4

PV Output Characteristics and Mathematical Models

I–V and P–V curves are commonly used to illustrate the outputs of PV cells, modules, strings, or arrays, as discussed in Section 1.6. Computer simulation is an important tool to reproduce the behavior of PV power systems in response to various environmental conditions and load disturbances. A computational model can be developed to recreate the PV generator output under variations in irradiance and temperature. There are various modeling approaches that have been presented in literature to represent PV output characteristics.

Doping creates an interface between two types of semiconductor material – p-type and n-type – inside a single crystal of a semiconductor. These p-n junctions are the elementary units of most semiconductor electronic devices: diodes, transistors, and integrated circuits. A crystalline-based PV cell is also constructed from a large area of silicon p-n junction, so a diode model is naturally used to represent the output characteristics of crystalline-based solar cells.

A model to represent crystalline-based PV cells is usually formed from the equivalent circuits. These models are usually categorized into two main types: single-diode models (SDMs) and double-diode models (DDMs), and their equivalent circuits are shown in Figures 4.1 and 4.2, respectively.

An ideal model for PV cells should have a current source in parallel with a diode. However, due to various non-ideal factors, the equivalent circuit of the standard SDM also includes one shunt resistor and one series resistor, as illustrated in Figure 4.1. In this book, the model is defined as the complete single diode model (CSDM) in order to distinguish it from other simplified single-diode models (SSDMs). The current–voltage characteristics according to the equivalent circuit are expressed as

$$i_{pv} = i_{ph} - i_d - \frac{v_d}{R_h} \tag{4.1}$$

The I–V characteristics of the p-n junction diode is nonlinear, and it can be represented in exponential form:

$$i_d = i_s \left[e^{\left(\frac{q v_d}{k T_c A_n}\right)} - 1 \right] \tag{4.2}$$

where v_d is equal to

$$v_d = v_{pv} + i_{pv} R_s \tag{4.3}$$

This is based on the theory of Shockley (1949). The symbol T_c is the absolute temperature of the p-n junction. The ideality factor of the diode, A_n, is a measure of how closely

Photovoltaic Power System: Modeling, Design, and Control, First Edition. Weidong Xiao.

Companion Website: www.wiley.com/go/xiao/pvpower

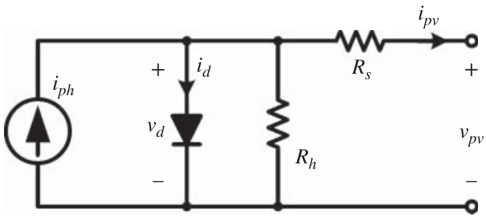

Figure 4.1 Equivalent circuit of single-diode model.

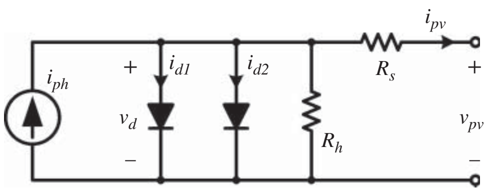

Figure 4.2 Equivalent circuit of double-diode model.

the diode follows the ideal diode equation, which is defined as $A_n = 1$. The value of A_n typically ranges from 1 to 2. Values of the p-n junction constants, model parameters, and variables are listed in Tables 4.1–4.3, because they are frequently used in the chapter. To represent the relation of v_{pv} and i_{pv}, or the I–V curve, five unknown parameters should be identified to give the CSDM: i_{ph}, i_s, A_n, R_S, and R_h. Thus, the model is sometimes referred to as the five-parameter model.

