ICCS200: Assignment 6

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4: Quick Sort Recurrence

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ii) $g(n) = \frac{f(n)}{n+1} = \frac{2}{n+1} + \frac{f(n-1)}{n} = \frac{2}{n+1} + g(n-1)$ and by using $f(0) = 0$ we know that $g(1) = 1$

iii)

$$g(n) = \tfrac{2}{n+1} + g(n-1)$$
$$g(n) = \tfrac{2}{n+1} + \tfrac{2}{n} + g(n-2)$$
$$g(n) = \tfrac{2}{n+1} + \tfrac{2}{n} + \tfrac{2}{n-1} + g(n-3)$$

so we can see that $g(n) = \frac{2}{n+1} + \frac{2}{n} + \frac{2}{n-1} + ... + \frac{2}{3} + 2 = \frac{2}{n+1} + \frac{2}{n} + \frac{2}{n-1} + ... + \frac{2}{3} + \frac{2}{2} + 2 - \frac{2}{2}$
$g(n) = 2H_{n+1} - 1$

iv) from $g(n) = \frac{f(n)}{n+1}$ so $f(n) = (n+1)g(n)$ and we know that $g(n) = 2H_{n+1} - 1$ and $H_{n+1} \leq 1 + ln(n+1)$ now let combine everything together.
$f(n) = (n+1)(2H_{n+1} - 1)$
$f(n) = 2nH_{n+1} - n + 2H_{n+1} - 1$
so the worst case of $f(n)$ is
$f(n) = 2n(1 + ln(n+1)) - n + 2(1 + ln(n+1)) - 1$
$f(n) = 2n + 2nln(n+1) - n + 2 + 2ln(n+1) - 1$
$f(n) = 2nln(n+1) + n + 2ln(n+1) + 1$
now we will show that $f(n) \in O(nln(n))$

$$\lim_{n\to\infty} \frac{f(n)}{nln(n)}$$

$$\lim_{n\to\infty} \frac{2nln(n+1) + n + 2ln(n+1) + 1}{nln(n)}$$

$$\lim_{n\to\infty} \frac{2nln(n+1)}{nln(n)} + \lim_{n\to\infty} \frac{n}{nln(n)} + \lim_{n\to\infty} \frac{2ln(n+1)}{nln(n)} + \lim_{n\to\infty} \frac{1}{nln(n)}$$

$$2 \lim_{n\to\infty} \frac{ln(n+1)}{ln(n)} + 0 + 0 + 0$$

as we know by simple application of L'Hospital rule that

$$\lim_{n\to\infty} \frac{ln(n+1)}{ln(n)} = 1$$

we will have that

$$\lim_{n\to\infty} \frac{f(n)}{nln(n)} = 2 < \infty$$

therefore, we can conclude that $f(n) \in O(nln(n))$