Stellar Variability Analysis Using χ^2 Minimization

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1 Introduction

Photometric time-series data provide a fundamental tool for identifying and characterizing variable stars. The aim of this project was to develop an automated pipeline for detecting periodic variability in stellar light curves and estimating their dominant periods. The analysis focuses on approximately sinusoidal variability and employs χ^2 minimization as a statistical criterion for model selection.

This work was carried out as an independent computational project and constitutes a complete analysis pipeline, from raw observational data to physical interpretation of the results.

2 Data

The data consist of photometric observations of stellar brightness collected at irregular time intervals. For each star, the dataset contains:

- observation times t_i,
- measured magnitudes m_i,
- associated uncertainties σ_i.

Each light curve is analyzed independently. As a preprocessing step, the weighted mean magnitude is computed using inverse-variance weights $w_i = 1/\sigma_i^2$ and subtracted from the data.

3 Method

3.1 Model

The stellar variability is modeled as a single-period sinusoidal signal:

$$m(t) = A + a \sin\left(\frac{2\pi t}{P}\right) + b \cos\left(\frac{2\pi t}{P}\right), \tag{1}$$

where P is the trial period and a, b are fitted coefficients.

For each trial period, the parameters a and b are obtained analytically by minimizing the χ^2 statistic.

3.2 χ^2 minimization

For a given period P, the χ^2 value is defined as:

$$\chi^2 = \sum_i \left(\frac{m_i - m(t_i)}{\sigma_i} \right)^2 . \tag{2}$$

The reduced χ^2,

$$\chi^2_{\mathrm{red}} = \frac{\chi^2}{N-2}, \tag{3}$$

is used as a measure of goodness of fit, where N is the number of observations.

3.3 Period search

A grid of trial periods is explored within a predefined range. For each period:

1. the best-fitting sinusoidal model is computed,
2. χ^2 and χ^2_{red} are evaluated,
3. the minimum χ^2 identifies the most likely period.

To mitigate daily aliasing effects, folded light curves are additionally inspected at $2P$.

3.4 Variability criteria

A star is classified as a candidate variable if it satisfies both:

- $\chi^2_{\mathrm{red}} < 3$,
- a contrast criterion

$$C = \frac{\langle \chi^2 \rangle - \chi^2_{\min}}{\langle \chi^2 \rangle} > 0.1. \tag{4}$$

These criteria balance sensitivity to real variability against robustness to noise.

4 Results

The pipeline was applied to a sample of stellar light curves. For each object, the following quantities were obtained:

- best-fitting period,
- fitted amplitudes,
- χ^2 and χ^2_{red},
- variability classification.

Only a subset of stars exhibited clear periodic variability. In several cases, visually plausible periods were rejected due to high χ^2_{red}, illustrating the importance of statistical validation beyond visual inspection.

5 Discussion

The method reliably identifies stars with approximately sinusoidal variability and sufficient signal-to-noise ratio. However, several limitations are evident:

- non-sinusoidal or multi-periodic variability is not captured by the model,
- sparse or irregular sampling can lead to ambiguous period estimates,
- $\chi^2_{\text{red}} > 1$ may reflect model inadequacy rather than poor data quality.

The analysis highlights the importance of combining automated selection criteria with physical interpretation and diagnostic evaluation.

6 Conclusions

This project demonstrates a complete, automated approach to detecting periodic stellar variability using χ^2 minimization. The pipeline integrates statistical modeling, numerical computation, and physical interpretation of observational data.

The work provides a foundation for more advanced analyses, such as non-sinusoidal modeling or multi-period searches, and represents an initial research-oriented project in time-domain astronomy.