CAS 741: SRS

Dynamical Systems: Multi-Pendulum

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Contents

Revision History

Table 1: Revision History

Date	Developer(s)	Change
September 28, 2018	Karol Serkis	First revision of document in landscape orientation for presentation
September 26, 2018	Karol Serkis	SRS presentation slides discussed with Dr. Spencer Smith

Reference Material

Mathematical Notation

This section describes the notation conventions used in this document.

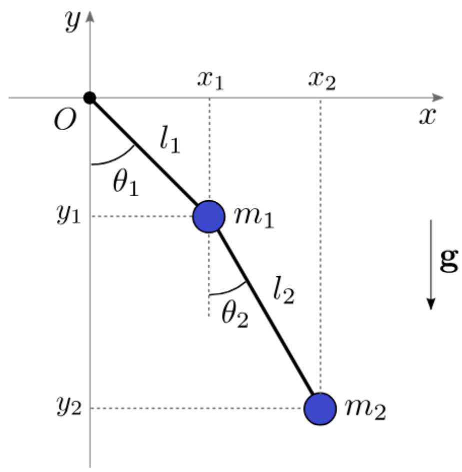


Figure 1: A simple gravity double pendulum (model assumes no friction or air resistance)

$$x_1 = l_1 \sin\theta_1 \qquad y_1 = -l_1 \cos\theta_1$$
$$x_2 = l_1 \sin\theta_1 + l_2 \sin\theta_2 \qquad y_2 = -l_1 \cos\theta_1 - l_2 \cos\theta_2$$

Table of Units

Throughout this document SI (Système International d'Unités) is employed as the unit system. In addition to the basic units, several derived units are used as described below. For each unit, the symbol is given followed by a description of the unit and the SI name.

symbol	**unit**	**SI**
m	length	metre
kg	mass	kilogram
s	time	second

Table of Symbols

The table that follows summarizes the symbols used in this document along with their units. The choice of symbols was made to be consistent with calculus, ordinary differentials (ODE), Lagrangian, kinematics etc. The symbols are listed in alphabetical order.

symbol	space	unit	description
θ	$\mathbb{N}$	–	amplitude from the pivot point
g	$\mathbb{N}$	–	gravitational constant
m_1	$\mathbb{N}$	–	mass of the 1st pendulum weight
m_2	$\mathbb{N}$	–	mass of the 2nd pendulum weight
m_n	$\mathbb{N}$	–	mass of the nth pendulum weight
x_1	$\mathbb{N}$	–	length of the 1st pendulum rod
x_2	$\mathbb{N}$	–	length of the 2nd pendulum rod
x_n	$\mathbb{N}$	–	length of the nth pendulum rod

Abbreviations and Acronyms

symbol	description
A	Assumption
DD	Data Definition
GD	General Definition
GS	Goal Statement
IM	Instance Model
LC	Likely Change
NF	Non-Functional Requirement
PS	Physical System Description
R	Requirement
SRS	Software Requirements Specification
T	Theoretical Model

1 Introduction

1.1 Purpose of Document

The purpose of this document is to describe the requirements for a Multi-Pendulum Simulation program, a multi-platform equivalent solution that only focuses on multi-pendulum simulations and tracking the chaotic motion of the system. It will allow users to generate diagrams and plot trajectories over time using two different ODE/DAE initial value problem solvers. In the case of a double pendulum you have a new system that is dynamic and chaotic and requires a set of coupled ordinary differential equation solvers. Once you introduce multiple pendula the system becomes chaotic and interesting to model and simulate. The simulation software will be created with multi-platform support.

The goals and models used in the Multi-Pendulum code will be provided, insuring assumptions and unambiguous definitions are itentified. This document is intended to be used as a reference to provide all information necessary to understand and verify the inputs to outputs. The SRS is abstract: the contents describe the problem being solved, but not how to solve it.

This document will be used as a starting point for subsequent development phases, including writing the design specification and the software verification and validation plan. The verification and validation plan will show the steps in the software documentation/implementation.

1.2 Scope of Requirements

The scope of the Multi-Pendulum Simulation program is limited to the generation of diagrams and plot trajectories that are possible to run and compute on a local system.

Assumptions: No server-side processing or parallel/distributed computing will be utilized. The multi-pendulum simulation will be a closed system. Air resistance and friction will not be considered for the simulation.

Question: Should the implementation be limited to local systems? Should a mobile-ready implementation be made for lower performance systems? What do I mean by multi-platform implementation?

1.3 Characteristics of Intended Reader

The intended reader is expected to have a minimum knowledge in mathematics and physics at undergraduate level. Simplification of some physical concepts are proposed to make the document technically accessible. Nevertheless, a basic knowledge in mathematics (calculus, differentials) and physics (kinematics, energy potential, Lagrangian) is recommended to get a deeper understanding of the document.

