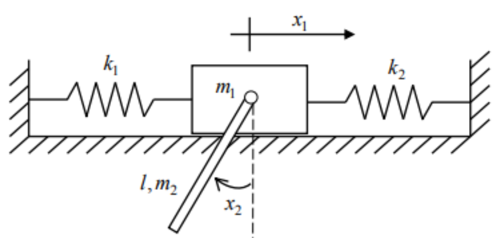
```
> lagrange := proc (n, q, r, L)
        local i, uzm_q, uzm_r, rel_r_q, Lq, Lr, Lrt;
        global row;
        uzm_q := seq(q[i]=q[i](t), i=1 .. n);
        uzm_r := seq(r[i]=r[i](t), i=1 .. n);
        for i to n do
            Lq[i] := subs( [uzm_q, uzm_r], diff (L, q[i]) );
            Lr[i] := subs( [uzm_q, uzm_r], diff (L, r[i]) );
       end do;
       for i to n do
           Lrt[i] := diff (Lr[i], t)
       end do;
       rel_r_q := seq(r[i](t) =diff (q[i](t), t), i=1 .. n);
       for i to  n do
           row[i] := subs(rel_r_q, Lrt[i]-Lq[i]=0)
       end do;
       seq(row[i], i=1 .. n)
   end proc;
```

```
lagrange := proc(n, q, r, L)                                                    (1)
   local i, uzm_q, uzm_r, rel_r_q, Lq, Lr, Lrt;
   global row;
   uzm_q := seq(q[i]=q[i](t), i=1 ..n);
   uzm_r := seq(r[i]=r[i](t), i=1 ..n);
   for i to n do
      Lq[i] := subs( [uzm_q, uzm_r], diff (L, q[i]) ); Lr[i] := subs( [uzm_q, uzm_r], diff (L,
      r[i]) )
   end do;
   for i to n do Lrt[i] := diff (Lr[i], t) end do;
   rel_r_q := seq(r[i](t) =diff (q[i](t), t), i=1 ..n);
   for i to n do row[i] := subs(rel_r_q, Lrt[i] − Lq[i]=0) end do;
   seq(row[i], i=1 ..n)
end proc
```

\>



\> $E := \frac{1}{2}(m[1]+m[2])\cdot v[1]^2 + \frac{1}{6}m[2]l^2\cdot v[2]^2 - \frac{1}{2}m[2]\cdot v[1]\cdot v[2]\cdot l\cdot\cos(x[2]);$

$U := -m[2]\cdot g\cdot\frac{l}{2}\cos(x[2]) + \frac{1}{2}(k[1]+k[2])\cdot x[1]^2;$

 # Note: In the above formulas, v1 and v2 denote the time derivatives of the corresponding generalized coordinates.

$$E := \frac{(m_1 + m_2)\, v_1^2}{2} + \frac{m_2\, l^2\, v_2^2}{6} - \frac{m_2\, v_1\, v_2\, l \cos(x_2)}{2}$$

$$U := -\frac{m_2\, g\, l \cos(x_2)}{2} + \frac{(k_1 + k_2)\, x_1^2}{2} \tag{2}$$

> L := E - U;

$$L := \frac{(m_1 + m_2)\, v_1^2}{2} + \frac{m_2\, l^2\, v_2^2}{6} - \frac{m_2\, v_1\, v_2\, l \cos(x_2)}{2} + \frac{m_2\, g\, l \cos(x_2)}{2} - \frac{(k_1 + k_2)\, x_1^2}{2} \tag{3}$$

> Lag1 := lagrange(2, x, v, L);

$$Lag1 := (m_1 + m_2) \left(\frac{d^2}{dt^2} x_1(t)\right) - \frac{m_2 \left(\frac{d^2}{dt^2} x_2(t)\right) l \cos(x_2(t))}{2} + \frac{m_2 \left(\frac{d}{dt} x_2(t)\right)^2 l \sin(x_2(t))}{2} + (k_1 + k_2)\, x_1(t) = 0, \frac{m_2\, l^2 \left(\frac{d^2}{dt^2} x_2(t)\right)}{3} - \frac{m_2 \left(\frac{d^2}{dt^2} x_1(t)\right) l \cos(x_2(t))}{2} + \frac{m_2\, g\, l \sin(x_2(t))}{2} = 0 \tag{4}$$

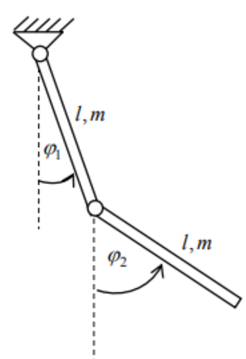
> row[1];

$$(m_1 + m_2) \left(\frac{d^2}{dt^2} x_1(t)\right) - \frac{m_2 \left(\frac{d^2}{dt^2} x_2(t)\right) l \cos(x_2(t))}{2} + \frac{m_2 \left(\frac{d}{dt} x_2(t)\right)^2 l \sin(x_2(t))}{2} + (k_1 + k_2)\, x_1(t) = 0 \tag{5}$$

> row[2];

$$\frac{m_2\, l^2 \left(\frac{d^2}{dt^2} x_2(t)\right)}{3} - \frac{m_2 \left(\frac{d^2}{dt^2} x_1(t)\right) l \cos(x_2(t))}{2} + \frac{m_2\, g\, l \sin(x_2(t))}{2} = 0 \tag{6}$$

