> #Ex.1 Solving heat equation of the form (partial differential equation) :

$PDE := \frac{\partial}{\partial t} u(t, x) = \frac{1}{\mathrm{Pi}^2} \cdot \frac{\partial^2}{\partial x^2} u(t, x);$

$$PDE := \frac{\partial}{\partial t} u(t, x) = \frac{\frac{\partial^2}{\partial x^2} u(t, x)}{\pi^2} \qquad \textbf{(1)}$$

> # initial and boundary conditions

$conds := \{u(0, x) = x + \sin(\mathrm{Pi} \cdot x) + \sin(2 \cdot \mathrm{Pi} \cdot x), u(t, 0) = 0, u(t, 1) = 1\};$

$$conds := \{u(0, x) = x + \sin(\pi x) + \sin(2 \pi x), u(t, 0) = 0, u(t, 1) = 1\} \qquad \textbf{(2)}$$

> # **Remark :** pdsolve returns *module* structure

```
ans := pdsolve(PDE, conds, numeric);
```

$$ans := \textbf{module}(\ ) \ \ldots \ \textbf{end module} \qquad \textbf{(3)}$$

> # Find the value of u at t = 1, x = 0.5

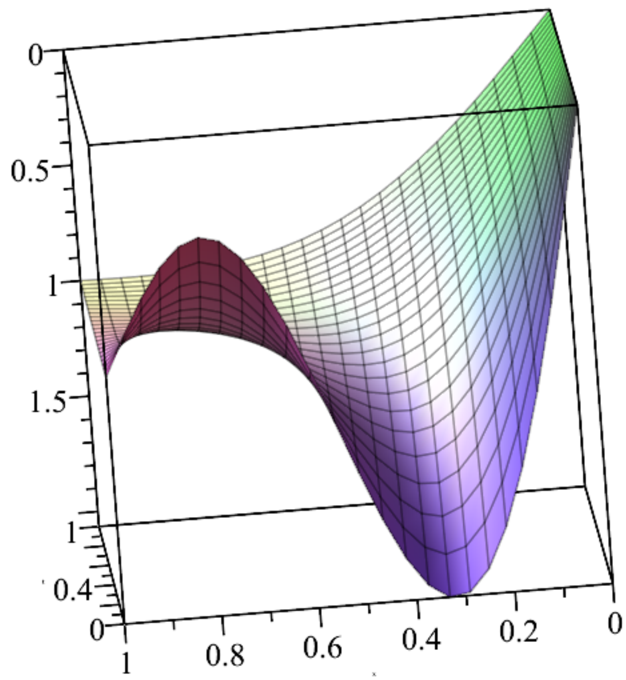
```
U := ans:-value( );
U(1, 0.5);
```

$$U := \textbf{proc}(\ ) \ \ldots \ \textbf{end proc}$$

$$[t = 1., x = 0.5, u(t, x) = 0.868559670607543] \qquad \textbf{(4)}$$

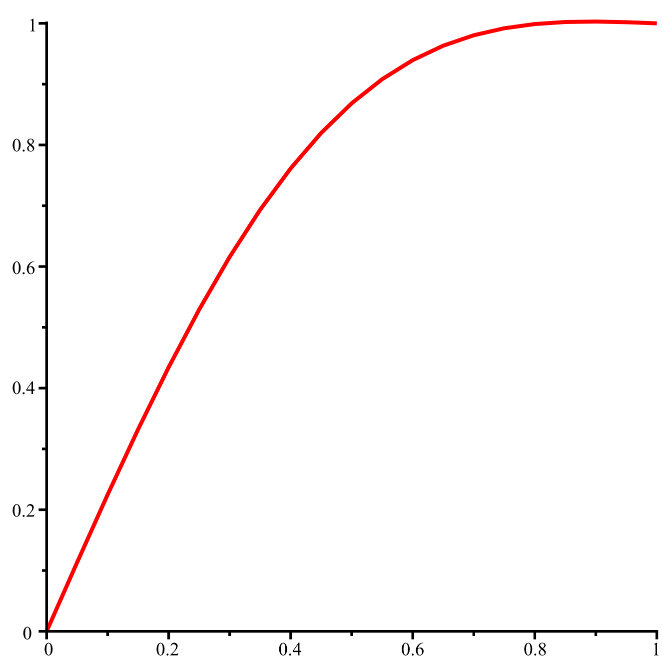
> # Plot the solution as a 3D plot for 0 < t < 1, 0 < x < 1

```
ans:-plot3d(t = 0 ..1, x = 0 ..1, axes = boxed);
```