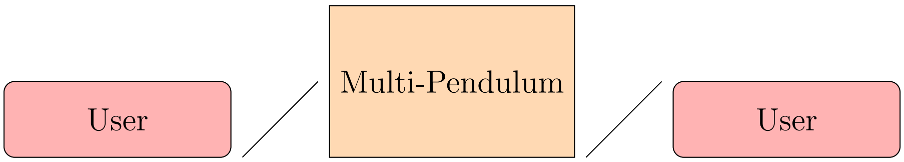
1.4 Organization of Document

- The organization of this document follows the template for an SRS for scientific computing software proposed by Dr. Spencer Smith.
- The presentation follows the standard pattern of presenting goals, theories, definitions, and assumptions.
- The goal statements are refined to the theoretical models, and the theoretical models to the instance models. The data definitions are used to support the definitions of the different models.

2 General System Description

This section identifies the interfaces between the system and its environment, describes the user characteristics and lists the system constraints.

2.1 System Context



- User Responsibilities:
  - Ensure that the input data is fits the system model.
  - Ensure that the input data is within scope.
- Multi-Pendulum Simulation program Responsibilities:
  - Detect data type mismatch, such as a string of characters instead of a floating point number.
  - Determine if the inputs satisfy the required physical and software constraints.
  - Solve the system of equations arising from the input data to generate the output data.
  - Generate a plot of the output data.

2.2 User Characteristics

The end user of Multi-Pendulum Simulation program should have an understanding of first year undergraduate math and physics. Less understanding of physics and math are required to use the software.

2.3 System Constraints

There are no system constraints.

3 Specific System Description

This section first presents the problem description, which gives a high-level view of the problem to be solved. This is followed by the solution characteristics specification, which presents the assumptions, theories, definitions and finally the instance models.

3.1 Problem Description

A simple gravity pendulum has very easy to system to model and consists of a weight suspended from a pivot and the weight is given enough space to swing freely. To simplify the model we assume no air resistance with a frictionless pivot. The model and calculations for the simple gravity pendulum are well defined and only require simple derivations and differential solvers.

Multi-Pendulum Simulation program will produce a simulation given a set of equilibrium constants and input.

3.1.1 Terminology and Definitions

This subsection provides a list of terms that are used in the subsequent sections and their meaning, with the purpose of reducing ambiguity and making it easier to correctly understand the requirements:

Lagrangian: the $L = T - V$, where T and V are the kinetic and potential energies of the system respectively.

Equilibrium position: the pendulum rod and weight position in its resting state.

3D Cartesian coordinate system: the pendulum rod and weight swing from a pivot position origin (x, y, z)

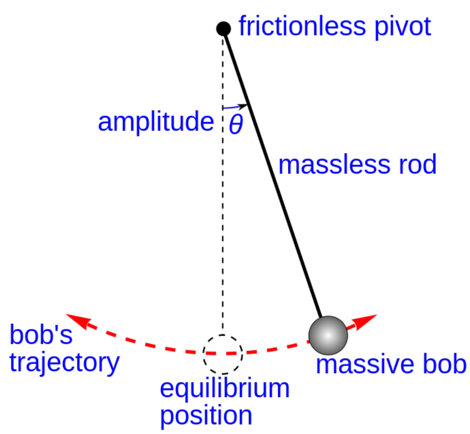


Figure 2: A simple gravity pendulum where the model assumes no friction or air resistance

3.1.2 Physical System Description

The physical system of Multi-Pendulum Simulation program includes the following elements:

PS1: Simulate an n-rod Multi-Pendulum system with no friction and no air resistance in a 3D space.

