make the two target closer.
The output of this task is is a set of images of the power spectrum of the received signal in presence of two targets. Each time make the two targets closer. The engineer should also comment the results.

The output of this task is a figure with the SNR on the x axis and the MSE on the estimate of the DoA on the y axis.

4 DoA with backprojection

It's possible to estimate the direction of arrival also by back projecting the received signal into a predefined 2D Field of View and then sum coherently in each position in space.

- Generate a 2D FOV of the same size as the one in Section 2.1 with a wisely selected pixel spacing (keep in mind that back projection is slow, and a wrong pixel density can result in a huge computational burden. It's also suggested to use the `meshgrid()` function);
- Perform the back projection by taking each complex number representing the signal received, projecting it on each pixel of the FOV and summing coherently. In other words, for a point in P

$$s_P = \sum_{i=1}^{MN} s_i D(i, P) e^{+j\frac{4\pi}{\lambda}D(i,P)} \tag{2}$$

  Where s_P is the reconstructed signal in P, i is the virtual antenna number, $D(i, P)$ is the distance from the i^{th} virtual antenna to the point in P and s_i is the signal received at the i^{th} antenna.

- take the absolute value of the entire backprojection grid.

The output of this task is a figure showing the grid with the computed backprojected signal at each position. The maximum should be in the direction of arrival of the signal. Can you give an intuitive explanation on how the backprojection works?

A Review of DoA estimation with standard uniform linear array (ULA)

Let suppose to have a uniform linear array (ULA) composed by N elements as in Figure 4.

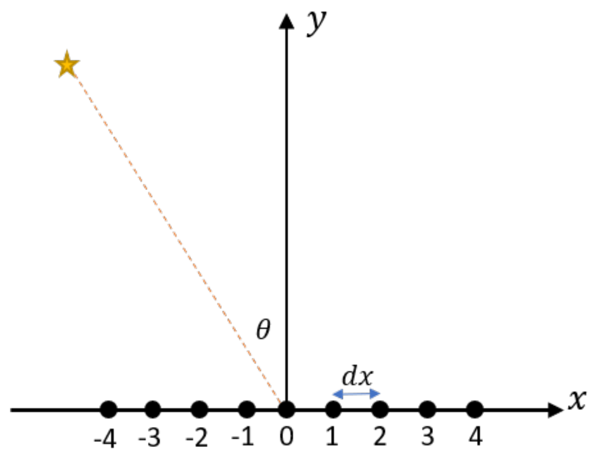


Figure 4: Geometry of a uniform linear array with $N = 9$ elements.

Let's suppose that each antenna, in turn, transmits a signal (i.e. a chirp) towards the target positioned in $\mathbf{P}$. The electromagnetic wave bounces back and the reflection arrives to the same antenna that transmitted the signal. Now matched filtering and detection are performed.
Each antenna repeats sequentially the same procedure (send, receive, filtering and detection). Keep in mind that the elements of the array could be physically separated antennas, a single ground based radar moving on a rail or a space borne radar moving in a straight orbit.
After matched filtering and detection, the signal model at the n^{th} antenna can be expressed as

$$s_{RX}^{n} = \rho_p e^{-j\frac{4\pi}{\lambda}(R - nd_x \sin(\theta))} \tag{3}$$

Where ρ_p is a complex number with amplitude proportional to the RCS (Radar Cross Section) of the target and phase equal to the back-scattering phase, λ is the wavelength of the system, n is the antenna number, d is the distance of the antenna elements (assumed constant) and θ is the angular position (DoA) of the target.
The first exponential refers to the travel path from the first antenna to the target, while the second exponential refers to the travel path of the echo from the target to the n^{th} antenna.
It is easy to see that equation 3 represents a spatially sampled complex sinusoid with sampling period equal to d_x.
The spatial frequency of this sinusoid is directly related to the angle of arrival θ by the formula

$$f = \frac{1}{2\pi}\frac{d}{dx}\sphericalangle s_{RX}^{n} = \frac{2\sin(\theta)}{\lambda} \tag{4}$$

where we assumed $x = nd_x$. It is interesting to notice that, when the target is placed end-fire (along the x axis), the resolution of the system is null, in fact, a sensitivity analysis shows that

$$\Delta f = \frac{\partial f}{\partial \theta}\Delta\theta = \frac{\cos(\theta)}{\lambda}\Delta\theta \tag{5}$$

For $\theta = \pi/2$ it is zero. This means that if we place a target end-fire and we move it a little bit, the array can't detect the movement since its sensibility is very poor in that direction.
At the opposite, the best resolution is obtained broadside for $\theta = 0$.

How can we quantify such resolution? Look at the way we are able to detect the angle of arrival. We take the samples captured by each antenna, we Fourier Transform and we look at the position of the peak. That frequency is related to the angle of arrival. The resolution in angle, therefore, is tightly linked to the frequency resolution.
From the basics of signal processing we know that if we have a sinusoid at frequency f_0 and we multiply it by a rectangular window of length L, in the frequency domain we have a `sinc` function centered in f_0 with the first zero at $f_0 + \frac{1}{L}$. In our case $L = Nd_x$ (the total length of the linear array) and the resolution in the (spatial) frequency domain is $\frac{1}{L}$. But we know the relation between frequency and angle from equation 4. This equation can be simplified for small angles becoming

$$f = \frac{2\sin(\theta)}{\lambda} \approx \frac{2\theta}{\lambda} \tag{6}$$

therefore

$$\rho_f = \frac{1}{L} = \frac{2\rho_\theta}{\lambda} \longrightarrow \rho_\theta = \frac{\lambda}{2L} \tag{7}$$

The angular resolution can now be converted into spatial resolution by the simple conversion

$$\rho_x = \rho_\theta R = \frac{\lambda}{2L} R \tag{8}$$

Where R is the array-to-target distance. Note that the resolution is independent on the spacing between the antennas. It depends only on the wavelength and the total length of the array. The spacing, instead, must be carefully tuned following Nyquist sampling theorem to avoid ambiguities (aliasing).

> You are probably used to sampling in time domain. The typical example is a sinusoid (continuous signal) sampled by an Analog to Digital Converter (ADC) to produce sampled signal. The sampling happens every dt seconds, the sampling frequency is $f_s = 1/d_t$ Hz and the sinusoid has frequency f_0 Hz.
> Is the case of the antenna array any different? No! The signal in the plane of the array (the x axis in our case) is still a sinusoid, The "sampler" is no more the ADC by the array itself, the signal is sampled every d_x (meters this time), the sampling frequency is $f_x = 1/d_x[1/m]$.