> #ZADANIE 1

> #a)

> $zad1 := taylor(\cos(3\,x), x = 0);$

$$zad1 := 1 - \frac{9}{2}\,x^2 + \frac{27}{8}\,x^4 + \mathrm{O}\left(x^6\right) \tag{1}$$

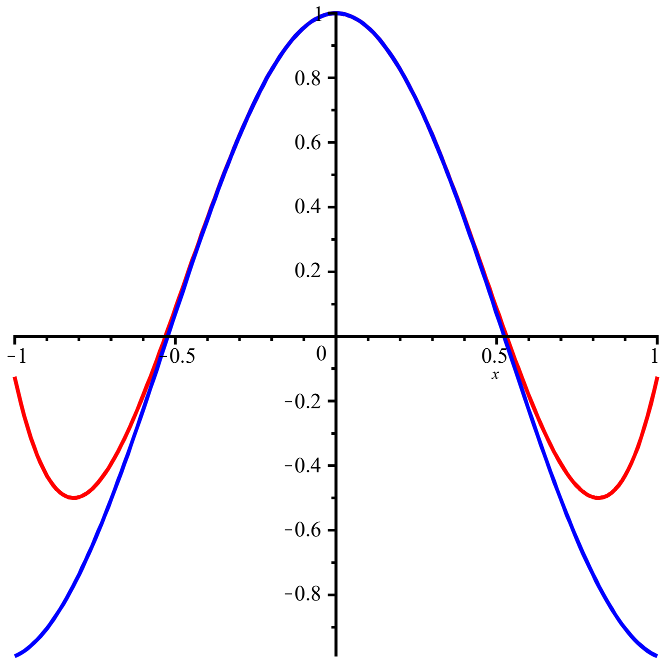
> #b)

> $with(plots):$

> $zad1 := convert(zad1, polynom, x);$

$$zad1 := 1 - \frac{9}{2}\,x^2 + \frac{27}{8}\,x^4 \tag{2}$$

> $plot([zad1, \cos(3\,x)], x = -1..1, color = [\text{"red"}, blue]);$



> #ZADANIE 2

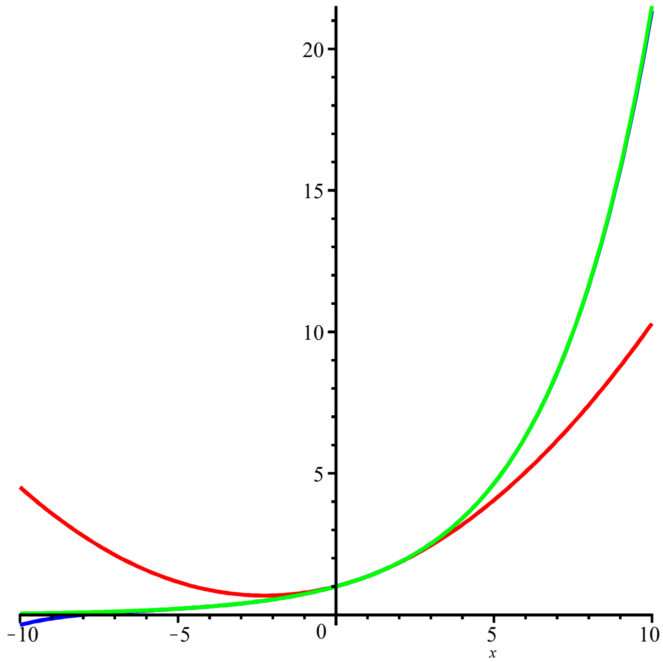
> #a)

> $szereg1 := taylor\left(\frac{e^x}{2^x}, x = 1, 3\right);$

$$szereg1 := \frac{e}{2} + \left(\frac{e}{2} - \frac{e\ln(2)}{2}\right)(x-1) + \left(\frac{e}{4} - \frac{e\ln(2)^2}{4} + \left(-\frac{e}{2} + \frac{e\ln(2)}{2}\right)\ln(2)\right)(x-1)^2 + \mathrm{O}\left((x-1)^3\right) \tag{3}$$

> $szereg2 := taylor\left(\frac{e^x}{2^x}, x = 1, 8\right):$

> #b)
$szereg1 := convert(szereg1, polynom, x):$
$szereg2 := convert(szereg2, polynom, x):$
$plot\left(\left[szereg1, szereg2, \frac{e^x}{2^x}\right], x, color = [\text{"red"}, \text{"blue"}, \text{"green"}]\right);$



> #c)

>

> $f1 := x \rightarrow \frac{e^x}{2^x}$: $f2 := unapply(szereg1, x)$: $f3 := unapply(szereg2, x)$:

> $evalf(f1(-5))$; $evalf(f2(-5))$; $evalf(f3(-5))$;

$$0.2156143040$$
$$1.16035138$$
$$0.21193317 \tag{4}$$

> #ZADANIE 3

> #a)

> $g1 := mtaylor\left(x^2 \ln(y), \left[x = \pi, y = \frac{\pi}{2}\right], 3\right)$;

$$g1 := \pi^2 \ln\left(\frac{\pi}{2}\right) + 2\pi\left(y - \frac{\pi}{2}\right) + 2\pi(x-\pi)\ln\left(\frac{\pi}{2}\right) - 2\left(y - \frac{\pi}{2}\right)^2 + 4(x-\pi)\left(y - \frac{\pi}{2}\right) + (x-\pi)^2\ln\left(\frac{\pi}{2}\right) \tag{5}$$