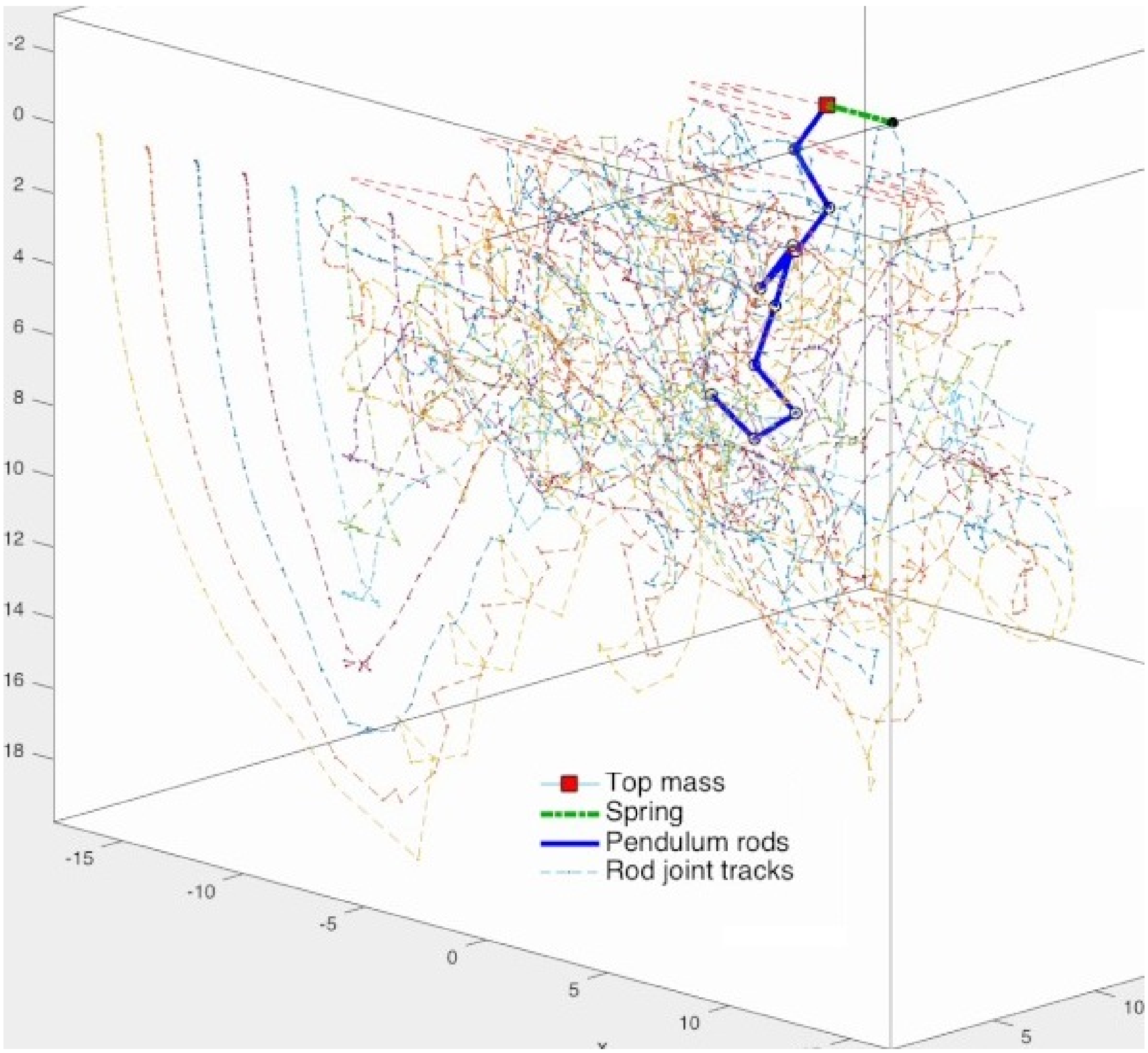


Figure 3: An example of dynamical and chaoitc system with Spring-Mass-Multi-Pendulum

3.1.3 Goal Statements

Given the user input and the initial state of the Multi-Pendulum Simulation the goal statements are:

GS1: Generate a plot of the movment of the pendula from equilibrium state of rest and show logged statistics over time to the user.

3.2 Solution Characteristics Specification

3.2.1 Assumptions

This section simplifies the original problem and helps in developing the theoretical model by filling in the missing information for the physical system. The numbers given in the square brackets refer to the theoretical model [T], general definition [GD], data definition [DD], instance model [IM], or likely change [LC], in which the respective assumption is used.

A1: All generated simulation diagrams will fit the mathematical model and scope.

A2: The user knows what the simulation model is for and inputs weights and lengths according to possible simulation characteristics

3.2.2 Theoretical Models

This section focuses on the general equations and laws that Multi-Pendulum Simulation program is based on.

Number	T1
Label	**Double Pendulum Pivot rod**
Equation	$$x_1 = l_1 \sin\theta_1 \qquad y_1 = -l_1 \cos\theta_1$$ $$x_2 = l_1 \sin\theta_1 + l_2 \sin\theta_2 \qquad y_2 = -l_1 \cos\theta_1 - l_2 \cos\theta_2$$
Description	Simple cartesian coordinate model system solution
Source	–
Ref. By	–

Number	T2
Label	**Double Pendulum Kinetic Energy**
Equation	$$T = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2$$ $$= \frac{1}{2}m_1(\dot{x}_1^2 + \dot{y}_1^2) + \frac{1}{2}m_2(\dot{x}_2^2 + \dot{y}_2^2)$$ $$= \frac{1}{2}m_1 l_1^2 \dot{\theta}_1^2 + \frac{1}{2}m_2 \left[l_1^2\dot{\theta}_1^2 + l_2^2\dot{\theta}_2^2 + 2l_1 l_2 \dot{\theta}_1\dot{\theta}_2 \cos(\theta_1 - \theta_2)\right]$$
Description	Simple cartesian coordinate model system solution
Source	–
Ref. By	–

