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Klaus Schittkowski

More Test Examples for Nonlinear Programming Codes

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Preface

This collection of 188 nonlinear programming test examples is a supplement of the test problem collection published by Hock and Schittkowski [2]. As in the former case, the intention is to present an extensive set of nonlinear programming problems that were used by other authors in the past to develop, test or compare optimization algorithms. There is no distinction between an "easy" or "difficult" test problem, since any related classification must depend on the underlying algorithm and test design. For instance, a nonlinear least squares problem may be solved easily by a special purpose code within a few iterations, but the same problem can be unsolvable for a general nonlinear programming code due to ill-conditioning. Thus one should consider both collections as a possible offer to choose some suitable problems for a specific test frame.

One difference between the new collection and the former one published by Hock and Schittkowski [2], is the attempt to present some more realistic or "real world" problems. Moreover a couple of nonlinear least squares test problems were collected which can be used e.g. to test data fitting algorithms. The presentation of the test problems is somewhat simplified and numerical solutions are computed only by one nonlinear programming code, the sequential quadratic programming algorithm NLPQL of Schittkowski [3]. But both test problem collections are implemeted in the same way in form of special FORTRAN-subroutines, so that the same test programs can be used. Again the implemeted FORTRAN-subroutines can be obtained from the author on a magnetic tape.

Chapter I shows how the test problems are documented. A classification scheme is defined allowing a formal description of the general type of the problem functions. Detailed information on the usage of the corresponding FORTRAN-subroutines is found in Chapter II for those who received them from the author on a magnetic tape. A list of some characteristic problem data is presented in Chapter III, and the sub-

sequent one contains the detailed description of all test problems using a fixed documentation scheme. Two appendices describe some problem data that did not fit into the documentation scheme, and list some numerical performance results of the used nonlinear programming code.

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Chapter I

THE DOCUMENTATION SCHEME

The development of algorithms for solving the nonlinear programming problem

$$\begin{array}{lll} & \min f(x) & \\ & g_j(x) \geq 0, & j = 1,\ldots,m_1 \\ x \in \mathbb{R}^n: & g_j(x) = 0, & j = m_1+1,\ldots,m \\ & x_l \leq x \leq x_u & \end{array} \qquad \text{(NLP)}$$

with continuously differentiable functions f and $g_1,\ldots,g_m$ requires the availability of test examples. The intention is to present a collection of optimization problems which can be used by a test designer to choose from either to develop a new algorithm or to become familar with an existing code written by somebody else. All problems were found in the literature and have been used in the past to test or compare optimization software.

To be able to characterize the basic structure of the problem functions, the classification scheme of Bus [1] is extended. Each problem is described by a sequence of letters and numbers of the form

OCD-Kr-s.

The following lists give all possible abbreviations which could replace the letters O, C, D, and K.

O	Information on objective function
C	Constant objective function
L	Linear objective function
Q	Quadratic objective function
S	Sum of squares
P	Generalized polynomial objective function
G	General objective function

C	Information on constraint functions
U	Unconstrained problem
B	Upper and lower bounds only
L	Linear constraint functions
Q	Quadratic constraint functions
P	Generalized polynomial constraint functions
G	General constraint functions

D	Regularity of the problem
R	Regular problem
I	Irregular problem

K	Information on the solution
T	Exact solution known ("theoretical" problem)
P	Exact solution not known ("practical" problem)

A problem is called a regular one, if the first and second derivatives of all problem functions exist in the feasible region, otherwise an irregular one. $k = p$ means that the solution of the problem can be obtained only numerically, i.e. that neither the exact numerical values nor arithmetic expressions are available. Most of these problems possess some practical background. The number r informs which partial derivatives are available in the corresponding subroutine package. In most cases, we have $r = 1$ indicating that first derivatives can be retrieved analytically. In all other cases, r is set to zero showing that gradients are not included. Finally the number s is replaced by the current serial number within the class or test problems identified by OCD-Kr.

To give an example, consider the problem

$$x_1, x_2: \begin{array}{l} \min -x_2 \\ 1 + x_1 - 2x_2 \geq 0 \\ x_1^2 + x_2^2 - 1 = 0 \\ x_1 \geq 0. \end{array}$$

Since the exact solution is given by (0.6,0.8) and since it is the second problem of its class, we classify this problem by

LQR - T1 - 2.

All test problems are documented in a predetermined format which contains the following information:

PROBLEM: The field contains the problem number and, if available, a specific name of the problem or a few words describing the practical background.

CLASSIFICATION: Using the classification scheme of the last section, the problem is identified in the form OCD-Kr-s. If several problems are distinguished only by some parameters not changing the classification scheme, we present this number in the form OCD-Kr-$(s_1,\ldots,s_k)$.

SOURCE: Some references indicate where the problem was found and where additional information or test results are given.

NUMBER OF VARIABLES: Contains the dimension of the problem n.

NUMBER OF CONSTRAINTS: Contains the number of all inequality constraints (m_1), of all equality constraints $(m-m_1)$, and of all upper and lower bounds of the variables (b). If there are linear restrictions, their number is presented in brackets behind (m_1) or $(m-m_1)$, respectively.

OBJECTIVE FUNCTION: The objective function f(x) is defined.

CONSTRAINTS: The restrictions are described in the form

$$
\begin{aligned}
g_1(x) &\geq 0 \\
&\vdots \\
g_{m_1}(x) &\geq 0 \\
g_{m_1+1}(x) &= 0 \\
&\vdots \\
g_m(x) &= 0.
\end{aligned}
$$

If the description of the problem functions requires an extensive set of constant data, then they are summarized in Appendix A.

LOWER BOUNDS: Definition of lower bounds for the variables, i.e. of the vector x_l.

UPPER BOUNDS: Definition of upper bounds for the variables, i.e. of the vector x_u.

START: The starting point x_o and the corresponding objective function value $f(x_o)$ are presented together with the information, whether x_o is feasible or not.

SOLUTION: This part of the documentation scheme contains the solution data of a test problem:

x^*: Optimal solution (exact or computed)

$f(x^*)$: Objective function value

$r(x^*)$: Sum of constraint violations.

To give an example, consider the test problem defined above. Since it got the number 217 within the series it is described in the following format:

PROBLEM: 217

CLASSIFICATION: LQR-T1-2

SOURCE: 63

NUMBER OF VARIABLES : N = 2

NUMBER OF CONSTRAINTS: M1 = 1(1) ,M-M1 = 1 ,B = 1

OBJECTIVE FUNCTION :

$$f(x) = -x_2$$

CONSTRAINTS :

$$g_1(x) = 1 + x_1 - 2x_2 \geq 0$$

$$g_2(x) = x_1^2 + x_2^2 - 1 = 0$$

LOWER BOUNDS : XL = (0 , -)

UPPER BOUNDS : XU = (- , -)

START: X = (10, 10)

F(X) = -10 (NOT FEASIBLE)

SOLUTION: X* = (0.6 , 0.8)

F(X*) = -0.8

R(X*) = 0

Chapter II

USAGE OF THE FORTRAN SUBROUTINES

This chapter describes the organization of the FORTRAN subroutines and informs the user on the way how to execute the test problems. Since it is assumed that at least a subset of the problems is used within a series of test runs for different optimization programs, the problems are coded in a very flexible manner. For example, it is possible to execute an arbitrary subset of the restrictions. To distinguish between linear and nonlinear constraints, we have to define a fixed succession of the restrictions in the following way:

$$g_j(x) \geq 0, \quad j = 1,\ldots,m_{11}, \qquad \text{linear functions,}$$
$$g_j(x) \geq 0, \quad j = m_{11}+1,\ldots,m_1, \qquad \text{nonlinear functions,}$$
$$g_j(x) = 0, \quad j = m_1+1,\ldots,m_{21}, \qquad \text{linear functions,}$$
$$g_j(x) = 0, \quad j = m_{21}+1,\ldots,m, \qquad \text{nonlinear functions.}$$

A test problem is set up by

```
call TPno(MODE),
```

where no is replaced by the actual problem number. The parameter MODE describes the five possible operations of the subroutine:

MODE = 1: The driving program will be provided with all information necessary to initialize an optimization program for the solution of the test problem, i.e. dimension, type and number of constraints, upper and lower bounds, starting point, derivatives of the linear constraints, and, in particular, the exact or computed optimal solution.

MODE = 2: The objective function f(x) is computed at a current iterate x.

MODE = 3: The gradient of the objective function will be computed.

MODE = 4: A predetermined subset of the constraints $g_1(x),\ldots,g_m(x)$ is evaluated at the iterate x.

MODE = 5: The gradients of a predetermined subset of the nonlinear constraints are computed.

The information on the test problem is delivered in the following common-blocks which have to be defined in the driving program with appropriate array dimensions:

COMMON/L1/N,NILI,NINL,NELI,NENL: A call of TPno(1) gives on return the data:

N	Dimension of the problem, i.e. n.
NILI	Number of linear inequalities, i.e. m_{11}.
NINL	Number of nonlinear inequalities, i.e. m_1-m_{11}.
NELI	Number of linear equalities, i.e. $m_{21}-m_1$.
NENL	Number of nonlinear equalities, i.e. $m-m_{21}$.