> $g2 := mtaylor\left(x^2 \ln(y), \left[x = \pi, y = \frac{\pi}{2}\right], 9\right)$:

> #b)
$g1 := convert(g1, polynom, [x, y])$:

> $g2 := convert(g2, polynom, [x, y])$:

> $g0 := (x, y) \rightarrow x^2 \ln(y)$: $g1 := unapply(g1, [x, y])$: $g2 := unapply(g2, [x, y])$:

> $evalf(g0(5, 3))$; $evalf(g1(5, 3))$; $evalf(g2(5, 3))$;

$$27.46530722$$
$$26.80844329$$

(6)

$$27.44484350 \quad (6)$$

> #ZADANIE 4

> #a)
int(ln(x^x), x);

$$\ln(x^x)\,x - \frac{x^2 \ln(x)}{2} - \frac{x^2}{4} \quad (7)$$

> #b)
$int\left(\cos^2(x)\sin(2x), x = 0 .. \frac{\pi}{2}\right);$

$$\frac{1}{2} \quad (8)$$

> #c)
$int\left(int\left((x+y)^2, y = -3\,\mathrm{sqrt}\left(1 - \frac{x^2}{4}\right) .. 3\,\mathrm{sqrt}\left(1 - \frac{x^2}{4}\right)\right), x = -2 .. 2\right);$

$$\frac{39\,\pi}{2} \quad (9)$$

> #ZADANIE 5
#a)

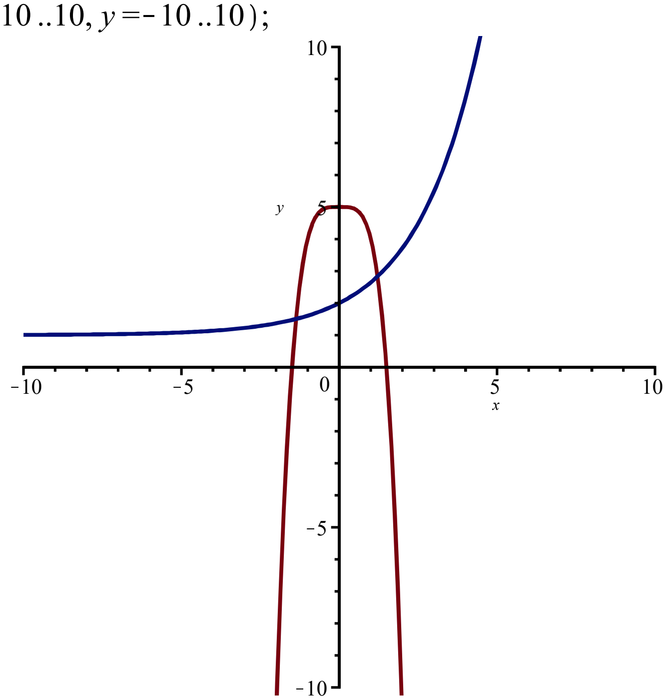
> $y1 := -x^4 + 5;$

$$y1 := -x^4 + 5 \quad (10)$$

> $y2 := e^{\frac{x}{2}} + 1;$

$$y2 := e^{\frac{x}{2}} + 1 \quad (11)$$

> plot([y1, y2], x = -10 .. 10, y = -10 .. 10);



> #b)

> $a := fsolve(y1 = y2, x = -5..0);$
$b := fsolve(y1 = y2, x = 0..5);$

$$a := -1.367316426$$
$$b := 1.213133062 \tag{12}$$

> #c)
$int(int(1, y = y2..y1), x = a..b);$

$$6.181768501 \tag{13}$$

> #ZADANIE 6

> $evalf\left(int\left(\sin\left(x^x\right), x = 0..1\right)\right);$

$$0.7029578376 \tag{14}$$

> #ZADANIE 7

> $trapezy :=$ **proc** (Y, n, h)
$wzor := \frac{h}{2}\,(Y[0] + 2 \cdot add(Y[i] + Y[n], i = 1..n - 1));$
end proc;

Warning, (in trapezy) `wzor` is implicitly declared local

$trapezy :=$ **proc**(Y, n, h) (15)
local $wzor, i;$
$wzor := 1/2 * h * (Y[0] + 2 * add(Y[i] + Y[n], i = 1..n - 1))$
end proc

> #ZADANIE 8

> $f := x \rightarrow \ln\left(x^2 + 1\right):$

> $n := 10:$ # liczba podprzedzialów (liczba węzlów - 1)

> $a := 0. : b := 4. : h := \frac{(b - a)}{n}:$

> $X := Array(0..n, [seq(a + h \cdot i, i = 0..n)]):$

> $Y := map(f, X):$

> $t := trapezy(Y, n, h);$

$$t := 15.62368702 \tag{16}$$

> $sym := int\left(\ln\left(x^2 + 1\right), x = a..b\right);$

$$sym := 5.984488704 \tag{17}$$

> $blad\_wzgledny := \frac{\mathrm{abs}(sym - t)}{sym} \cdot 100;$

$$blad\_wzgledny := 161.0697052 \tag{18}$$

>