The DDM includes one current source, two diodes, and shunt and series resistances, as shown in Figure 4.2. The output current of the DDM is expressed as

$$i_{pv} = i_{ph} - i_{d1} - i_{d2} - \frac{v_d}{R_h} \tag{4.4}$$

where the currents in the two diodes can be expressed as:

$$i_{d1} = i_{s1}\left[e^{\left(\frac{qv_d}{kT_cA_{n1}}\right)} - 1\right] \tag{4.5}$$

$$i_{d2} = i_{s2}\left[e^{\left(\frac{qv_d}{kT_cA_{n2}}\right)} - 1\right] \tag{4.6}$$

and where the diode voltage is

$$v_d = v_{pv} + i_{pv}R_s \tag{4.7}$$

Table 4.1 PV model coefficients.

Symbols	Term definition	Value
E_{STC}	Irradiance at STC*	1000 W/m^2
k	Boltzmann constant	1.38×10^{-23} J/K
q	Charge	1.6×10^{-19} C
T_{CS}	PV cell temperature at STC*	298°K
V_{TCS}	Thermal voltage of p-n junction at STC*	25.7 mV

*Standard test conditions (STC): 1000 W/m^2, AM1.5 standard reference spectrum, and 298 K (or 25°C).

Table 4.2 PV model parameters and constants.

Symbols	Definition	Unit
α_T	Temperature coefficient on PV current	(A/A)/K
β_T	Temperature coefficient on PV voltage	(V/V)/K
γ_T	Irradiance coefficient on PV power	(W/W)·m^2/W
ν_T	Irradiance coefficient on PV voltage	(V/V)·m^2/W
A_n	Diode ideality factor in the SDM	n/a
A_{n1}	Diode ideality factor 1 in the double-diode model	n/a
A_{n2}	Diode ideality factor 2 in the double-diode model	n/a
I_{MS}	PV current at the maximum power point (MPP) at the STC	A
I_{ph}	PV photon current at the STC	A
I_{SCS}	PV short-circuit current at the STC	A
I_{SS}	PV short-circuit current at the STC	A
P_{MPP}	PV power at the maximum power point (MPP) at the STC	W
R_s	Series resistance	Ω
R_h	Shunt resistance	Ω
V_{MS}	PV voltage at the maximum power point at the STC	V
V_{OCS}	PV open-circuit voltage at the STC	V

Table 4.3 PV model variables.

Symbols	Definition	Unit
E_a	Solar irradiance	W/m^2
I_M	Instant MPP current	A
i_{ph}	PV photon current	A
i_{pv}	PV cell output current	A
i_d	Diode current	A
i_s	Diode reverse-bias saturation current	A
T_C	PV cell temperature	K
v_d	Diode voltage	V
V_M	Instant MPP voltage	V
v_{OC}	PV open-circuit voltage	V
v_{pv}	PV cell terminal voltage	V
v_t	Thermal voltage of p–n junction	V

Seven unknown parameters must be identified in the DDM: i_{ph}, i_{s1}, i_{s2}, A_1, A_2, R_S, and R_h. The model coefficients and variables are as set out in Tables 4.1–4.3. The DDM is commonly considered to be a more comprehensive model of PV cell output characteristics than the SDM. Two more tuning parameters appear in the model than in the SDM, in order to accurately reproduce the output characteristics of PV cells. However, the model is not commonly used due to its complexity (Huang et al. 2016;

Mahmoud et al. 2013). The model in (4.4) includes two independent terms for diode current, which increases the complexity for parameter identification and computer simulation. Improper tuning of the parameters can prevent the claimed advantage of accuracy being achieved. Parameter identification for the DDM is also very sensitive to initial conditions (Romero-Cadaval et al. 2013).

Due to the complexity of the DDM, the SDM is normally used for simulating PV power systems, because it offers a reasonable trade-off between simplicity and accuracy. For this reason, the DDM will not be further discussed.

