

# Probabilistic Model

(Teaching Inspired by Research)

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UNIVERSITÄT ZU LÜBECK  
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- ① Introduction
- ② Bayes Decision Theory
- ③ Statistical Document Modelling
- ④ Conclusion

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## Contents of the Course

Week	Lecture	Practical Exercises
1	(05/04) Introduction to Medical Information Retrieval (MIR)	(05/04) Introduction to Python
2	(12/04) Main Components and Classification of MIR Systems	(12/04) Introduction to Python
3	(19/04) Metadata in Medical Information Retrieval Systems	(19/04) CBIR in Medical Applications
4	(26/04) No Lecture due to a Business Trip	(26/04) CBIR in Medical Applications
5	(03/05) Set Theoretic Model: Boolean Retrieval	(03/05) CBIR in Medical Applications
6	(10/05) Set Theoretic Model: Fuzzy Retrieval	(10/05) Flask Tutorial
7	(17/05) Vector Space Model: Similarity Measures	(17/05) Flask Tutorial

## Contents of the Course

<b>8</b>	<b>(24/05)</b> Vector Space Model: Distance Functions	<b>(24/05)</b> HTML
<b>9</b>	<b>(31/05)</b> Vector Space Model: Latent Semantic Indexing	<b>(31/05)</b> HTML
<b>10</b>	<b>(07/06)</b> Probabilistic Model	<b>(07/06)</b> HTML
<b>11</b>	<b>(14/06)</b> Text-based Retrieval of Medical Information	<b>(14/06)</b> Deep Learning
<b>12</b>	<b>(21/06)</b> Audio-based Retrieval of Medical Information	<b>(21/06)</b> Deep Learning
<b>13</b>	<b>(28/06)</b> Image-based Retrieval of Medical Information	<b>(28/06)</b> Relevance Feedback
<b>14</b>	<b>(05/07)</b> Demonstrators from Current Research Projects	<b>(05/07)</b> Relevance Feedback
<b>15</b>	<b>(12/07)</b> Summary and Conclusions	<b>(12/07)</b> Evaluation 

## Probabilistic Model – Optimum Effectiveness

*“If a reference retrieval system’s response to each request is a ranking of the documents in the collection in order of **decreasing probability of relevance** to the user who submitted the request, where the **probabilities are estimated as accurately as possible** on the basis of whatever data have been made available to the system for this purpose, **the overall effectiveness of the system** to its user **will be the best** that is obtainable on the basis of those data.”*

[van Rijsbergen 1979]

## Probability of Relevance Based on Historical Data

The probability of relevance can be modelled using historical data, if available:

$$P(R = 1 | \mathbf{d}_i, \mathbf{q}) = \frac{N_{\mathbf{q}, \mathbf{d}_i, R=1}}{N_{\mathbf{q}, \mathbf{d}_i, R \in \{0,1\}}} \quad .$$

## Probability of Relevance for Unseen Queries and Documents

For unseen queries and/or documents, we can assume the following approximation:

$$P(R = 1 | \mathbf{d}_i, \mathbf{q}) \approx p(\mathbf{q} | \mathbf{d}_i, R = 1) \quad .$$



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## A Priori and A Posteriori Probability

- A priori probability:  $P(\mathbf{d}_i)$ .
- A posteriori probability:  $P(\mathbf{d}_i|\mathbf{q})$ .

## Likelihood Density Function

- The likelihood density function  $p(\mathbf{q}|\mathbf{d}_i)$  describes how vectors  $\mathbf{q}$  are distributed within  $\mathbf{d}_i$ .
- It is usually trained from examples.

## Bayes Decision Theory for the Retrieval Problem

### Known:

- Documents:  $\mathbf{d}_i$
- A priori probabilities:  $P(\mathbf{d}_i)$
- Likelihood density functions:  $p(\mathbf{q}|\mathbf{d}_i)$
- A query to be processed:  $\mathbf{q} = (q_1, q_2, \dots, q_l)^T$

### Unknown:

- A posteriori probabilities:  $P(\mathbf{d}_i|\mathbf{q})$

## Computation of the A Posteriori Probability

- Using the **Bayes rule** we obtain:

$$P(\mathbf{d}_i|\mathbf{q}) = \frac{p(\mathbf{q}|\mathbf{d}_i)P(\mathbf{d}_i)}{p(\mathbf{q})} \quad .$$

## Final Retrieval Result Based on Bayes Decision Theory

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## Statistical Modelling of Text Documents

What is the probability of the next word:

$$p(\text{house}|\text{this is the}) = ? \quad p(\text{did}|\text{this is the}) = ?$$

# **Statistical Modelling of One-Dimensional Time Signals**



# Statistical Modelling of Images

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## Final Statements

- If documents can be represented as probability density functions over possible queries, the statistical approaches used for supervised classification can also be applied for retrieval.
- Multimedia documents of different kind (text, audio, image, etc.) usually require different techniques for the statistical modelling of their contents.