> # Plot the solution as a 2D plot for t = 1 and 0 < x < 1

$W1 := ans{:}\text{-}plot(t=1, x=0..1, thickness=1, linestyle=solid);$

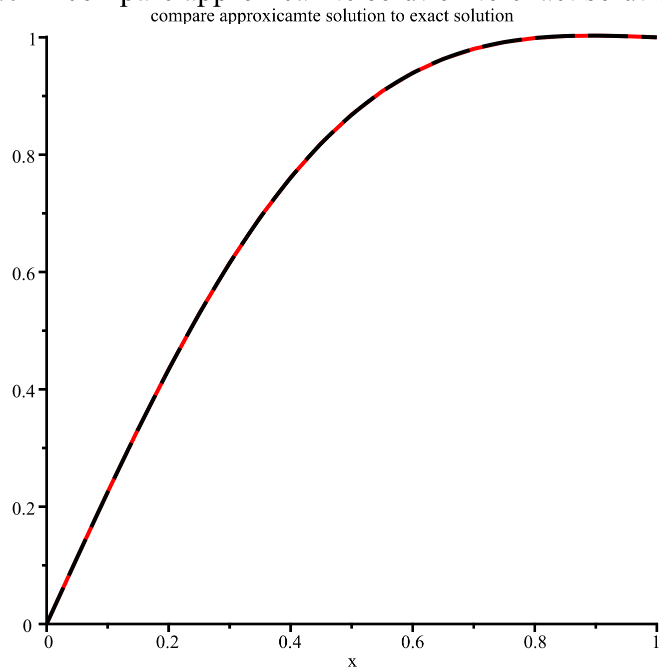


> # Comparing graphically the plot above with exact solution,

which has the form : $u = x + e^{-t}\sin(\pi x) + e^{-4\,t}\sin(2\;\pi x)$

```
with(plots) :
W2 := plot(eval(x + exp(-t)·sin(Pi·x) + exp(-4·t)·sin(2 Pi·x), t = 1), x = 0 ..1, color = black,
    linestyle = dash) :
```

```
> display(W1, W2, title = "compare approxicamte solution to exact solution" );
```



> #Ex.2 Solving wave equation of the form
·(partial differential equation) :

$$row2 := \frac{\partial^2}{\partial t^2} u(t, x) = \frac{\partial^2}{\partial x^2} u(t, x);$$

$$row2 := \frac{\partial^2}{\partial t^2}\; u(t, x) = \frac{\partial^2}{\partial x^2}\; u(t, x) \tag{5}$$

> # initial and boundary conditions

```
conds2 := {u(0, x) = sin(Pi·x), D[1](u)(0, x) = 0, u(t, 0) = 0 , u(t, 1) = 0};
```

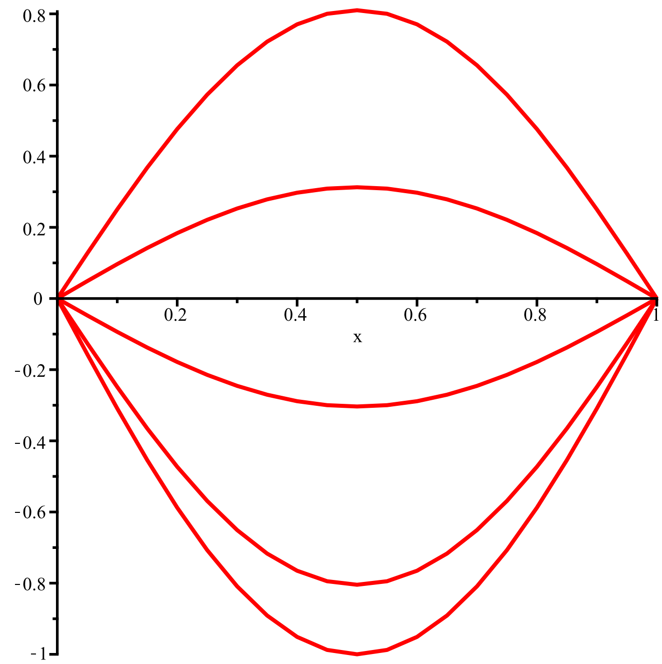
$$conds2 := \left\{u(0, x) = \sin\left(\pi\, x\right), u(t, 0) = 0, u(t, 1) = 0, \mathrm{D}_1(u)(0, x) = 0\right\} \tag{6}$$

```
> ans2 := pdsolve(row2, conds2, numeric);
```

