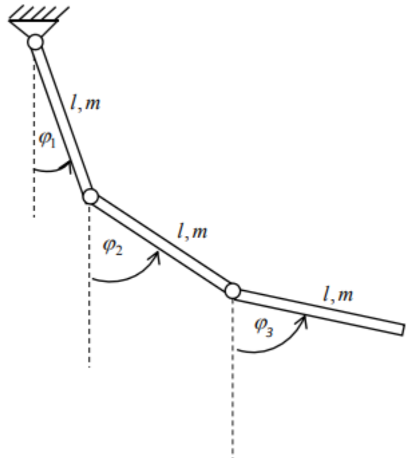
> # triple pendulum scheme:



> # lagrange procedure: *https://github.com/karolklimonczykk/Deriving-differential-equations*

```
lagrange := proc (n, q, r, L)
    local i, uzm_q, uzm_r, rel_r_q, Lq, Lr, Lrt;
    global row;
    uzm_q := seq(q[i]=q[i](t), i=1 .. n);
    uzm_r := seq(r[i]=r[i](t), i=1 .. n);
    for i to n do
        Lq[i] := subs([uzm_q, uzm_r], diff(L, q[i]));
        Lr[i] := subs([uzm_q, uzm_r], diff(L, r[i]));
    end do;
    for i to n do
        Lrt[i] := diff(Lr[i], t)
    end do;
    rel_r_q := seq(r[i](t) = diff(q[i](t), t), i=1 .. n);
    for i to n do
        row[i] := subs(rel_r_q, Lrt[i]-Lq[i]=0)
    end do;
    seq(row[i], i=1 .. n)
end proc:
```

> # potential energy

$$E := \frac{7}{6}\cdot m\cdot l^2\cdot \omega[1]^2 + \frac{3}{2}\cdot m\cdot l^2\omega[1]\cdot\omega[2]\cdot\cos(\varphi[1]-\varphi[2]) + \frac{2}{3}\cdot m\cdot l^2\cdot\omega[1]\cdot\omega[3]\cdot\cos(\varphi[3]-\varphi[1]) + \frac{1}{2}\cdot m\cdot l^2\cdot\omega[2]\cdot\omega[3]\cdot\cos(\varphi[3]-\varphi[2]) + \frac{1}{6}\cdot m\cdot l^2\cdot\omega[3]^2;$$

$$E := \frac{7\,m\,l^2\,\omega_1^2}{6} + \frac{3\,m\,l^2\,\omega_1\,\omega_2\cos(\varphi_1-\varphi_2)}{2} + \frac{2\,m\,l^2\,\omega_1\,\omega_3\cos(-\varphi_3+\varphi_1)}{3} + \frac{m\,l^2\,\omega_2\,\omega_3\cos(-\varphi_3+\varphi_2)}{2} + \frac{m\,l^2\,\omega_3^2}{6} \tag{1}$$

> # kinetic energy

$$U := -\frac{5}{2}\cdot m\cdot g\cdot l\cdot\cos(\varphi[1]) - \frac{3}{2}\cdot m\cdot g\cdot l\cdot\cos(\varphi[2]) - \frac{1}{2}\cdot m\cdot g\cdot l\cdot\cos(\varphi[3]);$$

(2)

$$U := -\frac{5\, m\, g\, l \cos(\varphi_1)}{2} - \frac{3\, m\, g\, l \cos(\varphi_2)}{2} - \frac{m\, g\, l \cos(\varphi_3)}{2} \tag{2}$$

> $L := E - U;$

$$L := \frac{7\, m\, l^2\, \omega_1^2}{6} + \frac{3\, m\, l^2\, \omega_1\, \omega_2 \cos(\varphi_1 - \varphi_2)}{2} + \frac{2\, m\, l^2\, \omega_1\, \omega_3 \cos(-\varphi_3 + \varphi_1)}{3} + \frac{m\, l^2\, \omega_2\, \omega_3 \cos(-\varphi_3 + \varphi_2)}{2} + \frac{m\, l^2\, \omega_3^2}{6} + \frac{5\, m\, g\, l \cos(\varphi_1)}{2} + \frac{3\, m\, g\, l \cos(\varphi_2)}{2} + \frac{m\, g\, l \cos(\varphi_3)}{2} \tag{3}$$

```
> # note: Note: In the above relationships, ω1, ω2 and ω3 denote
   generalized velocities,  corresponding to the coordinates φ1,
   φ2 and φ3
> # Deriving differential equations
```

