① $\frac{\partial c}{\partial t} = D \cdot \left[\frac{\partial^2 c}{\partial r^2} + \frac{1}{r} \frac{\partial c}{\partial r} \right] - k \cdot c$

② $C_m = e^{-\lambda t} + A \cdot r^3$

③ $\frac{\partial C_m}{\partial t} = -\lambda e^{-\lambda t}$

$\frac{\partial C}{\partial r} = 3Ar^2$

$\frac{\partial^2 C}{\partial r^2} = 6Ar$

④ $\frac{\partial C_m}{\partial t} = D \cdot \left[\frac{\partial^2 C_m}{\partial r^2} + \frac{1}{r} \frac{\partial C_m}{\partial r} \right] - k \cdot C_m + S(r,t)$

$S(r,t) = -\lambda e^{-\lambda t} - D \left[6Ar + \frac{3Ar^2}{r} \right] + k \cdot (e^{-\lambda t} + A \cdot r^3)$

$S(r,t) = (k-\lambda) e^{-\lambda t} - (9 \cdot D \cdot A) r + (k \cdot A) r^3$

⑤ Steady-State solution

$S(r) = -(9 \cdot D \cdot A) r + (k \cdot A) \cdot r^3$

$0 = D \cdot \frac{1}{r} \frac{d}{dr} \left(r \frac{dC_m}{dr} \right) - kC_m - (9 \cdot D \cdot A) r + (k \cdot A) \cdot r^3$

$0 = D \cdot \frac{d}{dr} \left(r \frac{dC_m}{dr} \right) - kC_m r - (9 \cdot D \cdot A) r^2 + (k \cdot A) \cdot r^4$

$0 = D \cdot \left(r \frac{dC_m}{dr} \right) - kC_m \int r \, dr - (9 \cdot D \cdot A) \int r^2 dr + (k \cdot A) \int r^4 dr$

$0 = D \cdot \left(r \frac{dC_m}{dr} \right) - kC_m \cdot \frac{r^2}{2} - (9 \cdot D \cdot A) \frac{r^3}{3} + (k \cdot A) \frac{r^5}{5} + B_1$

⑥ BC1: $\frac{dC_m}{dr} = 0$ at $r = 0 \longrightarrow B_1 = 0$

$0 = D \cdot \frac{dC_m}{dr} - kC_m \frac{r}{2} - (9 \cdot D \cdot A) \frac{r^2}{3} + (k \cdot A) \frac{r^4}{5}$

$0 = D \int \frac{dC_m}{dr} dr - kC_m \int \frac{r}{2} dr - (9 \cdot D \cdot A) \int \frac{r^2}{3} dr + (k \cdot A) \int \frac{r^4}{5} dr$

$0 = D \cdot C_m - kC_m \cdot \frac{r^2}{4} - (9 \cdot D \cdot A) \frac{r^3}{9} + (k \cdot A) \frac{r^5}{25} + B_2$

7 BC2: $C_m = C_e$ at $r = R$

$$B_2 = k C_e \cdot \frac{R^2}{4} - D \cdot C_e + (9 \cdot D \cdot A)\frac{R^3}{9} - (k \cdot A)\frac{R^5}{25}$$

$$B_2 = C_e\left(\frac{kR^2}{4} - D\right) + A \cdot R^3\left(D - \frac{k \cdot R^2}{25}\right)$$

8 $$0 = C_m\left(D - \frac{kr^2}{4}\right) - A \cdot r^3\left(D - \frac{kr^2}{25}\right) + B_2$$

$$C_m = \frac{1}{D - kr^2/4} \cdot \left[Ar^3\left(D - \frac{kr^2}{25}\right) - B_2\right]$$

$$C_m = \frac{1}{D - kr^2/4} \cdot \left[Ar^3\left(D - \frac{kr^2}{25}\right) - B_2\right]$$

where,

$$B_2 = C_e\left(\frac{kR^2}{4} - D\right) + A \cdot R^3\left(D - \frac{k \cdot R^2}{25}\right)$$