AE6310: Optimization for the Design of Engineered Systems

Assignment 1: Unconstrained Optimization Theory and Line Search Methods

Due: February 6th, 2020

Answer the following questions. Organize your work and be careful to properly answer all parts of each question. Points will be deducted for unorganized presentation of results.

1. (20 points) Compute the standard form of the following quadratic functions. Sketch them and find any minimizers.

   (a) $f(x_1, x_2) = x_1^2 + x_1x_2 + x_2^2 - x_1 - x_2$

   (b) $f(x_1, x_2) = x_1^2 + 2x_1x_2 + x_2^2 - x_1 - x_2$

   (c) $f(x_1, x_2) = 3x_1^2 + 2x_1x_2 + 3x_2^2 - x_1 - x_2$

   (d) $f(x_1, x_2) = 4x_1^2 + 6x_1x_2 + 2x_2^2 + x_1 + x_2$

2. (20 points) Plot the following functions and find and characterize their critical points. Justify your answers.

   (a) $f(x_1, x_2) = x_1^4 + x_2^4 + 1 - x_1^2 + x_2^2$

   (b) $f(x_1, x_2) = x_1^2 + x_2^2 + 2x_1x_2$

   (c) $f(x_1, x_2) = 4x_1^2 + x_2^2 + x_1x_2 + x_1$

   (d) $f(x_1, x_2) = x_1^4 + x_2^2 + 2x_1x_2 - x_1 - x_2$

3. (20 points) Consider the function $f(x) = x(1-x)^2(x-3)$ on the interval $x \in [-1, 4]$. From the point $x = 0$, find a descent direction.

   (a) Plot the function and the intervals in x that satisfy the sufficient decrease condition for $c_1 = 0.01$, $c_1 = 0.1$ and $c_1 = 0.5$ from the point $x = 0$ on a single plot.

   (b) Plot the function and the intervals that satisfy the second Wolfe condition for $c_2 = 0.9$, $c_2 = 0.5$ and $c_2 = 0.1$ from the point $x = 0$ on a single plot.

   (c) Plot the function and the intervals that satisfy the strong Wolfe conditions for the constants $c_1 = 0.01, c_2 = 0.9$, and $c_1 = 0.1, c_2 = 0.5$ from the point $x = 0$ on a single plot.

   Tip: Use these results to test your line search algorithm for question 3.

4. (40 points) The following question is based on the following two functions:

$$f(x_1, x_2) = x_1^4 + x_2^4 + 1 - x_1^2 + x_2^2$$
$$f(x_1, x_2) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$$

   (a) Implement the steepest descent method and the conjugate gradient method using either a sufficient decrease or strong Wolfe line search.

   (b) Compare the performance of these two algorithms from different starting points. Comment on the difference in performance you observe.