> *lagrange*$(3, \varphi, \omega, L)$;

$$\frac{7\, m\, l^2 \left(\frac{d^2}{dt^2} \varphi_1(t)\right)}{3} + \frac{3\, m\, l^2 \left(\frac{d^2}{dt^2} \varphi_2(t)\right) \cos(\varphi_1(t) - \varphi_2(t))}{2} - \frac{3\, m\, l^2 \left(\frac{d}{dt} \varphi_2(t)\right) \left(\frac{d}{dt} \varphi_1(t) - \frac{d}{dt} \varphi_2(t)\right) \sin(\varphi_1(t) - \varphi_2(t))}{2} + \frac{2\, m\, l^2 \left(\frac{d^2}{dt^2} \varphi_3(t)\right) \cos(-\varphi_3(t) + \varphi_1(t))}{3} - \frac{2\, m\, l^2 \left(\frac{d}{dt} \varphi_3(t)\right) \left(-\frac{d}{dt} \varphi_3(t) + \frac{d}{dt} \varphi_1(t)\right) \sin(-\varphi_3(t) + \varphi_1(t))}{3} + \frac{3\, m\, l^2 \left(\frac{d}{dt} \varphi_1(t)\right) \left(\frac{d}{dt} \varphi_2(t)\right) \sin(\varphi_1(t) - \varphi_2(t))}{2} + \frac{2\, m\, l^2 \left(\frac{d}{dt} \varphi_1(t)\right) \left(\frac{d}{dt} \varphi_3(t)\right) \sin(-\varphi_3(t) + \varphi_1(t))}{3} + \frac{5\, m\, g\, l \sin(\varphi_1(t))}{2} = 0, \tag{4}$$

$$\frac{3\, m\, l^2 \left(\frac{d^2}{dt^2} \varphi_1(t)\right) \cos(\varphi_1(t) - \varphi_2(t))}{2} - \frac{3\, m\, l^2 \left(\frac{d}{dt} \varphi_1(t)\right) \left(\frac{d}{dt} \varphi_1(t) - \frac{d}{dt} \varphi_2(t)\right) \sin(\varphi_1(t) - \varphi_2(t))}{2} + \frac{m\, l^2 \left(\frac{d^2}{dt^2} \varphi_3(t)\right) \cos(-\varphi_3(t) + \varphi_2(t))}{2}$$

$$-\frac{m l^2 \left(\frac{d}{dt} \varphi_3(t)\right) \left(-\frac{d}{dt} \varphi_3(t) + \frac{d}{dt} \varphi_2(t)\right) \sin\left(-\varphi_3(t) + \varphi_2(t)\right)}{2}$$

$$-\frac{3 m l^2 \left(\frac{d}{dt} \varphi_1(t)\right) \left(\frac{d}{dt} \varphi_2(t)\right) \sin\left(\varphi_1(t) - \varphi_2(t)\right)}{2}$$

$$+\frac{m l^2 \left(\frac{d}{dt} \varphi_2(t)\right) \left(\frac{d}{dt} \varphi_3(t)\right) \sin\left(-\varphi_3(t) + \varphi_2(t)\right)}{2} + \frac{3 m g l \sin\left(\varphi_2(t)\right)}{2} = 0,$$

$$\frac{2 m l^2 \left(\frac{d^2}{dt^2} \varphi_1(t)\right) \cos\left(-\varphi_3(t) + \varphi_1(t)\right)}{3}$$

$$-\frac{2 m l^2 \left(\frac{d}{dt} \varphi_1(t)\right) \left(-\frac{d}{dt} \varphi_3(t) + \frac{d}{dt} \varphi_1(t)\right) \sin\left(-\varphi_3(t) + \varphi_1(t)\right)}{3}$$

$$+\frac{m l^2 \left(\frac{d^2}{dt^2} \varphi_2(t)\right) \cos\left(-\varphi_3(t) + \varphi_2(t)\right)}{2}$$

$$-\frac{m l^2 \left(\frac{d}{dt} \varphi_2(t)\right) \left(-\frac{d}{dt} \varphi_3(t) + \frac{d}{dt} \varphi_2(t)\right) \sin\left(-\varphi_3(t) + \varphi_2(t)\right)}{2}$$