>



> $E2 := \frac{2}{3} \cdot m \cdot l^2 \omega[1]^2 + \frac{1}{6} \cdot m \cdot l^2 \cdot \omega[2]^2 + \frac{1}{2} \cdot m \cdot l^2 \cdot \omega[1] \cdot \omega[2] \cdot \cos(\varphi[1] - \varphi[2]);$

$U2 := -\frac{3}{2} \cdot m \cdot g \cdot l \cdot \cos(\varphi[1]) - \frac{1}{2} \cdot m \cdot g \cdot l \cdot \cos(\varphi[2]);$

$$E2 := \frac{2\, m\, l^2\, \omega_1^2}{3} + \frac{m\, l^2\, \omega_2^2}{6} + \frac{m\, l^2\, \omega_1\, \omega_2 \cos(\varphi_1 - \varphi_2)}{2}$$

$$U2 := -\frac{3\, m\, g\, l \cos(\varphi_1)}{2} - \frac{m\, g\, l \cos(\varphi_2)}{2} \tag{7}$$

> $L2 := E2 - U2;$

$$L2 := \frac{2\, m\, l^2\, \omega_1^2}{3} + \frac{m\, l^2\, \omega_2^2}{6} + \frac{m\, l^2\, \omega_1\, \omega_2 \cos(\varphi_1 - \varphi_2)}{2} + \frac{3\, m\, g\, l \cos(\varphi_1)}{2} + \frac{m\, g\, l \cos(\varphi_2)}{2} \tag{8}$$

> $Lag2 := lagrange(2, \varphi, \omega, L2);$

$$Lag2 := \frac{4\, m\, l^2 \left(\frac{d^2}{dt^2}\, \varphi_1(t)\right)}{3} + \frac{m\, l^2 \left(\frac{d^2}{dt^2}\, \varphi_2(t)\right) \cos(\varphi_1(t) - \varphi_2(t))}{2} - \frac{m\, l^2 \left(\frac{d}{dt}\, \varphi_2(t)\right) \left(\frac{d}{dt}\, \varphi_1(t) - \frac{d}{dt}\, \varphi_2(t)\right) \sin(\varphi_1(t) - \varphi_2(t))}{2} + \frac{m\, l^2 \left(\frac{d}{dt}\, \varphi_1(t)\right) \left(\frac{d}{dt}\, \varphi_2(t)\right) \sin(\varphi_1(t) - \varphi_2(t))}{2} + \frac{3\, m\, g\, l \sin(\varphi_1(t))}{2} = 0,$$
$$\frac{m\, l^2 \left(\frac{d^2}{dt^2}\, \varphi_2(t)\right)}{3} + \frac{m\, l^2 \left(\frac{d^2}{dt^2}\, \varphi_1(t)\right) \cos(\varphi_1(t) - \varphi_2(t))}{2} - \frac{m\, l^2 \left(\frac{d}{dt}\, \varphi_1(t)\right) \left(\frac{d}{dt}\, \varphi_1(t) - \frac{d}{dt}\, \varphi_2(t)\right) \sin(\varphi_1(t) - \varphi_2(t))}{2} - \frac{m\, l^2 \left(\frac{d}{dt}\, \varphi_1(t)\right) \left(\frac{d}{dt}\, \varphi_2(t)\right) \sin(\varphi_1(t) - \varphi_2(t))}{2} + \frac{m\, g\, l \sin(\varphi_2(t))}{2} = 0 \tag{9}$$

> $row[1];$
> $row[2];$

$$\frac{4\, m\, l^2 \left(\frac{d^2}{dt^2}\, \varphi_1(t)\right)}{3} + \frac{m\, l^2 \left(\frac{d^2}{dt^2}\, \varphi_2(t)\right) \cos(\varphi_1(t) - \varphi_2(t))}{2} - \frac{m\, l^2 \left(\frac{d}{dt}\, \varphi_2(t)\right) \left(\frac{d}{dt}\, \varphi_1(t) - \frac{d}{dt}\, \varphi_2(t)\right) \sin(\varphi_1(t) - \varphi_2(t))}{2} + \frac{m\, l^2 \left(\frac{d}{dt}\, \varphi_1(t)\right) \left(\frac{d}{dt}\, \varphi_2(t)\right) \sin(\varphi_1(t) - \varphi_2(t))}{2} + \frac{3\, m\, g\, l \sin(\varphi_1(t))}{2} = 0$$

(10)

$$\frac{m\,l^2\left(\frac{d^2}{dt^2}\,\varphi_2(t)\right)}{3} + \frac{m\,l^2\left(\frac{d^2}{dt^2}\,\varphi_1(t)\right)\cos\left(\varphi_1(t) - \varphi_2(t)\right)}{2} - \frac{m\,l^2\left(\frac{d}{dt}\,\varphi_1(t)\right)\left(\frac{d}{dt}\,\varphi_1(t) - \frac{d}{dt}\,\varphi_2(t)\right)\sin\left(\varphi_1(t) - \varphi_2(t)\right)}{2} - \frac{m\,l^2\left(\frac{d}{dt}\,\varphi_1(t)\right)\left(\frac{d}{dt}\,\varphi_2(t)\right)\sin\left(\varphi_1(t) - \varphi_2(t)\right)}{2} + \frac{m\,g\,l\sin\left(\varphi_2(t)\right)}{2} = 0 \tag{10}$$

>