$ans2 :=$ **module( ) ... end module** (7)

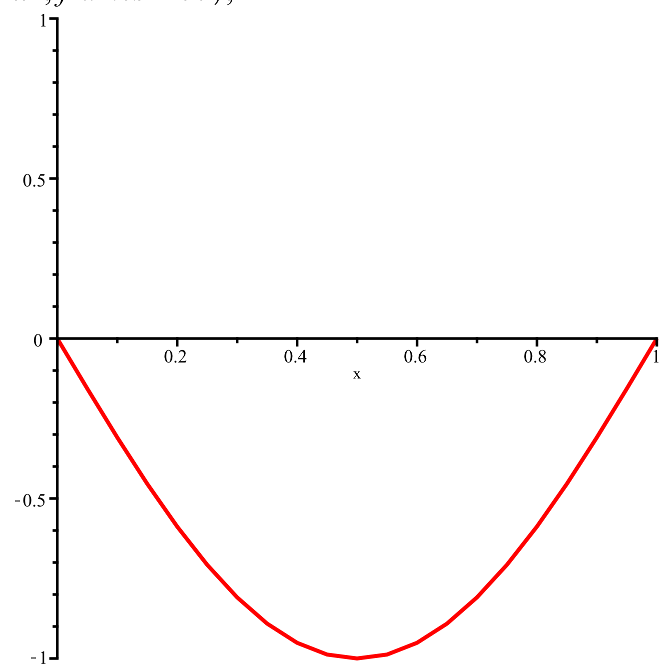
> # Built the profiles of the solution at t = 0.2, 0.4, 0.6, 0.8, 1 for 0 < x < 1. Plot all profiles in one figure.

```
> temp := 0 :
for i from 1 to 5 do:
 temp := temp + 0.2;
 P[i] := ans2:-plot(t = temp, x = 0 ..1);
 end do:
> display(seq(P[i], i = 1 ..5))
```



> **# animation of solution in time = 0 .. 1**

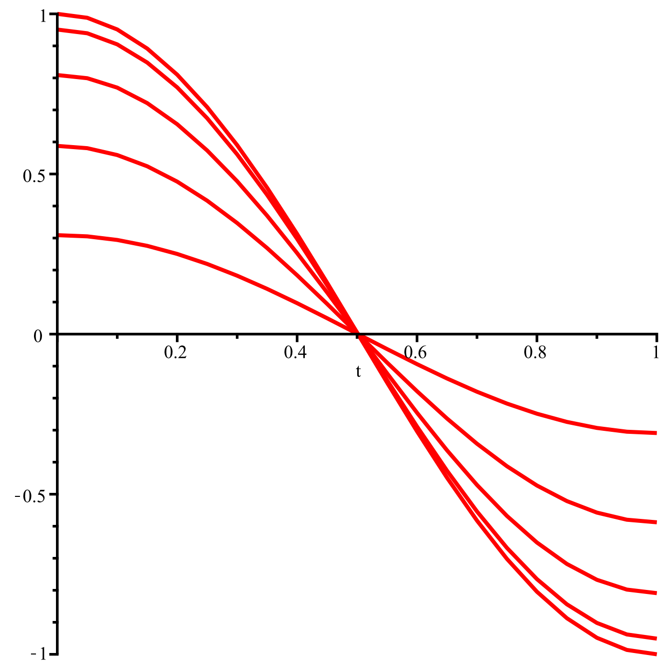
$ans2$:-$animate(t = 0 ..1, frames = 60)$;



> **# Built the profiles of the solution at x = 0.1, 0.2, 0.3, 0.4, 0.5 for 0 < t < 1. Plot all profiless in one figure.**

> $temp := 0$:
for i **from** 1 **to** 5 **do**:
$temp := temp + 0.1$;
$P2[i] := ans2$:-$plot(t = 0 ..1, x = temp)$;
end do:

> $display(seq(P2[i], i = 1 ..5))$



1
0.5
0
-0.5
-1
0.2
0.4
0.6
0.8
1
t

>