COMMON/L2/X(n): For MODE = 1, X will be set to the starting point x_o. For MODE > 1, X must contain the argument x for which the problem functions or derivatives are to be computed.

COMMON/L3/G(m): For all indices j with INDEX1(j) = .TRUE., G(j) is set to the j-th constraint value $g_j(x)$. Action only for MODE = 4.

COMMON/L4/GF(n): Contains the gradient of the objective function on return, i.e. GF(i) = $\frac{\partial}{\partial x_i}f(x)$, i = 1,...,n. Action only for MODE = 3.

COMMON/L5/GG(m,n): For MODE = 1, all constant partial derivatives are stored in GG. In particular, the rows $1,\ldots,m_{11}$ and $m_1+1,\ldots,m_{21}$ of GG store the constant derivatives of the linear constraints. For MODE = 5, the j-th row of GG defined by INDEX2 = .TRUE. will be replaced by the gradient of the j-th restriction, i.e. GG(j,i) = $\frac{\partial}{\partial x_i}g_j(x)$ if this term is not constant. Since all array dimensions of the common blocks are defined by the exact values of n or m, respectively, we recommend to define GG as a one-dimensional array in the driving program and to use it there in the form

$$GG((i-1)*m+j) = \frac{\partial}{\partial x_i}g_j(x).$$

COMMON/L6/FX: For MODE = 2, FX contains the objective function value f(x) on return.

COMMON/L9/INDEX1(m): The logical array INDEX1 has to be predetermined by the user before a call of TPno(4) and defines the restrictions which are to be computed in the case MODE = 4. INDEX1 is not changed by the subroutine.

COMMON/L10/INDEX2(m): The logical array INDEX2 has to be predetermined by the user before a call of TPno(5), and defines the gradients of the nonlinear restrictions which are to be computed in the case MODE = 5. INDEX2 is not changed during a call of TPno.

COMMON/L11/LXL(n): The logical array LXL informs about the existence of lower bounds. If there is a lower bound for the i-th variable, LXL(i) is set to .TRUE. during a call of TPno(1). Otherwise, we find LXL(i) = .FALSE..

COMMON/L12/LXU(n): Same for the existence of upper bounds.

COMMON/L13/XL(n): If LXL(i) = .TRUE., XL(i) is replaced by a lower bound for the i-th variable during a call of TPno(1).

COMMON/L14/XU(n): If LXU(i) = .TRUE., XU(i) is set to an upper bound for the i-th variable during a call of TPno(1).

COMMON/L15/LSUM: If the objective function is a sum of l squared functions, then LSUM stores the value l when executing TPno(1).

COMMON/L16/F(l): When calling TPno with MODE = 2, the l-dimensional array F contains the function values $f_i(x)$, x taken from /L2/, if f(x) is the sum of l squared functions $f_1(x),\ldots,f_l(x)$.

COMMON/L17/DF(l,n): Execution of TPno(3) for a sum of squares objective function will fill the array DF(l,n) with partial derivatives of the l functions $f_1(x),\ldots,f_l(x)$, i.e.

$$DF(i,j) = \frac{\partial}{\partial x_j} f_i(x), \quad i = 1,\ldots,l, \quad j = 1,\ldots,n,$$

where x is found in /L2/.

COMMON/L20/LEX,NEX,FEX,XEX(NEX·n): L20 gives information on the optimal solution of the problem and is set during a call of TPno(1). If LEX = .FALSE., a precise solution is not known a priori and x* stores the best computed solution known to the author. Otherwise, we have LEX = .TRUE.. NEX gives the number of all optimal solutions. NEX = -1 indicates that infinitely many solutions are present. FEX contains the minimal objective function value and XEX the j-th optimal solution in XEX(n·(j-1)+i), i = 1,...,n, j = 1,...,NEX. In the case NEX = -1, XEX contains only one arbitrary solution.

To give an example, we consider our standard test problem TP217. The corresponding FORTRAN-subroutine TP217(MODE) is implemented then as follows:

```
      SUBROUTINE TP217(MODE)
      COMMON/L1/N,NILI,NINL,NELI,NENL
      COMMON/L2/X(2)
      COMMON/L3/G(2)
      COMMON/L4/GF(2)
      COMMON/L5/GG(2,2)
      COMMON/L6/FX
      COMMON/L9/INDEX1
      COMMON/L10/INDEX2
      COMMON/L11/LXL
      COMMON/L12/LXU
      COMMON/L13/XL(2)
      COMMON/L14/XU(2)
      COMMON/L20/LEX,NEX,FEX,XEX(2)
      LOGICAL LEX,LXL(2),LXU(2),INDEX1(2),INDEX2(2)
      REAL*8 X,G,GF,GG,FX,XL,XU,FEX,XEX
      GOTO(1,2,3,4,5),MODE
    1 N=2
      NILI=1
      NINL=0
      NELI=0
      NENL=1
      X(1)=10.D+0
      X(2)=10.D+0
      LXU(1)=.FALSE.
      LXU(2)=.FALSE.
      LXL(1)=.TRUE.
      LXL(2)=.FALSE.
      XL(1)=0.D+0
      GG(1,1)= 1.D+0
      GG(1,2)= -2.D+0
      LEX=.TRUE.
      NEX=1
      FEX=-0.8D+0
      XEX(1)=0.6D+0
      XEX(2)=0.8D+0
      RETURN
    2 FX=-X(2)
      RETURN
```

```
3 GF(1)=0.D+0
  GF(2)=-1.D+0
  RETURN
4 IF (INDEX1(1)) G(1)=1.D+0+X(1)-2.D+0*X(2)
  IF (INDEX1(2)) G(2)=X(1)**2+X(2)**2-1.D+0
  RETURN
5 IF (.NOT.INDEX2(2)) GOTO 8
  GG(2,1)=2.D+0*X(1)
  GG(2,2)=2.D+0*X(2)
8 RETURN
  END
```

Chapter III

CONDENSED INFORMATION ON THE TEST PROBLEMS

To give a first survey of the test examples and their solution properties, a comprehensive list of all problems is presented in Table 1. Besides the current problem number and the classification number OCD-Kr-s as described in Chapter I, we report the dimension n, the number of all inequality constraints m_1, the number of all equality constraints $m-m_1$, and the number of all bounds b. If linear restrictions exist, their number is given in the brackets behind m_1 or $m-m_1$, respectively. The column headed by x_o gives the information, whether the starting point x_o is feasible (T) or not (F). Moreover, the objective function value $f(x^*)$ is displayed, where x^* denotes either the exact solution, if known in advance, or a solution computed numerically by the nonlinear programming code NLPQL. Any numerical data not given in full length, are assumed to be exact, i.e. the remaining digits are zero.

Some information on the test problem classes is gathered in Table 2 where for each test problem class defined by OCD, the numbers of "theoretical" (T) and "practical" (P) problems are listed together with the corresponding serial numbers. Similarly, Table 3 shows an index of all test problems with respect to the type of the objective function and Table 4 an index with respect to the type of the constraints. To facilitate the search for test problems investigated by a specific author, Table 5 gives a list of all authors and of all corresponding test problems.