$$+\frac{m l^2 \left(\frac{d^2}{dt^2} \varphi_3(t)\right)}{3} - \frac{2 m l^2 \left(\frac{d}{dt} \varphi_1(t)\right) \left(\frac{d}{dt} \varphi_3(t)\right) \sin\left(-\varphi_3(t) + \varphi_1(t)\right)}{3}$$

$$-\frac{m l^2 \left(\frac{d}{dt} \varphi_2(t)\right) \left(\frac{d}{dt} \varphi_3(t)\right) \sin\left(-\varphi_3(t) + \varphi_2(t)\right)}{2} + \frac{m g l \sin\left(\varphi_3(t)\right)}{2} = 0$$

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> # Numerical solution of the obtained sysyem of equestions of
  motion, assuming : m = 1, l = 0.5, g = 9.81 and initial
  conditions
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> $m := 1 : l := 0.5 : g := 9.81 : ICs := \varphi[1](0) = \frac{Pi}{6}, \varphi[2](0) = \frac{Pi}{3}, \varphi[3](0) = \frac{Pi}{2},$
> $D(\varphi[1])(0) = 0, D(\varphi[2])(0) = 0, D(\varphi[3])(0) = 0;$

$$ICs := \varphi_1(0) = \frac{\pi}{6}, \varphi_2(0) = \frac{\pi}{3}, \varphi_3(0) = \frac{\pi}{2}, D(\varphi_1)(0) = 0, D(\varphi_2)(0) = 0, D(\varphi_3)(0) = 0 \tag{5}$$

> $L1 := lagrange(3, \varphi, \omega, L);$

$$L1 := 0.5833333334 \frac{d^2}{dt^2} \varphi_1(t) + 0.3750000000 \left(\frac{d^2}{dt^2} \varphi_2(t)\right) \cos\left(\varphi_1(t) - \varphi_2(t)\right) \tag{6}$$

$$-0.3750000000 \left(\frac{d}{dt} \varphi_2(t)\right) \left(\frac{d}{dt} \varphi_1(t) - \frac{d}{dt} \varphi_2(t)\right) \sin\left(\varphi_1(t) - \varphi_2(t)\right)$$

$$+0.1666666667 \left(\frac{d^2}{dt^2} \varphi_3(t)\right) \cos\left(-\varphi_3(t) + \varphi_1(t)\right) - 0.1666666667 \left(\frac{d}{dt} \varphi_3(t)\right) \Bigg($$

$$- \frac{d}{dt} \varphi_3(t) + \frac{d}{dt} \varphi_1(t)) \sin(-\varphi_3(t) + \varphi_1(t)) + 0.3750000000 (\frac{d}{dt} \varphi_1(t)) (\frac{d}{dt}$$

$$\varphi_2(t)) \sin(\varphi_1(t) - \varphi_2(t)) + 0.1666666667 (\frac{d}{dt} \varphi_1(t)) (\frac{d}{dt} \varphi_3(t)) \sin(-\varphi_3(t)$$

$$+ \varphi_1(t)) + 12.26250000 \sin(\varphi_1(t)) = 0, 0.3750000000 (\frac{d^2}{dt^2} \varphi_1(t)) \cos(\varphi_1(t)$$

$$- \varphi_2(t)) - 0.3750000000 (\frac{d}{dt} \varphi_1(t)) (\frac{d}{dt} \varphi_1(t) - \frac{d}{dt} \varphi_2(t)) \sin(\varphi_1(t) - \varphi_2(t))$$

$$+ 0.1250000000 (\frac{d^2}{dt^2} \varphi_3(t)) \cos(-\varphi_3(t) + \varphi_2(t)) - 0.1250000000 (\frac{d}{dt} \varphi_3(t)) ($$

$$- \frac{d}{dt} \varphi_3(t) + \frac{d}{dt} \varphi_2(t)) \sin(-\varphi_3(t) + \varphi_2(t)) - 0.3750000000 (\frac{d}{dt} \varphi_1(t)) (\frac{d}{dt}$$

$$\varphi_2(t)) \sin(\varphi_1(t) - \varphi_2(t)) + 0.1250000000 (\frac{d}{dt} \varphi_2(t)) (\frac{d}{dt} \varphi_3(t)) \sin(-\varphi_3(t)$$