Number	T3
Label	**Double Pendulum Potential Energy**
Equation	$$\begin{aligned} V &= m_1 g y_1 + m_2 g y_2 \\ &= -m_1 g l_1 \cos\theta_1 - m_2 g(l_1 \cos\theta_1 + l_2 \cos\theta_2) \\ &= -(m_1 + m_2) g l_1 \cos\theta_1 - m_2 g l_2 \cos\theta_2 \end{aligned}$$
Description	Simple cartesian coordinate model system solution
Source	–
Ref. By	–

Number	T4
Label	**Double Pendulum Lagrangian ($L = T - V$)**
Equation	$$\begin{aligned} L = &\frac{1}{2}(m_1 + m_2) l_1^2 \dot{\theta}_1^2 + \frac{1}{2} m_2 l_2^2 \dot{\theta}_2^2 + m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \cos(\theta_1 - \theta_2) \\ &+(m_1 + m_2) g l_1 \cos\theta_1 + m_2 g l_2 \cos\theta_2 \end{aligned}$$
Description	Simple cartesian coordinate model system solution
Source	–
Ref. By	–

3.2.3 General Definitions

We will use the Lagrangian and ODEs. No need for general defininitions in current documentation.

3.2.4 Data Definitions

This section collects and defines all the data needed to build the instance models. Currently no data definitions as implementation has not started yet.

3.2.5 Instance Models

This section transforms the problem defined in problem description into one which is expressed in mathematical terms.

Label	**Double Pendulum Lagrangian ($L = T - V$)**
Equation	$$L = \frac{1}{2}(m_1 + m_2)l_1^2\dot{\theta}_1^2 + \frac{1}{2}m_2 l_2^2\dot{\theta}_2^2 + m_2 l_1 l_2 \dot{\theta}_1\dot{\theta}_2 \cos(\theta_1 - \theta_2)$$ $$+(m_1 + m_2)gl_1 \cos\theta_1 + m_2 g l_2 \cos\theta_2$$
Description	Simple cartesian coordinate model system solution
Source	–
Ref. By	–

3.2.6 Data Constraints

The data constraints on the input and output variables, respectively. The column for physical constraints gives the physical limitations on the range of values that can be taken by the variable. The column for software constraints restricts the range of inputs to reasonable values. The constraints are conservative, to give the user of the model the flexibility to experiment with unusual situations. The column of typical values is intended to provide a feel for a common scenario. The uncertainty column provides an estimate of the confidence with which the physical quantities can be measured. This information would be part of the input if one were performing an uncertainty quantification exercise.

- Constraint on gravity: g = $9.8m/s^2$

3.2.7 Properties of a Correct Solution

A correct solution must satisfy the system of non-linear equations described. The user will also be able to judge the results based on the knowledge about the model and input.

4 Requirements

This section provides the functional requirements, the business tasks that the software is expected to complete, and the nonfunctional requirements, the qualities that the software is expected to exhibit.

4.1 Functional Requirements

R1: Multi-Pendulum Simulation program will take the following inputs:

1. The initial mass of the weights.
2. The inital length of the rods.

R2: Multi-Pendulum Simulation program will ensure that the inputs do not violate the constraints specified in the Data Contraints section:

1. Multi-Pendulum Simulator program will generate diagrams with and plot lines and timeline of logged movement.
2. The timeline of swings of the pendulum will be logged and eventually return to a resting state in equilibrium

4.2 Nonfunctional Requirements

Multi-Pendulum Simulator program will be try to be small and simple, so performance is not a priority. Any reasonable implementation will be very quick and use minimal storage. Rather than performance, the non-functional requirement priorities are correctness, understandability, reusability, maintainability, and portability.

NF1: Multi-Pendulum Simulator program aceess axis labels & 3D cartesian coordinates.

5 Likely Changes

LC1: Generation of diagrams using distributed/parallel computing

References

- [1] Pendulum
  `https://en.wikipedia.org/wiki/Pendulum`
- [2] Pendulum (mathematics)
  `https://en.wikipedia.org/wiki/Pendulum_(mathematics)`
- [3] Double Pendulum
  `https://en.wikipedia.org/wiki/Double_pendulum`
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  `http://www.cas.mcmaster.ca/~nedialk/daets/`
- [5] Multi-body Lagrangian Simulations
  `https://www.youtube.com/channel/UCCuLchOx0W0yoNE9KOCYlVQ`
- [6] The double pendulum: Lagrangian formulation
  `https://diego.assencio.com/?index=1500c66ae7ab27bb0106467c68feebc6`