Zad 3

$X \sim N(0,1)$ czyli $f(x) = \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{x^2}{2}} \quad x \in R$

Niech $Y = X^2$

$F_Y(t) = P(Y<t) = P(X^2<t) = P(-\sqrt{t} < X < \sqrt{t}) =$

$= F_X(\sqrt{t}) - F_X(-\sqrt{t}) = F_X(\sqrt{t}) - (1 - F_X\sqrt{t})) = 2F_X(\sqrt{t}) - 1$

$f_X(X) = (F_X(X))'$

$f_Y(y) = (2F_X(\sqrt{t}) - 1)' =$

$$= 2 \cdot \frac{1}{2\sqrt{t}} \cdot \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{t}{2}} = \frac{1}{\sqrt{t}} \cdot \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{t}{2}}$$

Zad 5

Niech $Y = X^2$, gdzie $X \sim N(0,1)$

$1^\circ\ y<0,\ P(Y<y) = 0$
$2^\circ\ y \geq 0,\ P(Y<y) = 2F_X(\sqrt{t}) - 1$

$$f_Y(y) = (2F_X(\sqrt{y}))' - 0 = 2\left(\int_{\infty}^{\sqrt{y}} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt\right)' = \left(2 \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{y}{2}} \cdot \sqrt{y}\right)' =$$

$$= 2 \cdot \frac{1}{\sqrt{2}\sqrt{\pi}} e^{-\frac{y}{2}} \left(\frac{1}{2} y^{-\frac{1}{2}}\right) = \frac{1}{\sqrt{2} \cdot \Gamma(\frac{1}{2})} y^{-\frac{1}{2}} \cdot e^{-\frac{y}{2}}$$

$p - 1 = -\frac{1}{2} \Rightarrow p = \frac{1}{2}$
$-by = -\frac{y}{2} \Rightarrow b = \frac{1}{2}$

Stąd

$X \sim Gamma(b, p),\ b = \frac{1}{2},\ p = \frac{1}{2}$