CAS 741: Problem Statement

Dynamical Systems: Multi-Pendulum Simulation (MPSim)

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Table 1: Revision History

Date	Developer(s)	Change
September 17, 2018	Karol Serkis	First revision of document
September 14, 2018	Karol Serkis	Problem Idea proposed & discussed with Dr. Spencer Smith
December 18, 2018	Karol Serkis	Problem Statement & bibliography fixed, & program name fixed

Problem

A simple gravity pendulum has very easy to system to model and consists of a weight suspended from a pivot and the weight is given enough space to swing freely. To simplify the model we assume no air resistance and a friction-less pivot [1]. The model and calculations for the simple gravity pendulum are well defined and only require ordinary differential equation (ODE) solvers as well as Lagrangian mechanics equations [2]. Below is an example of a single pendulum to demonstrate the idea:

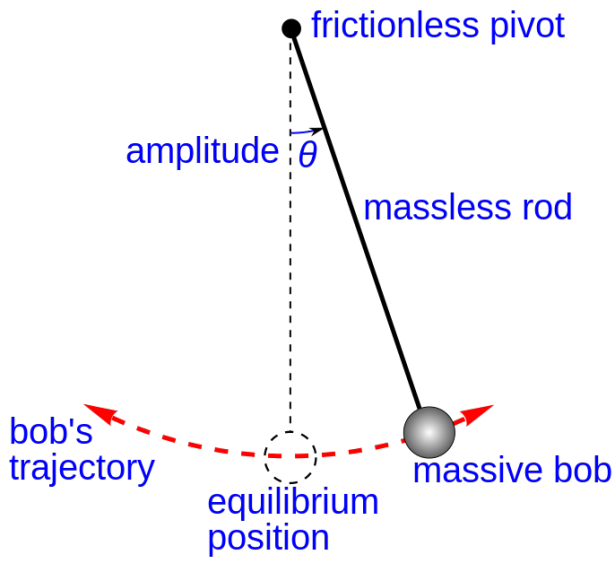

Figure 1: A simple gravity pendulum: friction-less pivot & no air resistance [1]

However, once you attach a pendulum to the bottom of another pendulum in the case of a double pendulum you have a new system that is dynamic and chaotic and requires a set of coupled ordinary

differential equation solvers [3]. Once you introduce multiple pendulums the system becomes chaotic and interesting to model and simulate.

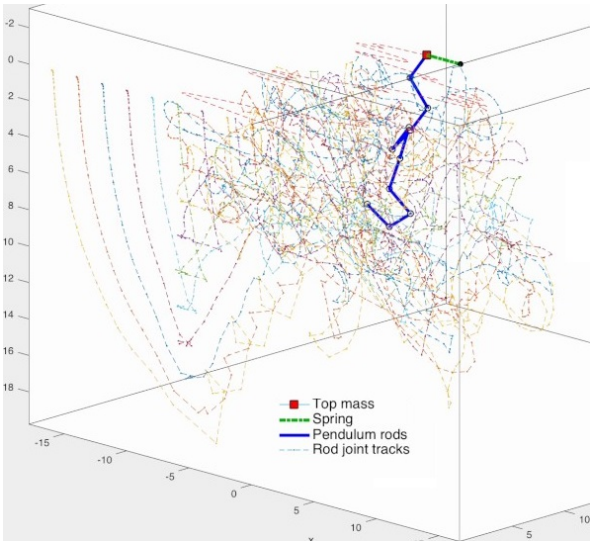

Figure 2: An example of dynamical and chaotic system with Spring-Mass-Multi-Pendulum [5]

Proposed Solution

A proposed software solution will produce a simulation of a multi-pendulum system. Inspiration for this problem came from Dr. Ned Nedialkov's Multi-body Lagrangian Simulations using DAETS (Differential-Algebraic Equations by Taylor Series). DAETS is a C++ package for solving initial value problems for DAE systems [4].

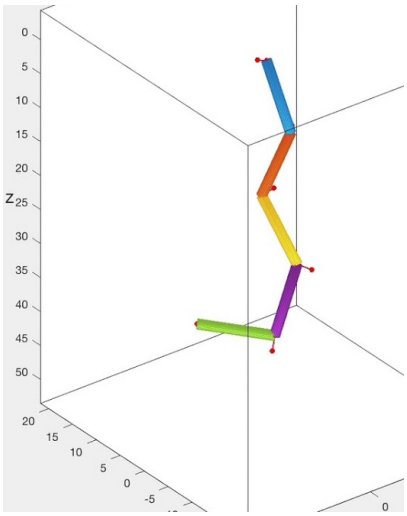

Figure 3: Simulation of Multi-Pendulum system using DAETS [5]

The proposed software is to develop a multi-platform equivalent solution that only focuses on multi-pendulum simulations and tracking the chaotic motion of the system. It will allow users to track kinetic and

potential energy over time using ODE/DAE initial value problem solvers, and display a simulation showing the animated trajectory of the pendulums [4]. To make things simpler to animate and calculate we can restrict to a rigid-body system, where the mass of each pendulum rod is located at the center of gravity of each rod [3].

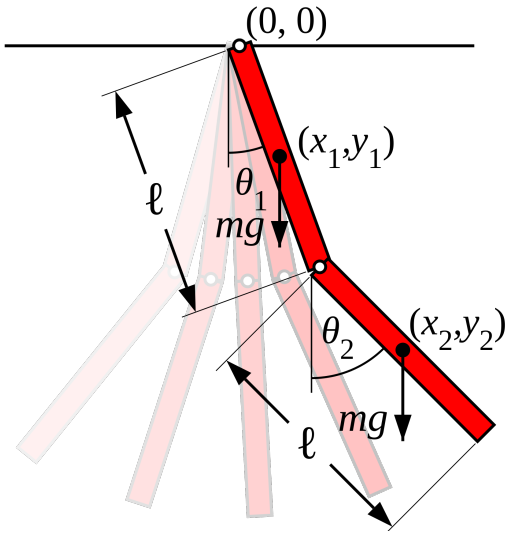

Figure 4: An example of rigid-body double pendulum [3]

Context

Environment

The simulation software will be created with multi-platform support in mind and be compatible with Windows 10, Mac OS, Linux, etc. In order to achieve this, Python will be used for development and implementation of the simulation software.

Stakeholders

Specific stakeholders include:

- Karol Serkis
- Dr. Spencer Smith
- Dr. Ned Nedialov
- Students of CAS 741
- Individuals studying or working in fields related to physics

References

[1] Wikipedia - Pendulum,
https://en.wikipedia.org/wiki/Pendulum

[2] Wikipedia - Pendulum (mathematics),
https://en.wikipedia.org/wiki/Pendulum_(mathematics)

[3] Wikipedia - Double Pendulum,
https://en.wikipedia.org/wiki/Double_pendulum

[4] Differential-Algebraic Equations by Taylor Series,
http://www.cas.mcmaster.ca/~nedialk/daets/

[5] Multi-body Lagrangian Simulations,
https://www.youtube.com/channel/UCCuLchOxOWOyoNE9KOCYlVQ