TP	OCD-KR-S	N	M1	M-M1	B	XO	F(X*)
201	QUR-T1-1	2	0	0	0	T	0.
202	SUR-T1-1	2	0	0	0	T	0.
203	SUR-T1-2	2	0	0	0	T	0.
204	SUR-T1-3	2	0	0	0	T	0.183601
205	SUR-T1-4	2	0	0	0	T	0.
206	PUR-T1-1	2	0	0	0	T	0.
207	PUR-T1-2	2	0	0	0	T	0.
208	PUR-T1-3	2	0	0	0	T	0.
209	PUR-T1-4	2	0	0	0	T	0.
210	PUR-T1-5	2	0	0	0	T	0.
211	PUR-T1-6	2	0	0	0	T	0.
212	PUR-T1-7	2	0	0	0	T	0.
213	PUR-T1-8	2	0	0	0	T	0.
214	GUR-T1-1	2	0	0	0	T	0.
215	LQR-T1-1	2	1	0	1	T	0.
216	PQR-T1-1	2	0	1	0	F	1.
217	LQR-T1-2	2	1(1)	1	1	F	-0.8
218	LQR-T1-3	2	1	0	1	T	0.
219	LPR-T1-1	4	0	2	0	F	-1.
220	LPR-T1-2	2	1	0	0	T	1.
221	LPR-T1-3	2	1	0	2	T	-1.
222	LPR-T1-4	2	1	0	2	F	-1.5
223	LGR-T1-1	2	2	0	4	T	-0.83403
224	QLR-T1-1	2	4(4)	0	4	T	-304.
225	QQR-T1-1	2	5	0	0	T	2.
226	QQR-T1-2	2	2	0	2	T	-0.5
227	QQR-T1-3	2	2	0	0	T	1.
228	QQR-T1-4	2	2(1)	0	0	T	-3.
229	PBR-T1-1	2	0	0	4	T	0.
230	LPR-T1-5	2	2	0	0	F	0.375
231	PLR-T1-1	2	2(2)	0	0	T	0.
232	PLR-T1-2	2	3(3)	0	2	T	-1.
233	PQR-T1-2	2	1	0	0	T	0.
234	PQR-T1-3	2	1	0	4	F	-0.8
235	PQR-T1-4	3	0	1	0	T	0.04
236	GQR-T1-1	2	2	0	4	F	-58.9034
237	GQR-T1-2	2	3	0	3	F	-58.9034
238	GQR-T1-3	2	3	0	2	F	-58.9034
239	GQR-T1-4	2	1	0	4	F	-58.9034
240	QUR-T1-2	3	0	0	0	T	0.
241	SUR-T1-5	3	0	0	0	T	0.
242	SBR-T1-1	3	0	0	6	T	0.
243	SUR-T1-6	3	0	0	0	T	0.79660
244	SUR-T1-7	3	0	0	0	T	0.
245	SUR-T1-8	3	0	0	0	T	0.
246	SUR-T1-9	3	0	0	0	T	0.
247	GBR-T1-2	3	0	0	3	F	0.
248	LQR-T1-4	3	1(1)	1	0	T	-0.8
249	QQR-T1-5	3	1	0	1	T	1.
250	PLR-T1-3	3	2(2)	0	6	T	-3300.
251	PLR-T1-4	3	1(1)	0	6	T	-3456.
252	PQR-T1-5	3	0	1	1	F	0.04
253	GLR-T1-1	3	1(1)	0	3	T	87.3794
254	GQR-T1-5	3	0	2	1	F	-1.73205
255	QUR-T1-3	4	0	0	0	T	0.

Table 1: List of all test problems (continued)

TP	OCD-KR-S	N	M1	M-M1	B	XO	F(X*)
256	PUR-T1-9	4	0	0	0	T	0.
257	PBR-T1-2	4	0	0	2	F	0.
258	PUR-T1-10	4	0	0	0	T	0.
259	PBR-T1-3	4	0	0	1	T	0.
260	PUR-T1-11	4	0	0	0	T	0.
261	SUI-T1-1	4	0	0	0	T	0.
262	LLR-T1-1	4	3(3)	1(1)	4	T	-10.
263	LPR-T1-6	4	2	2	0	F	-1.
264	QQR-T1-6	4	3	0	0	T	-44.
265	GLR-T1-2	4	0	2(2)	4	F	0.97474
266	SUR-T1-10	5	0	0	0	T	1.
267	SUR-T1-11	5	0	0	0	T	0.
268	QLR-T1-2	5	5(5)	0	0	T	0.
269	SLR-T1-1	5	0	3(3)	0	F	4.09302
270	PQR-T1-6	5	1	0	4	T	-1.
271	SUR-T1-12	6	0	0	0	T	0.
272	SUR-T1-13	6	0	0	0	T	0.
273	PUR-T1-12	6	0	0	0	T	0.
274	QUR-T1-4	2	0	0	0	T	0.
275	QUR-T1-5	4	0	0	0	T	0.
276	QUR-T1-6	6	0	0	0	T	0.
277	LLR-T1-2	4	4(4)	0	4	F	5.07619
278	LLR-T1-3	6	6(6)	0	6	F	7.83852
279	LLR-T1-4	8	8(8)	0	8	F	10.6059
280	LLR-T1-5	10	10(10)	0	10	F	13.3754
281	GUR-T1-2	10	0	0	0	T	0.
282	SUR-T1-14	10	0	0	0	T	0.
283	PUR-T1-13	10	0	0	0	T	0.
284	LQR-T1-5	15	10	0	0	T	-1840.
285	LQR-T1-6	15	10	0	0	T	-8252.
286	SUR-T1-15	20	0	0	0	T	0.
287	PUR-T1-14	20	0	0	0	T	0.
288	SUR-T1-16	20	0	0	0	T	0.
289	GUR-T1-3	30	0	0	0	T	0.
290	PUR-T1-15	2	0	0	0	T	0.
291	PUR-T1-16	10	0	0	0	T	0.
292	PUR-T1-17	30	0	0	0	T	0.
293	PUR-T1-18	50	0	0	0	T	0.
294	SUR-T1-17	6	0	0	0	T	0.
295	SUR-T1-18	10	0	0	0	T	0.
296	SUR-T1-19	16	0	0	0	T	0.
297	SUR-T1-20	30	0	0	0	T	0.
298	SUR-T1-21	50	0	0	0	T	0.
299	SUR-T1-22	100	0	0	0	T	0.
300	QUR-T1-7	20	0	0	0	T	-20.
301	QUR-T1-8	50	0	0	0	T	-50.
302	QUR-T1-9	100	0	0	0	T	-100.
303	PUR-T1-19	20	0	0	0	T	0.
304	PUR-T1-20	50	0	0	0	T	0.
305	PUR-T1-21	100	0	0	0	T	0.
307	SUR-P1-1	2	0	0	2	T	124.360
308	SUR-P1-2	2	0	0	0	T	0.77319
309	PUR-P1-1	2	0	0	0	T	-3.98717
311	PUR-P1-2	2	0	0	0	T	0.

Table 1: List of all test problems (continued)

TP	OCD-KR-S	N	M1	M-M1	B	XO	F(X*)
312	PUR-P1-3	2	0	0	0	T	0.
314	GUR-P1-1	2	0	0	0	T	0.16904
315	LQR-P1-1	2	3(1)	0	0	T	-0.8
316	QQR-P1-1	2	0	1	0	F	334.315
317	QQR-P1-2	2	0	1	0	F	372.467
318	QQR-P1-3	2	0	1	0	F	412.750
319	QQR-P1-4	2	0	1	0	F	452.404
320	QQR-P1-5	2	0	1	0	F	485.531
321	QQR-P1-6	2	0	1	0	F	496.112
322	QQR-P1-7	2	0	1	0	F	499.96
323	QQR-P1-8	2	2(1)	0	2	T	3.79894
324	QQR-P1-9	2	2	0	1	F	5.
325	QQR-P1-10	2	2(1)	1	0	T	3.79134
326	QGR-P1-1	2	2	0	2	T	-79.8078
327	SQR-P1-1	2	1	0	2	T	0.03064
328	PBR-P1-1	2	0	0	4	F	1.74415
329	PQR-P1-1	2	3	0	4	T	-6961.81
330	PPI-P1-1	2	1	0	4	F	1.6205
331	GLI-P1-1	2	1(1)	0	3	T	4.258
332	GGR-P1-1	2	2	0	4	T	114.95
333	SUR-P1-3	3	0	0	0	T	0.0432
334	SUR-P1-4	3	0	0	0	T	0.8215E-2
335	LQR-P1-2	3	0	2	0	F	-0.4472E-2
336	LQR-P1-3	3	0	2(1)	0	F	-0.33789
337	QQR-P1-11	3	1	0	2	T	6.
338	QQR-P1-12	3	0	2(1)	0	F	-7.20570
339	PQR-P1-2	3	1	0	3	T	3.36168
340	PLR-P1-1	3	1(1)	0	1	F	-0.054
341	PQR-P1-3	3	1	0	3	T	-22.6274
343	PPR-P1-1	3	2	0	6	F	-5.68478
344	PPR-P1-2	3	0	1	0	F	0.03257
345	PPR-P1-3	3	0	1	0	F	0.03257
346	PPR-P1-4	3	2	0	6	F	-5.68478
347	GLR-P1-1	3	1	0	6	T	17374.6
348	GGR-PO-2	3	0	1	4	F	36.9708
350	SUR-P1-5	4	0	0	0	T	0.3075E-3
351	SUR-P1-6	4	0	0	0	T	318.581
352	SUR-P1-7	4	0	0	0	T	903.234
353	LGR-P1-1	4	2(1)	1(1)	4	T	-39.9337
354	PLR-P1-2	4	1(1)	0	4	T	0.11378
355	PPR-P1-5	4	0	1	4	F	69.6755
356	PGR-PO-1	4	5(1)	0	3	T	2.38116
357	GBR-PO-1	4	0	0	8	T	0.35845
358	SBR-P1-1	5	0	0	10	T	0.5460E-4
359	LLR-P1-1	5	14(14)	0	0	T	-5280420
360	QQR-P1-13	5	2	0	9	T	-5280340
361	QQR-P1-14	5	6	0	8	F	776412.
362	GLR-PO-2	5	4(4)	0	3	T	0.26230
364	GGR-PO-4	6	4(2)	0	6	T	0.06060
365	QGR-PO-2	7	5	0	4	F	23.3137
366	PPR-PO-6	7	14	0	14	T	-704.306
367	GQR-P1-1	7	3(2)	2(1)	7	T	-37.4130
368	PBR-P1-2	8	0	0	16	T	-0.74997
369	LPR-P1-1	8	6(3)	0	16	F	0.

