AE6310: Optimization for the Design of Engineered Systems
Assignment 3: Constraint Optimization Theory, Penalty Methods and Gradient Evaluation Techniques
Due: April 7th, 2020

Answer the following questions. Organize your work and be careful to properly answer all parts of each question. Points will be deducted for unorganized presentation of results.

1. (50 points) This question is based on the following optimization problem. Answer each question below for the following problem:

$$\begin{aligned} \min \quad & (x_1 + 2)^2 + 10(x_2 + 3)^2 \\ \text{subject to} \quad & x_1^2 + x_2^2 \leq 2 \end{aligned}$$

   (a) Using software such as scipy.minimzie or fmincon, find the constrained optimum and Lagrange multiplier values satisfying the KKT conditions.

   (b) Plot the contours of the quadratic penalty function for the penalty values $\rho = 5$, and $\rho = 10$

   (c) Plot the contours of the log-barrier penalty function for the penalty values $\mu = 0.5$ and $\mu = 0.1$

   (d) Using an optimization code, such as scipy.minimize or Matlab's fmincon, find the unconstrained optimizer for the quadratic and log-barrier penalty problems for a range of values of $\rho \in [1, 1000]$ and $\mu \in [1, 0.001]$. Using these results:

      i. Plot the number of objective function evaluations needed to find the unconstrained optimum vs. ρ for the quadratic penalty method, and vs. $1/\mu$ for the interior point method.

      ii. Plot the multiplier estimates vs. ρ and vs. $1/\mu$ ($\lambda \approx \rho c(x)$ for the quadratic penalty, and $\lambda \approx -\mu/c(x)$ for the log-barrier)

2. (50 points) You are given the following system of governing equations where x_1, x_2, and x_3 are the design variables, and u_1 and u_2 are the state variables. Note that the design variables must be positive.

$$R(x, u) = \begin{bmatrix} (x_1 + x_3)u_1 + (x_3 - x_2)u_2 - 1 \\ (x_3 - x_2)u_1 + (x_1 + x_2 - x_3)u_2 - 1 \end{bmatrix} = 0$$

   The output function of interest is given as

$$f(x, u) = x_1 x_2 + \sqrt{u_1 u_2}$$

   Create a function which takes as input the three design variables, solves the governing equations and returns the value of the function of interest.

   (a) Pick nominal values for the design variables. Compare the relative error between forward-difference and a complex step derivative approximation using complex-step with a step size of $h = 10^{-30}$ as the exact derivative. Use a range of values of the step size from $h = 10^{-1}$ to $h = 10^{-30}$.

   (b) Formulate and write down the adjoint equations. Create a function to compute the total derivative using the adjoint method. Compare the accuracy of all three components of the gradient using complex-step.