4.1 Ideal Single-diode Model

Based on the p-n junction structure for both PV cell and diode, the ideal single-diode model (ISDM) is a current source in parallel with a diode, as shown in Figure 4.3. It can be considered the simplest SDM. In comparison with the CSDM shown in Figure 4.1, the series resistance and shunt resistance are removed. The mathematical expressions are therefore:

$$i_{pv} = i_{ph} - \underbrace{i_s \left[e^{\left(\frac{qv_{pv}}{kT_c A_n}\right)} - 1 \right]}_{i_d} \tag{4.8}$$

Without the resistances, three model parameters must be identified in the modeling process: the photon current (i_{ph}), the diode reverse bias saturation current (i_s), and the diode ideality factor (A_n). Three independent constraints are required to identify the three unknown parameters in (4.8).

4.1.1 Product Specification

A common source for identifying values for the PV model parameters is the product datasheets provided by PV cell or module manufacturers. The modeling process is to match the PV output characteristics with these data, which are prepared at STC. Table 4.4 illustrates one such specification. All data refer to the multi-crystalline solar cell, IM156B3, which is manufactured by MOTECH Industrial Inc. To avoid any confusion, it should be noted that the datasheet was downloaded in 2014 showing the version of October 2012. It might not be consistent with the latest data since the manufacturer often releases new product versions. The data that are useful for PV cell modeling and simulation include the STC values of the short-circuit current I_{SCS}, open-circuit voltage V_{OCS}, the operating voltage and current at the maximum power point (MPP) (V_{MS}, I_{MS}), the temperature coefficients (α_T, β_T, and γ_T), and the correction factors for irradiation on the electrical outputs. The symbols α_T, β_T, and γ_T are the

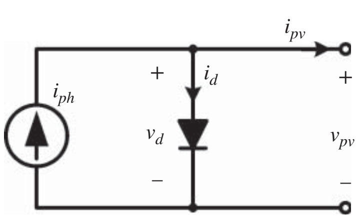

Figure 4.3 Equivalent circuits of the ideal single-diode model.

Table 4.4 Sample solar cell data.

Basic information					
Manufacturer	**Model**	**Cell material**	**Dimensions**		
MOTECH	IM156B3-164	Multi-crystalline	$156 \times 156 \pm 0.5$ mm		
Electrical performance at STC					
Efficiency	P_{MPP}	I_{MS}	V_{MS}	I_{SCS}	V_{OCS}
16.4 %	3.99 W	7.85 A	0.509 V	8.38 A	0.614 V
Temperature coefficients					
	α_T (%/°C)	β_T (%/°C)	γ_T (%/°C)		
	0.06	−0.33	−0.40		
Correction factors for irradiance					
	E_a (W/m^2)	Voltage correction (V/V)	Current correction (A/A)		
	1000	1.000	1.000		
	800	0.989	0.798		
	600	0.972	0.597		
	200	0.911	0.192		

All symbols shown above refer to Tables 4.2 and 4.3.

temperature coefficients for the correction of the PV cell output current, voltage, and power, respectively. Parameter identification is performed at the PV cell level since the cell is the fundamental unit for construction of modules and arrays.

4.1.2 Parameter Identification at Standard Test Conditions

ISDM parameters are initially determined at STC since the data are available from the product datasheet. For the case study, the modeling uses the data in Table 4.4.

When the terminal of the equivalent circuit, as shown in Figure 4.3, is shorted, the diode current, i_d, is equal to zero. The value of the photon current, i_{ph}, is equal to the short-circuit current, I_{SCS}, which is available from the product datasheet. Thus the photon current, i_{ph} becomes known at STC. Two unknowns remain in the ISDM to represent the diode.

When the terminal of the equivalent circuit, as shown in Figure 4.3, is opened, the output current of the PV cell, i_{pv}, is equal to zero. The value of the diode current, i_d, becomes equal to the photon current, i_{ph}, which is the I_{SCS} at STC. This can be expressed as in (4.9), which includes two unknown parameters of I_{SS} and A_n:

$$I_{SCS} = I_{SS}\left[e^{\left(\frac{V_{OCS}}{V_{TCS}A_n}\right)} - 1\right] \tag{4.9}$$