Table 1: List of all test problems (continued)

TP	OCD-KR-S	N	M1	M-M1	B	XO	F(X*)
370	SUR-PO-8	6	0	0	0	T	0.2297E-2
371	SUR-PO-9	9	0	0	0	T	0.1340E-5
372	SGR-P1-1	9	12	0	6	F	13390.1
373	SGR-P1-2	9	0	6	0	F	13390.1
374	LPR-P1-2	10	35	0	0	F	0.23326
375	SQR-P1-2	10	0	9(8)	0	F	-15.1610
376	GPR-PO-1	10	14	1(1)	20	T	-4430.09
377	GLR-PO-3	10	0	3(3)	20	F	-795.001
378	GGR-PO-5	10	0	3	0	F	-47.7632
379	SUR-P1-10	11	0	0	0	T	0.04014
380	PPR-PO-7	12	3	0	24	F	3.16858
381	LLR-P1-2	13	3(3)	1(1)	13	T	1.10149
382	LGI-P1-1	13	3	1(1)	13	T	1.03831
383	PLI-P1-1	14	0	1(1)	28	F	728565.
384	LQR-P1-4	15	10	0	0	T	-8310.26
385	LQR-P1-5	15	10	0	0	T	-8315.29
386	LQR-P1-6	15	11	0	0	T	-8164.37
387	LQR-P1-7	15	11	0	0	T	-8250.14
388	LQR-P1-8	15	15(4)	0	0	T	-5821.08
389	LQR-P1-9	15	15(4)	0	0	T	-5809.72
391	GUR-PO-2	30	0	0	0	T	0.
392	QLR-PO-1	30	45(45)	0	30	F	-1698880
393	PPR-PO-8	48	1	2(2)	72	T	0.86337
394	PQR-P1-5	20	0	1	0	F	1.91666
395	PQR-P1-6	50	0	1	0	F	1.91666

Table 1: List of all test problems

OCD	T	P	TP
LLR	5	2	262, 277, 278, 279, 280, 359, 381
LQR	6	9	215, 217, 218, 248, 284, 285, 315, 335, 336, 384, 385, 386, 387, 388, 389
LPR	6	2	219, 220, 221, 222, 230, 263, 369, 374
LGR	1	1	223, 353
LGI	0	1	382
QUR	9	0	201, 240, 255, 274, 275, 276, 300, 301, 302
QLR	2	1	224, 268, 392
QQR	6	14	225, 226, 227, 228, 249, 264, 316-325, 337, 338, 360, 361
QGR	0	2	326, 365
SUR	22	10	202-205, 241, 243-246, 266, 267, 271, 272, 282, 286, 288, 294-299, 307, 308, 333, 334, 350, 351, 352, 370, 371, 379
SUI	1	0	261
SBR	1	1	242, 358
SLR	1	0	269
SQR	0	2	327, 375
SGR	0	2	372, 373

Table 2: Test problem classes (continued)

OCD	T	P	TP
PUR	21	3	206-213, 256, 258, 260, 273, 283, 287, 290-293, 303, 304, 305, 309, 311, 312
PBR	3	2	229, 257, 259, 328, 368
PLR	4	2	231, 232, 250, 251, 340, 354
PLI	0	1	383
PQR	6	5	216, 233, 234, 235, 252, 270, 329, 339, 341, 394, 395
PPR	0	8	343-346, 355, 366, 380, 393
PPI	0	1	330
PGR	0	1	356
GUR	3	2	214, 281, 289, 314, 391
GBR	1	1	247, 357
GLR	2	3	253, 265, 347, 362, 377
GLI	0	1	331
GQR	5	1	236-239, 254, 367
GPR	0	1	376
GGR	0	4	332, 348, 364, 378

Table 2: Test problem classes

Type of objective functions	Test problems (TP)
Linear	215, 217-223, 230, 248, 262, 263, 277-280, 284, 285, 315, 335, 336, 353, 359, 369, 374, 381, 382, 384-389
Quadratic	201, 224-228, 240, 249, 255, 264, 268, 274-276, 300-302, 316-326, 337, 338, 360, 361, 365, 392
Sum of squares	202-205, 241-246, 261, 266, 267, 269, 271, 272, 282, 286, 288, 294-299, 307, 308, 327, 333, 334, 350-352, 358, 370-373, 375, 379
Generalized polynomial	206-213, 216, 229, 231-235, 250-252, 256-260, 270, 273, 283, 287, 290-293, 303-305, 309, 311, 312, 328-330, 339-341, 343-346, 354, 355, 356, 366, 368, 380, 383, 393-395
General	214, 236-239, 247, 253, 254, 265, 281, 289, 314, 331, 332, 347, 348, 357, 362, 364, 367, 376-378, 391

Table 3: Type of objective function

Type of constraint functions	Test problems (TP)
Unconstrained	201-214, 240, 241, 243-246, 255, 256, 258, 260, 261, 266, 267, 271-276, 281-283, 286-305, 307-309, 311, 312, 314, 333, 334, 351, 352, 370, 371, 379, 391
Upper and lower bounds	229, 242, 247, 257, 259, 328, 357, 358, 368
Linear	224, 231, 232, 250, 251, 253, 256, 262, 268, 269, 277-280, 331, 340, 347, 354, 359, 362, 377, 381, 383, 392
Quadratic functions	215-218, 225-228, 233-239, 248, 249, 252, 254, 264, 270, 284, 285, 315-325, 327, 329, 335-339, 341, 360, 361, 367, 375, 384-389, 394, 395
Generalized polynomials	219-222, 230, 263, 330, 343-346, 355, 366, 369, 374, 376, 380, 393
General	223, 326, 332, 348, 353, 356, 364, 365, 372, 373, 378, 382

Table 4: Type of constraint functions

Author	Test problems (TP)
Ballintijn, van der Hoek, Hoykaas	341, 372, 373
Bard	334
Bass	303-305
Beale	203, 205
Beightler, Ta-Chen, Rylander	330
Beltrami, Indusi	255, 258, 265, 309
Betts	203, 242
Biggs	244, 267
Box	242, 245, 251, 360
Bracken, McCormick	314, 381
Branin	213
Brent	203, 207-211, 245, 247, 256, 258, 274-280, 300-302, 370, 371
Chamberlain	227, 230
Charnes, Cooper	382
Colville	258, 260
Cornwell, Hutchinson, Minkoff, Schultz	236, 270, 285, 384-389
Cragg, Levy	261
Danskin	265
Davidon	352
Davies	341, 342
Duffin, Peterson, Zener	339

Table 5: Test problem authors (continued)

Author	Test problems (TP)
Eason, Fenton	229, 250, 328, 330, 332, 343, 346, 357
Enchorolt	223, 224
Eilers	333
Engvall	241
Fiacco, McCormick	221
Fletcher, Powell	247
Freudenstein, Roth	202
Gesing, Davison	223, 224, 226, 250, 315, 327, 329,
Gill, Murray	202, 203, 207-210, 212, 240-242, 247, 256, 260, 261, 307, 308, 334, 350, 352, 358, 379
Gould	327, 329
Gregory, Karney	300-302
Himmelblau	201, 203, 205-211, 228, 236-239, 246, 247, 253, 256, 259, 261, 311-314, 323, 325, 326, 336, 351
van der Hoek, Dijkshoorn	207-210, 274-276, 290-299
van der Hoek	207-210, 221, 231, 233, 251, 268, 274-276, 290-299, 341, 342, 360, 372, 373
Horntrum	215, 218, 264, 284
Jennrich, Sampson	307, 308
Mc Keown	204, 243, 266
Kowalik, Osborne	350, 370, 371

Table 5: Test problem authors (continued)

Author	Test problems (TP)
Leon	211
Levy, Guerra	219, 235, 269, 316-322, 324, 338, 345, 375
Lowe	381
Miele, Tietze, Levy	252, 254, 263
Mischke	328, 332
Oren	274-276, 290-299
Osborne	358, 379
Pierre	248, 262, 355
Pierre, Lowe	207-211, 222, 244, 248, 256, 262, 267, 290-299, 339, 340, 355, 367, 368, 381, 382
Pierre, Dudding	248, 258, 315, 374
Powell	256
Rosen, Krenser	284, 285, 384-389
Rosen, Suzuki	264
Rosenbrock	207-210, 229, 250, 294-299
Rosenbrock, Storey	211
Sandgren	229, 250, 328, 330, 332, 343, 348, 356, 357, 362, 364, 366, 369, 376, 380, 393
Schweigman	231, 233
Siddall	343, 346
Spanlang	207-210, 221, 331, 383, 392, 394, 395

Table 5: Test problem authors (continued)

Author	Test problems (TP)
Stoer	268
Walukiewicz	213, 214, 220, 225, 232, 234, 249, 257, 261, 269, 271-273, 281-283, 286-289, 327, 335, 337, 341, 344, 347, 353, 354, 359, 361, 365, 377, 378, 391
Witte, Holst	206-211
Wolfe	367, 368
Zakharov	207-211, 247, 256, 303-305
Zangwill	240, 339

Table 5: Test problem authors

Chapter IV

THE TEST PROBLEMS

Using the documentation scheme proposed in Chapter I, we present a detailed description of 188 problems for testing nonlinear programming algorithms. The corresponding FORTRAN-subroutines are available on request. Extensive lists of constant data, solution results or function evaluatives are shown in Appendix A.