$$+ \varphi_2(t)) + 7.357500000 \sin(\varphi_2(t)) = 0, 0.1666666667 (\frac{d^2}{dt^2} \varphi_1(t)) \cos(-\varphi_3(t)$$

$$+ \varphi_1(t)) - 0.1666666667 (\frac{d}{dt} \varphi_1(t)) (- \frac{d}{dt} \varphi_3(t) + \frac{d}{dt} \varphi_1(t)) \sin(-\varphi_3(t) + \varphi_1(t))$$

$$+ 0.1250000000 (\frac{d^2}{dt^2} \varphi_2(t)) \cos(-\varphi_3(t) + \varphi_2(t)) - 0.1250000000 (\frac{d}{dt} \varphi_2(t)) ($$

$$- \frac{d}{dt} \varphi_3(t) + \frac{d}{dt} \varphi_2(t)) \sin(-\varphi_3(t) + \varphi_2(t)) + 0.08333333334 \frac{d^2}{dt^2} \varphi_3(t)$$

$$- 0.1666666667 (\frac{d}{dt} \varphi_1(t)) (\frac{d}{dt} \varphi_3(t)) \sin(-\varphi_3(t) + \varphi_1(t)) - 0.1250000000 (\frac{d}{dt}$$

$$\varphi_2(t)) (\frac{d}{dt} \varphi_3(t)) \sin(-\varphi_3(t) + \varphi_2(t)) + 2.452500000 \sin(\varphi_3(t)) = 0$$

> $r := dsolve(\{L1, ICs\}, \{\varphi[1](t), \varphi[2](t), \varphi[3](t)\}, numeric, stepsize = 4);$

$$r := \textbf{proc}(x\_rkf45) \ \ldots \ \textbf{end proc} \quad \textbf{(7)}$$

> # Specifying the state of the system (angles **and** angular velocities) at time t = 1 :

$r(1);$

$$\Big[t = 1., \varphi_1(t) = 7.99082015187153\ 10^{54}, \frac{d}{dt} \varphi_1(t) = 7.73447392496997\ 10^{109}, \varphi_2(t) \quad \textbf{(8)}$$

$$= -7.75330259972629\ 10^{54}, \frac{d}{dt} \varphi_2(t) = -2.23266534542901\ 10^{110}, \varphi_3(t)$$

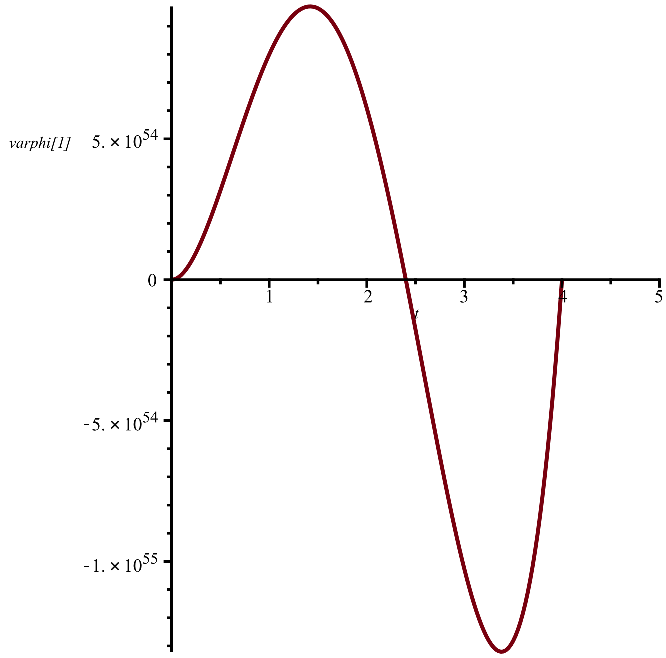
$$= 7.42762380977307\ 10^{54}, \frac{d}{dt} \varphi_3(t) = 3.76161711024048\ 10^{109} \Big]$$

> # Time courses of angles φ1, φ2, and φ3 within the time interval t∈[0, 5]:

$with(plots):$

$odeplot(r, t = 0..5);$

Warning, cannot evaluate the solution further right of 4., probably a singularity



varphi[1]
5. × 10^54
0
1
2
3
4
5
-5. × 10^54
-1. × 10^55