Asset Market Operations: Model Setup

1 Environment

Time is discrete, $t \in \{0, 1, 2, \ldots\}$. The economy consists of a representative household and a government.

2 Assets

There is a set $\mathcal{B} = \{1, \ldots, B\}$ of financial assets.

- $\boldsymbol{b}_t = (b_{1,t}, \ldots, b_{B,t})^\intercal \in \mathbb{R}^B$: household holdings
- $\boldsymbol{R}_t = (R_{1,t}, \ldots, R_{B,t})^\intercal \in \mathbb{R}^B_{++}$: gross returns

3 Shocks

Two exogenous shock processes:

Government issuance shifter:

$$\omega^G_{t+1} = \rho_G\, \omega^G_t + \sigma_G\, \varepsilon^G_{t+1}, \quad \varepsilon^G_{t+1} \sim \mathcal{N}(0, 1) \tag{1}$$

Asset weight shifter:

$$\omega^\alpha_{t+1} = \rho_\alpha\, \omega^\alpha_t + \sigma_\alpha\, \varepsilon^\alpha_{t+1}, \quad \varepsilon^\alpha_{t+1} \sim \mathcal{N}(0, 1) \tag{2}$$

Innovations ε^G_{t+1} and ε^α_{t+1} are independent.

4 Household

4.1 Endowment

The household receives a constant endowment $y > 0$ each period.

4.2 Preferences

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) + v(L_t) \right] \tag{3}$$

Consumption utility:

$$u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma}, \quad \sigma > 0 \tag{4}$$

Liquidity utility:

$$v(L) = \psi \cdot \frac{L^{1-\gamma} - 1}{1-\gamma}, \quad \gamma > 0, \quad \psi > 0 \tag{5}$$

4.3 Liquidity Aggregator

Baseline CES:

$$L_t = \left(\sum_{i=1}^{B} \alpha_{i,t} \, b_{i,t}^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}} \tag{6}$$

where $\rho > 0$ is the elasticity of substitution and $\alpha_{i,t} > 0$ with $\sum_i \alpha_{i,t} = 1$.

Nested CES extension:

Partition assets into K groups: $\mathcal{B} = \mathcal{G}_1 \cup \cdots \cup \mathcal{G}_K$.

Upper-level aggregator:

$$L_t = \left(\sum_{k=1}^{K} \alpha_{k,t} \, L_{k,t}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \tag{7}$$

Lower-level aggregator for group k:

$$L_{k,t} = \left(\sum_{i \in \mathcal{G}_k} \alpha_{i|k,t} \, b_{i,t}^{\frac{\rho_k-1}{\rho_k}} \right)^{\frac{\rho_k}{\rho_k-1}} \tag{8}$$

Parameters:

- $\eta > 0$: elasticity of substitution across groups
- $\rho_k > 0$: elasticity of substitution within group k
- $\alpha_{k,t} > 0$ with $\sum_k \alpha_{k,t} = 1$: group weights
- $\alpha_{i|k,t} > 0$ with $\sum_{i \in \mathcal{G}_k} \alpha_{i|k,t} = 1$: within-group weights

4.4 Asset Weights

Weights depend on the shock ω_t^α:

$$\alpha_{i,t} = \alpha_i(\omega_t^\alpha) \tag{9}$$

Parameterization (baseline CES):

$$\alpha_{i,t} = \frac{\bar{\alpha}_i \exp(\delta_i \, \omega_t^\alpha)}{\sum_{j=1}^{B} \bar{\alpha}_j \exp(\delta_j \, \omega_t^\alpha)} \tag{10}$$

where $\bar{\alpha}_i > 0$ are steady-state weights and $\delta_i \in \mathbb{R}$ are shock loadings.

Parameterization (nested CES):

Group weights:

$$\alpha_{k,t} = \frac{\bar{\alpha}_k \exp(\delta_k \, \omega_t^\alpha)}{\sum_{j=1}^{K} \bar{\alpha}_j \exp(\delta_j \, \omega_t^\alpha)} \tag{11}$$

Within-group weights:

$$\alpha_{i|k,t} = \frac{\bar{\alpha}_{i|k} \exp(\delta_{i|k} \, \omega_t^\alpha)}{\sum_{j \in \mathcal{G}_k} \bar{\alpha}_{j|k} \exp(\delta_{j|k} \, \omega_t^\alpha)} \tag{12}$$

Parameters:

- $\bar{\alpha}_k > 0$: steady-state weight on group k
- $\delta_k \in \mathbb{R}$: shock loading on group k weight
- $\bar{\alpha}_{i|k} > 0$: steady-state weight on asset i within group k
- $\delta_{i|k} \in \mathbb{R}$: shock loading on within-group weight for asset i

4.5 Budget Constraint

$$c_t + \mathbf{1}^\intercal \boldsymbol{b}_t = y + \boldsymbol{R}_t^\intercal \boldsymbol{b}_{t-1} - T_t \tag{13}$$

5 Government

5.1 Asset Positions

$$\boldsymbol{b}_t^G \in \mathbb{R}^B \tag{14}$$

- $b_{i,t}^G > 0$: government issues asset i
- $b_{i,t}^G < 0$: government holds asset i

5.2 Issuance Policy

Government positions depend on the shock ω_t^G:

$$\boldsymbol{b}_t^G = \boldsymbol{b}^G(\omega_t^G) \tag{15}$$

Parameterization:

$$b_{i,t}^G = \bar{b}_i^G + \phi_i \, \omega_t^G \tag{16}$$

where $\bar{b}_i^G$ is steady-state issuance and $\phi_i \in \mathbb{R}$ is the shock loading.

5.3 Budget Constraint

$$\mathbf{1}^\intercal \boldsymbol{b}_t^G + T_t = \boldsymbol{R}_t^\intercal \boldsymbol{b}_{t-1}^G \tag{17}$$

6 Market Clearing

Asset markets:

$$\boldsymbol{b}_t = \boldsymbol{b}_t^G \tag{18}$$

Goods market:

$$c_t = y \tag{19}$$

7 Equilibrium

Definition 1 (Competitive Equilibrium). *Given initial positions $\boldsymbol{b}_{-1}$ and shock processes $\{\omega_t^G, \omega_t^\alpha\}$, a competitive equilibrium is:*

- *Allocations:* $\{c_t, \boldsymbol{b}_t\}_{t\geq 0}$
- *Returns:* $\{\boldsymbol{R}_t\}_{t\geq 0}$
- *Government policy:* $\{\boldsymbol{b}_t^G, T_t\}_{t\geq 0}$

such that household and government optimize, and markets clear.

8 Parameters

Parameter	Description
Household	
β	Discount factor
σ	Risk aversion (consumption)
γ	Curvature of liquidity utility
ψ	Liquidity utility weight
y	Endowment
Liquidity aggregator (baseline)	
ρ	Elasticity of substitution
$\bar{\alpha}_i$	Steady-state weight on asset i
δ_i	Shock loading on weight for asset i
Liquidity aggregator (nested)	
η	Elasticity across groups
ρ_k	Elasticity within group k
$\bar{\alpha}_k$	Steady-state weight on group k
δ_k	Shock loading on group k weight
$\bar{\alpha}_{i\|k}$	Steady-state weight on asset i within group k
$\delta_{i\|k}$	Shock loading on within-group weight for asset i
Government	
$\bar{b}_i^G$	Steady-state position in asset i
ϕ_i	Shock loading on position for asset i
Shocks	
ρ_G	Persistence of issuance shock
σ_G	Volatility of issuance shock
ρ_α	Persistence of weight shock
σ_α	Volatility of weight shock