It was impossible to retain all solution results that were published in the sources together with the test problems. Particularly the implementation of more complex test problems defined by certain subprograms, was quite cumbersome. In some cases variables were undefined in the source or at least typing errors occured, so that even the computed objective function value with respect to the reported solution, was different from the published function value. For all "practical" test problems, it was therefore decided to present only numerical results obtained by the actual implementation, i.e. the nonlinear programming code NLPQL. It might be possible, that in the one or other case only a local solution was approximated and that a better global solution does exist.

PROBLEM:	201
CLASSIFICATION:	QUR-T1-1
SOURCE:	41
NUMBER OF VARIABLES :	N = 2
NUMBER OF CONSTRAINTS:	M1 = 0 ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = 4(x_1 - 5)^2 + (x_2 - 6)^2$$

CONSTRAINTS :

LOWER BOUNDS : XL = (- , -)

UPPER BOUNDS : XU = (- , -)

START:	X	= (8, 9)	
	F(X)	= 45	(FEASIBLE)
SOLUTION:	X*	= (5, 6)	
	F(X*)	= 0	
	R(X*)	= 0	

PROBLEM: 202

CLASSIFICATION: SUR-T1-1

SOURCE: 37/34

NUMBER OF VARIABLES : N = 2

NUMBER OF CONSTRAINTS: M1 = 0 ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = [-13 + x_1 - 2x_2 + 5x_2^2 - x_2^3]^2 + [-29 + x_1 - 14x_2 + x_2^2 + x_2^3]^2$$

CONSTRAINTS :

LOWER BOUNDS : XL = (- , -)

UPPER BOUNDS : XU = (- , -)

START: X = (15, -2)

F(X) = 1256 (FEASIBLE)

SOLUTION: X* = (5, 4)

F(X*) = 0

R(X*) = 0

PROBLEM: 203

CLASSIFICATION: SUR-T1-2

SOURCE: 41/5/14/37/8

NUMBER OF VARIABLES : N = 2

NUMBER OF CONSTRAINTS: M1 = 0 ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = u_1^2 + u_2^2 + u_3^2$$

$$u_i = c_i - x_1(1 - x_2^i)$$

$$c_1 = 1.5 \ , \quad c_2 = 2.25 \ , \quad c_3 = 2.625$$

CONSTRAINTS :

LOWER BOUNDS : XL = (- , -)

UPPER BOUNDS : XU = (- , -)

START: X = (2, 0.2)

F(X) = 0.529781 (FEASIBLE)

SOLUTION: X* = (3, 0.5)

F(X*) = 0

R(X*) = 0

PROBLEM:	204
CLASSIFICATION:	SUR-T1-3
SOURCE:	46
NUMBER OF VARIABLES :	N = 2
NUMBER OF CONSTRAINTS:	M1 = 0 ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = \sum_{i=1}^{3} f_i^2(x) = F^T F$$

$$F = A + Hx + 0.5x^T BxD$$

A,H,B,D : cf. Appendix A

CONSTRAINTS :

LOWER BOUNDS : XL = (- , -)

UPPER BOUNDS : XU = (- , -)

START:	X = (0.1, 0.1)	
	F(X) = 0.190380	(FEASIBLE)
SOLUTION:	X* = (0, 0)	
	F(X*) = 0.183601	
	R(X*) = 0	

PROBLEM: 205

CLASSIFICATION: SUR-T1-4

SOURCE: 5/41

NUMBER OF VARIABLES : N = 2

NUMBER OF CONSTRAINTS: $M^1 = 0$, $M-M^1 = 0$, $B = 0$

OBJECTIVE FUNCTION :

$$f(x) = [1.5 - x_1(1 - x_2)]^2 + [2.25 - x_1(1 - x_2^2)]^2 + [2.625 - x_1(1 - x_2^3)]^2$$

CONSTRAINTS :

LOWER BOUNDS : XL = (- , -)

UPPER BOUNDS : XU = (- , -)

START: X = (0, 0)

F(X) = 14.2031 (FEASIBLE)

SOLUTION: X* = (3, 0.5)

F(X*) = 0

R(X*) = 0

PROBLEM:	206
CLASSIFICATION:	PUR-T1-1
SOURCE:	81/41
NUMBER OF VARIABLES :	N = 2
NUMBER OF CONSTRAINTS:	M1 = 0 ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = (x_2 - x_1^2)^2 + 100(1 - x_1)^2$$

CONSTRAINTS :

LOWER BOUNDS : XL = (- , -)

UPPER BOUNDS : XU = (- , -)

START:	X	= (-1.2 , 1)	
	F(X)	= 484.194	(FEASIBLE)
SOLUTION:	X*	= (1, 1)	
	F(X*)	= 0	
	R(X*)	= 0	

PROBLEM:	207 (Banana Function)
CLASSIFICATION:	PUR-T1-2
SOURCE:	42/63/81/14/41/83/71/43/77/37
NUMBER OF VARIABLES :	N = 2
NUMBER OF CONSTRAINTS:	M1 = 0 ,M-M1 = 0 ,S = 0
OBJECTIVE FUNCTION :	$f(x) = (x_2 - x_1^2)^2 + (1 - x_1)^2$
CONSTRAINTS :	
LOWER BOUNDS :	XL = (- , -)
UPPER BOUNDS :	XU = (- , -)
START:	X = (-1.2 , 1) F(X) = 5.03360 (FEASIBLE)
SOLUTION:	X* = (1, 1) F(X*) = 0 R(X*) = 0

PROBLEM:	208 (Banana Function)
CLASSIFICATION:	PUR-T1-3
SOURCE:	42/63/81/14/41/83/71/43/77/37
NUMBER OF VARIABLES :	N = 2
NUMBER OF CONSTRAINTS:	M1 = 0 ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$$

CONSTRAINTS :

LOWER BOUNDS : XL = (- , -)

UPPER BOUNDS : XU = (- , -)

START:	X = (-1.2 , 1)	
	F(X) = 24.2	(FEASIBLE)
SOLUTION:	X* = (1, 1)	
	F(X*) = 0	
	R(X*) = 0	

PROBLEM:	209 (Banana Function)
CLASSIFICATION:	PUR-T1-4
SOURCE:	42/63/81/14/41/83/71/43/77/37
NUMBER OF VARIABLES :	N = 2
NUMBER OF CONSTRAINTS:	M1 = 0 ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = 10^4(x_2 - x_1^2)^2 + (1 - x_1)^2$$

CONSTRAINTS :

LOWER BOUNDS : XL = (- , -)

UPPER BOUNDS : XU = (- , -)

START: X = (-1.2 , 1)

F(X) = 1940.84 (FEASIBLE)

SOLUTION: X* = (1, 1)

F(X*) = 0

R(X*) = 0

PROBLEM:	210 (Banana Function)
CLASSIFICATION:	PUR-T1-5
SOURCE:	42/63/81/14/41/83/71/43/77/37
NUMBER OF VARIABLES :	N = 2
NUMBER OF CONSTRAINTS:	M1 = 0 ,M-M1 = 0 ,B = 0
OBJECTIVE FUNCTION :	$f(x) = 10^6(x_2 - x_1^2)^2 + (1 - x_1)^2$
CONSTRAINTS :	
LOWER BOUNDS :	XL = (- , -)
UPPER BOUNDS :	XU = (- , -)
START:	X = (-1.2 , 1)
	F(X) = 193605 (FEASIBLE)
SOLUTION:	X* = (1, 1)
	F(X*) = 0
	R(X*) = 0

PROBLEM: 211

CLASSIFICATION: PUR-T1-6

SOURCE: 81/41/72/48/14/83/63

NUMBER OF VARIABLES : N = 2

NUMBER OF CONSTRAINTS: M1 = 0 ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = 100(x_2 - x_1^3)^2 + (1 - x_1)^2$$

CONSTRAINTS :

LOWER BOUNDS : XL = (- , -)

UPPER BOUNDS : XU = (- , -)

START: X = (-1.2 , 1)

F(X) = 749.038 (FEASIBLE)

SOLUTION: X* = (1, 1)

F(X*) = 0

R(X*) = 0

PROBLEM:	212
CLASSIFICATION:	PUR-T1-7
SOURCE:	37/13
NUMBER OF VARIABLES :	N = 2
NUMBER OF CONSTRAINTS:	M1 = 0 ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = [4(x_1 + x_2)]^2 + [4(x_1 + x_2) + (x_1 - x_2) * ((x_1 - 2)^2 + x_2^2 - 1)]^2$$

CONSTRAINTS :

LOWER BOUNDS : XL = (- , -)

UPPER BOUNDS : XU = (- , -)

START:	X = (2, 0)	
	F(X) = 100	(FEASIBLE)
SOLUTION:	X* = (0, 0)	
	F(X*) = 0	
	R(X*) = 0	

PROBLEM: 213

CLASSIFICATION: PUR-T1-8

SOURCE: 79

NUMBER OF VARIABLES : N = 2

NUMBER OF CONSTRAINTS: M1 = 0 ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = [\ 10(x_1 - x_2)^2 + (x_1 - 1)^2\ ]^4$$

CONSTRAINTS :

LOWER BOUNDS : XL = (- , -)

UPPER BOUNDS : XU = (- , -)

START: X = (3, 1)

F(X) = 0.374810E+07 (FEASIBLE)

SOLUTION: X* = (1, 1)

F(X*) = 0

R(X*) = 0

PROBLEM:	214
CLASSIFICATION:	GUR-T1-1
SOURCE:	79
NUMBER OF VARIABLES :	N = 2
NUMBER OF CONSTRAINTS:	M1 = 0 ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = [10(x_1 - x_2)^2 + (x_1 - 1)^2]^{1/4}$$

CONSTRAINTS :

LOWER BOUNDS : XL = (- , -)

UPPER BOUNDS : XU = (- , -)

START:	X	= (-1.2 , 1)	
	F(X)	= 2.70122	(FEASIBLE)
SOLUTION:	X*	= (1, 1)	
	F(X*)	= 0	
	R(X*)	= 0	

PROBLEM:	215
CLASSIFICATION:	LQR-T1-1
SOURCE:	44
NUMBER OF VARIABLES :	N = 2
NUMBER OF CONSTRAINTS:	M1 = 1 ,M-M1 = 0 ,B = 1

OBJECTIVE FUNCTION :

$$f(x) = x_2$$

CONSTRAINTS :

$$g_1(x) = x_2 - x_1^2 \geq 0$$

LOWER BOUNDS :	XL = (0 , -)
UPPER BOUNDS :	XU = (- , -)
START:	X = (1, 1)
	F(X) = 1 (FEASIBLE)
SOLUTION:	X* = (0, 0)
	F(X*) = 0
	R(X*) = 0

PROBLEM: 216

CLASSIFICATION: PQR-T1-1

SOURCE: 79

NUMBER OF VARIABLES : N = 2

NUMBER OF CONSTRAINTS: M1 = 0 ,M-M1 = 1 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = 100(x_1^2 - x_2)^2 + (x_1 - 1)^2$$

CONSTRAINTS :

$$g_1(x) = x_1(x_1 - 4) - 2x_2 + 12 = 0$$

LOWER BOUNDS : XL = (- , -)

UPPER BOUNDS : XU = (- , -)

START: X = (-1.2 , 1)

F(X) = 24.2 (NOT FEASIBLE)

SOLUTION: X* = (2, 4)

F(X*) = 1

R(X*) = 0

PROBLEM:	217
CLASSIFICATION:	LQR-T1-2
SOURCE:	63
NUMBER OF VARIABLES :	N = 2
NUMBER OF CONSTRAINTS:	M1 = 1(1) ,M-M1 = 1 ,B = 1

OBJECTIVE FUNCTION :

$$f(x) = -x_2$$

CONSTRAINTS :

$$g_1(x) = 1 + x_1 - 2x_2 \geq 0$$

$$g_2(x) = x_1^2 + x_2^2 - 1 = 0$$

LOWER BOUNDS : XL = (0 , -)

UPPER BOUNDS : XU = (- , -)

START:	X	= (10, 10)	
	F(X)	= -10	(NOT FEASIBLE)
SOLUTION:	X*	= (0.6 , 0.8)	
	F(X*)	= -0.8	
	R(X*)	= 0	

PROBLEM: 218

CLASSIFICATION: LQR-T1-3

SOURCE: 44

NUMBER OF VARIABLES : N = 2

NUMBER OF CONSTRAINTS: M1 = 1 ,M-M1 = 0 ,B = 1

OBJECTIVE FUNCTION :

$$f(x) = x_2$$

CONSTRAINTS :

$$g_1(x) = x_2 - x_1^2 \geq 0$$

LOWER BOUNDS : XL = (- , 0)

UPPER BOUNDS : XU = (- , -)

START: X = (9,100)

F(X) = 100 (FEASIBLE)

SOLUTION: X* = (0, 0)

F(X*) = 0

R(X*) = 0

PROBLEM: 219

CLASSIFICATION: LPR-T1-1

SOURCE: 49

NUMBER OF VARIABLES : N = 4

NUMBER OF CONSTRAINTS: M1 = 0 ,M-M1 = 2 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = -x_1$$

CONSTRAINTS :

$$g_1(x) = x_2 - x_1^3 - x_3^2 = 0$$

$$g_2(x) = x_1^2 - x_2 - x_4^2 = 0$$

LOWER BOUNDS : XL = (- , - , - , -)

UPPER BOUNDS : XU = (- , - , - , -)

START: X = (10, 10, 10, 10)

F(X) = -10 (NOT FEASIBLE)

SOLUTION: X* = (1, 1, 0, 0)

F(X*) = -1

R(X*) = 0

PROBLEM:	220
CLASSIFICATION:	LPR-T1-2
SOURCE:	79
NUMBER OF VARIABLES :	N = 2
NUMBER OF CONSTRAINTS:	M1 = 0 ,M-M1 = 1 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = x_1$$

CONSTRAINTS :

$$g_1(x) = (x_1 - 1)^3 - x_2 = 0$$

LOWER BOUNDS : XL = (1 , 0)

UPPER BOUNDS : XU = (- , -)

START:	X = (25000 , 25000)	
	F(X) = -25000	(FEASIBLE)
SOLUTION:	X* = (1, 0)	
	F(X*) = 1	
	R(X*) = 0	

PROBLEM: 221

CLASSIFICATION: LPR-T1-3

SOURCE: 31/43/77

NUMBER OF VARIABLES : N = 2

NUMBER OF CONSTRAINTS: M1 = 1 ,M-M1 = 0 ,B = 2

OBJECTIVE FUNCTION :

$$f(x) = -x_1$$

CONSTRAINTS :

$$g_1(x) = (1 - x_1)^3 - x_2 \geq 0$$

LOWER BOUNDS : XL = (0 , 0)

UPPER BOUNDS : XU = (- , -)

START: X = (0.25 ,0.25)

F(X) = -0.25 (FEASIBLE)

SOLUTION: X* = (1, 0)

F(X*) = -1

R(X*) = 0

PROBLEM:	222
CLASSIFICATION:	LPR-T1-4
SOURCE:	63
NUMBER OF VARIABLES :	N = 2
NUMBER OF CONSTRAINTS:	$M_1 = 1$, $M-M_1 = 0$, $B = 2$

OBJECTIVE FUNCTION :

$$f(x) = -x_1$$

CONSTRAINTS :

$$g_1(x) = 0.125 - x_2 + (1 - x_1)^3 \geq 0$$

LOWER BOUNDS : XL = (0 , 0)

UPPER BOUNDS : XU = (- , -)

START: X = (1.3 , 0.2)

F(X) = -1.3 (NOT FEASIBLE)

SOLUTION: X* = (1.5, 0)

F(X*) = -1.5

R(X*) = 0

PROBLEM:	223
CLASSIFICATION:	LGR-T1-1
SOURCE:	35/28
NUMBER OF VARIABLES :	N = 2
NUMBER OF CONSTRAINTS:	M1 = 2 ,M-M1 = 0 ,B = 4

OBJECTIVE FUNCTION :

$$f(x) = -x_1$$

CONSTRAINTS :

$$g_1(x) = \exp[\exp(x_1)] \geq 0$$

$$g_2(x) = x_2 - \exp[\exp(x_1)] \geq 0$$

LOWER BOUNDS : XL = (0 , 0)

UPPER BOUNDS : XU = (10, 10)

START:	X	= (0.1, 3.3)	
	F(X)	= -0.1	(FEASIBLE)
SOLUTION:	X*	= (0.834 , 10)	[= (ln(ln(10)),10)]
	F(X*)	= -0.834032	[= -ln(ln(10))]
	R(X*)	= 0	

PROBLEM:	224
CLASSIFICATION:	QLR-T1-1
SOURCE:	35/28
NUMBER OF VARIABLES :	N = 2
NUMBER OF CONSTRAINTS:	M1 = 4(4) ,M-M1 = 0 ,B = 4

OBJECTIVE FUNCTION :

$$f(x) = 2x_1^2 + x_2^2 - 48x_1 - 40x_2$$

CONSTRAINTS :

$$g_1(x) = x_1 + 3x_2 \geq 0$$
$$g_2(x) = 18 - x_1 - 3x_2 \geq 0$$
$$g_3(x) = x_1 + x_2 \geq 0$$
$$g_4(x) = 8 - x_1 - x_2 \geq 0$$

LOWER BOUNDS :	XL = (0 , 0)	
UPPER BOUNDS :	XU = (6 , 6)	
START:	X = (0.1, 0.1)	
	F(X) = -8.77	(FEASIBLE)
SOLUTION:	X* = (4, 4)	
	F(X*) = -304	
	R(X*) = 0	

PROBLEM:	225
CLASSIFICATION:	QQR-T1-1
SOURCE:	79
NUMBER OF VARIABLES :	N = 2
NUMBER OF CONSTRAINTS:	M1 = 5 ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = x_1^2 + x_2^2$$

CONSTRAINTS :

$$g_1(x) = x_1 + x_2 - 1 \geq 0$$
$$g_2(x) = x_1^2 + x_2^2 - 1 \geq 0$$
$$g_3(x) = 9x_1^2 + x_2^2 - 9 \geq 0$$
$$g_4(x) = x_1^2 - x_2 \geq 0$$
$$g_5(x) = x_2^2 - x_1 \geq 0$$

LOWER BOUNDS :	XL = (- , -)		
UPPER BOUNDS :	XU = (- , -)		
START:	X	= (3, 1)	
	F(X)	= 10	(FEASIBLE)
SOLUTION:	X*	= (1, 1)	
	F(X*)	= 2	
	R(X*)	= 0	

PROBLEM:	226
CLASSIFICATION:	QQR-T1-2
SOURCE:	35
NUMBER OF VARIABLES :	N = 2
NUMBER OF CONSTRAINTS:	M1 = 2 ,M-M1 = 0 ,B = 2

OBJECTIVE FUNCTION :

$$f(x) = -x_1 x_2$$

CONSTRAINTS :

$$g_1(x) = x_1^2 + x_2^2 \geq 0$$

$$g_2(x) = 1 - x_1^2 - x_2^2 \geq 0$$

LOWER BOUNDS : XL = (0 , 0)

UPPER BOUNDS : XU = (- , -)

START:	X	= (0.8, 0.05)	
	F(X)	= -0.04	(FEASIBLE)
SOLUTION:	X*	= (0.7071 ,0.7071)	[= (1/√2, 1/√2)]
	F(X*)	= -0.5	
	R(X*)	= 0	

PROBLEM: 227

CLASSIFICATION: QQR-T1-3

SOURCE: 16

NUMBER OF VARIABLES : N = 2

NUMBER OF CONSTRAINTS: M1 = 2 ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = (x_1 - 2)^2 + (x_2 - 1)^2$$

CONSTRAINTS :

$$g_1(x) = -x_1^2 + x_2 \geq 0$$
$$g_2(x) = x_1 - x_2^2 \geq 0$$

LOWER BOUNDS : XL = (- , -)

UPPER BOUNDS : XU = (- , -)

START: X = (0.5, 0.5)

F(X) = 2.5 (FEASIBLE)

SOLUTION: X* = (1, 1)

F(X*) = 1

R(X*) = 0

PROBLEM: 228

CLASSIFICATION: QQR-T1-4

SOURCE: 41

NUMBER OF VARIABLES : N = 2

NUMBER OF CONSTRAINTS: M1 = 2(1) ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = x_1^2 + x_2$$

CONSTRAINTS :

$$g_1(x) = -x_1 - x_2 + 1 \geq 0$$
$$g_2(x) = -(x_1^2 + x_2^2) + 9 \geq 0$$

LOWER BOUNDS : XL = (- , -)

UPPER BOUNDS : XU = (- , -)

START: X = (0, 0)

F(X) = 0 (FEASIBLE)

SOLUTION: X* = (0, -3)

F(X*) = -3

R(X*) = 0

PROBLEM: 229

CLASSIFICATION: PBR-T1-1

SOURCE: 27/71/26/73

NUMBER OF VARIABLES : N = 2

NUMBER OF CONSTRAINTS: M1 = 0 ,M-M1 = 0 ,B = 4

OBJECTIVE FUNCTION :

$$f(x) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$$

CONSTRAINTS :

LOWER BOUNDS : XL = (-2, -2)

UPPER BOUNDS : XU = (2, 2)

START: X = (-1.2 , 1)

F(X) = 24.2 (FEASIBLE)

SOLUTION: X* = (1, 1)

F(X*) = 0

R(X*) = 0

PROBLEM:	230 (Chamberlain-problem)
CLASSIFICATION:	LPR-T1-5
SOURCE:	16
NUMBER OF VARIABLES :	N = 2
NUMBER OF CONSTRAINTS:	M1 = 2 ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = x_2$$

CONSTRAINTS :

$$g_1(x) = -2x_1^2 + x_1^3 + x_2 \geq 0$$

$$g_2(x) = -2(1 - x_1)^2 + (1 - x_1)^3 + x_2 \geq 0$$

LOWER BOUNDS :	XL = (- , -)	
UPPER BOUNDS :	XU = (- , -)	
START:	X = (0, 0)	
	F(X) = 0	(NOT FEASIBLE)
SOLUTION:	X* = (0.5,0.375)	
	F(X*) = 0.375	
	R(X*) = 0	

PROBLEM:	231
CLASSIFICATION:	PLR-T1-1
SOURCE:	74/43
NUMBER OF VARIABLES :	N = 2
NUMBER OF CONSTRAINTS:	M1 = 2(2) ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$$

CONSTRAINTS :

$$g_1(x) = \frac{1}{3}x_1 + x_2 + 0.1 \geq 0$$

$$g_2(x) = -\frac{1}{3}x_1 + x_2 + 0.1 \geq 0$$

LOWER BOUNDS :	XL = (- , -)
UPPER BOUNDS :	XU = (- , -)
START:	X = (-1.2 , 1)
	F(X) = 24.2 (FEASIBLE)
SOLUTION:	X* = (1, 1)
	F(X*) = 0
	R(X*) = 0

PROBLEM: 232

CLASSIFICATION: PLR-T1-2

SOURCE: 79

NUMBER OF VARIABLES : N = 2

NUMBER OF CONSTRAINTS: M1 = 3(3) ,M-M1 = 0 ,S = 2

OBJECTIVE FUNCTION :

$$f(x) = -[9 - (x_1 - 3)^2](x_2^3/27\sqrt{3})$$

CONSTRAINTS :

$$g_1(x) = x_1/\sqrt{3} - x_2 \geq 0$$

$$g_2(x) = x_1 + \sqrt{3}x_2 \geq 0$$

$$g_3(x) = 6 - x_1 - \sqrt{3}x_2 \geq 0$$

LOWER BOUNDS : XL = (0 , 0)

UPPER BOUNDS : XU = (- , -)

START: X = (2, 0.5)

F(X) = -0.0213833 (FEASIBLE)

SOLUTION: X* = (3, 1.732) [= (3,√3)]

F(X*) = -1

R(X*) = 0

PROBLEM: 233

CLASSIFICATION: PQR-T1-2

SOURCE: 74/43

NUMBER OF VARIABLES : N = 2

NUMBER OF CONSTRAINTS: M1 = 1 ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$$

CONSTRAINTS :

$$g_1(x) = x_1^2 + x_2^2 - 0.25 \geq 0$$

LOWER BOUNDS : XL = (- , -)

UPPER BOUNDS : XU = (- , -)

START: X = (1.2 , 1)

F(X) = 19.4 (FEASIBLE)

SOLUTION: X* = (1, 1)

F(X*) = 0

R(X*) = 0

PROBLEM: 234

CLASSIFICATION: PQR-T1-3

SOURCE: 79

NUMBER OF VARIABLES : N = 2

NUMBER OF CONSTRAINTS: M1 = 1 ,M-M1 = 0 ,B = 4

OBJECTIVE FUNCTION :

$$f(x) = (x_2 - x_1)^4 - (1 - x_1)$$

CONSTRAINTS :

$$g_1(x) = -x_1^2 - x_2^2 + 1 \geq 0$$

LOWER BOUNDS : XL = (0.2, 0.2)

UPPER BOUNDS : XU = (2, 2)

START: X = (0, 0)

F(X) = -1 (NOT FEASIBLE)

SOLUTION: X* = (0.2, 0.2)

F(X*) = -0.8

R(X*) = 0

PROBLEM: 235

CLASSIFICATION: PQR-T1-4

SOURCE: 49

NUMBER OF VARIABLES : N = 3

NUMBER OF CONSTRAINTS: M1 = 0 ,M-M1 = 1 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = 0.01(x_1 - 1)^2 + (x_2 - x_1^2)^2$$

CONSTRAINTS :

$$g_1(x) = x_1 + x_3^2 + 1 = 0$$

LOWER BOUNDS : XL = (- , - , -)

UPPER BOUNDS : XU = (- , - , -)

START: X = (-2, 3, 1)

F(X) = 1.09 (FEASIBLE)

SOLUTION: X* = (-1, 1, 0)

F(X*) = 0.04

R(X*) = 0

PROBLEM: 236

CLASSIFICATION: GQR-T1-1

SOURCE: 41/19

NUMBER OF VARIABLES : N = 2

NUMBER OF CONSTRAINTS: M1 = 2 ,M-M1 = 0 ,B = 4

OBJECTIVE FUNCTION :

$$-f(x) = B_1 + B_2x_1 + B_3x_1^2 + B_4x_1^3 + B_5x_1^4 + B_6x_2 + B_7x_1x_2$$
$$+ B_8x_1^2x_2 + B_9x_1^3x_2 + B_{10}x_1^4x_2 + B_{11}x_2^2 + B_{12}x_2^3 + B_{13}x_2^4$$
$$B_{14}\left[\frac{1.0}{x_2+1.0}\right] + B_{15}x_1^2x_2^2 + B_{16}x_1^3x_2^2 + B_{17}x_1^3x_2^3 + B_{18}x_1x_2^2$$
$$+ B_{19}x_1x_2^3 + B_{20}\{\exp[0.0005x_1x_2]\}$$

B_i: cf. Appendix A

CONSTRAINTS :

$$g_1(x) = x_1x_2 - 700.00 \geq 0$$
$$g_2(x) = x_2 - 5.0(x_1/25.0)^2 \geq 0$$

LOWER BOUNDS : XL = (0, 0)

UPPER BOUNDS : XU = (75, 65)

START: X = (90, 10)

F(X) = 82.8278 (NOT FEASIBLE)

SOLUTION: X* = (75, 65)

F(X*) = -58.9034

R(X*) = 0

PROBLEM: 237

CLASSIFICATION: GQR-T1-2

SOURCE: 41

NUMBER OF VARIABLES : N = 2

NUMBER OF CONSTRAINTS: $M^1 = 3$, $M-M^1 = 0$, $B = 3$

OBJECTIVE FUNCTION :

$$
\begin{aligned}
-f(x) = {} & B_1 + B_2x_1 + B_3x_1^2 + B_4x_1^3 + B_5x_1^4 + B_6x_2 + B_7x_1x_2 \\
& + B_8x_1^2x_2 + B_9x_1^3x_2 + B_{10}x_1^4x_2 + B_{11}x_2^2 + B_{12}x_2^3 + B_{13}x_2^4 \\
& + B_{14}\left[\frac{1.0}{x_2+1.0}\right] + B_{15}x_1^2x_2^2 + B_{16}x_1^3x_2^2 + B_{17}x_1^3x_2^3 + B_{18}x_1x_2^2 \\
& + B_{19}x_1x_2^3 + B_{20}\{\exp[0.0005x_1x_2]\}
\end{aligned}
$$

B_i: cf. Appendix A

CONSTRAINTS :

$$g_1(x) = x_1x_2 - 700.00 \geq 0$$

$$g_2(x) = x_2 - 5.0(x_1/25.0)^2 \geq 0$$

$$g_3(x) = (x_2 - 50.0)^2 - 5.0(x_1 - 55.0) \geq 0$$

LOWER BOUNDS : XL = (54, -)

UPPER BOUNDS : XU = (75, 65)

START: X = (95, 10)

F(X) = 86.0371 (NOT FEASIBLE)

SOLUTION: X* = (75, 65)

F(X*) = -58.9034

R(X*) = 0

PROBLEM: 238

CLASSIFICATION: GQR-T1-3

SOURCE: 41

NUMBER OF VARIABLES : N = 2

NUMBER OF CONSTRAINTS: M1 = 3 ,M-M1 = 0 ,B = 2

OBJECTIVE FUNCTION :

$$
\begin{aligned}
-f(x) = {} & B_1 + B_2x_1 + B_3x_1^2 + B_4x_1^3 + B_5x_1^4 + B_6x_2 + B_7x_1x_2 \\
& + B_8x_1^2x_2 + B_9x_1^3x_2 + B_{10}x_1^4x_2 + B_{11}x_2^2 + B_{12}x_2^3 + B_{13}x_2^4 \\
& + B_{14}\left[\frac{1.0}{x_2+1.0}\right] + B_{15}x_1^2x_2^2 + B_{16}x_1^3x_2^2 + B_{17}x_1^3x_2^3 + B_{18}x_1x_2^2 \\
& + B_{19}x_1x_2^3 + B_{20}\{\exp[0.0005x_1x_2]\}
\end{aligned}
$$

B_i: cf. Appendix A

CONSTRAINTS :

$$g_1(x) = x_1x_2 - 700.00 \geq 0$$

$$g_2(x) = x_2 - 5.0(x_1/25.0)^2 \geq 0$$

$$g_3(x) = (x_2 - 50.0)^2 - 5.0(x_1 - 55.0) \geq 0$$

LOWER BOUNDS : XL = (- , -)

UPPER BOUNDS : XU = (75, 65)

START: X = (95, 10)

F(X) = 86.0371 (NOT FEASIBLE)

SOLUTION: X* = (75, 65)

F(X*) = -58.9034

R(X*) = 0

PROBLEM: 239

CLASSIFICATION: GQR-T1-4

SOURCE: 41

NUMBER OF VARIABLES : N = 2

NUMBER OF CONSTRAINTS: M1 = 1 ,M-M1 = 0 ,B = 4

OBJECTIVE FUNCTION :

$$
\begin{aligned}
-f(x) = {} & B_1 + B_2x_1 + B_3x_1^2 + B_4x_1^3 + B_5x_1^4 + B_6x_2 + B_7x_1x_2 \\
& + B_8x_1^2x_2 + B_9x_1^3x_2 + B_{10}x_1^4x_2 + B_{11}x_2^2 + B_{12}x_2^3 + B_{13}x_2^4 \\
& + B_{14}\left[\frac{1.0}{x_2+1.0}\right] + B_{15}x_1^2x_2^2 + B_{16}x_1^3x_2^2 + B_{17}x_1^3x_2^3 + B_{18}x_1x_2^2 \\
& + B_{19}x_1x_2^3 + B_{20}\{\exp[0.0005x_1x_2]\}
\end{aligned}
$$

B_i: cf. Appendix A

CONSTRAINTS :

$$g_1(x) = x_1x_2 - 700.0 \geq 0$$

LOWER BOUNDS : XL = (0, 0)

UPPER BOUNDS : XU = (75, 65)

START: X = (95, 10)

F(X) = 86.0371 (NOT FEASIBLE)

SOLUTION: X* = (75, 65)

F(X*) = -58.9034

R(X*) = 0

PROBLEM: 240

CLASSIFICATION: QUR-T1-2

SOURCE: 37/85

NUMBER OF VARIABLES : N = 3

NUMBER OF CONSTRAINTS: M 1 = 0 ,M-M 1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = (x_1 - x_2 + x_3)^2 + (-x_1 + x_2 + x_3)^2 + (x_1 + x_2 - x_3)^2$$

CONSTRAINTS :

LOWER BOUNDS : XL = (- , - , -)

UPPER BOUNDS : XU = (- , - , -)

START: X = (100 , -1 , 2.5)

F(X) = 29726.8 (FEASIBLE)

SOLUTION: X* = (0, 0, 0)

F(X*) = 0

R(X*) = 0

PROBLEM: 241

CLASSIFICATION: SUR-T1-5

SOURCE: 37/30

NUMBER OF VARIABLES : N = 3

NUMBER OF CONSTRAINTS: M1 = 0 ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = \sum_{i=1}^{5} f_i^2(x)$$

$$f_1(x) = x_1^2 + x_2^2 + x_3^2 - 1$$

$$f_2(x) = x_1^2 + x_2^2 + (x_3 - 2)^2 - 1$$

$$f_3(x) = x_1 + x_2 + x_3 - 1$$

$$f_4(x) = x_1 + x_2 - x_3 + 1$$

$$f_5(x) = x_1^3 + 3x_2^2 + (5x_3 - x_1 + 1)^2 - 36$$

CONSTRAINTS :

LOWER BOUNDS : XL = (- , - , -)

UPPER BOUNDS : XU = (- , - , -)

START: X = (1, 2, 0)

F(X) = 629 (FEASIBLE)

SOLUTION: X* = (0, 0, 1)

F(X*) = 0

R(X*) = 0

PROBLEM: 242

CLASSIFICATION: SBR-T1-1

SOURCE: 37/8/11

NUMBER OF VARIABLES : N = 3

NUMBER OF CONSTRAINTS: M1 = 0 ,M-M1 = 0 ,B = 6

OBJECTIVE FUNCTION :

$$f(x) = \sum_{i=1}^{10} [(\exp(-x_1 t_i) - \exp(-x_2 t_i)) - x_3(\exp(-t_i) - \exp(-10t_i))]^2$$

$$t_i = 0.1i \ , \quad i = 1,\ldots,10$$

CONSTRAINTS :

LOWER BOUNDS : XL = (0, 0, 0)

UPPER BOUNDS : XU = (10, 10, 10)

START: X = (2.5, 10 , 10)

F(X) = 275.831 (FEASIBLE)

SOLUTION: X* = (1, 10, 1)

F(X*) = 0

R(X*) = 0

PROBLEM: 243

CLASSIFICATION: SUR-T1-6

SOURCE: 46

NUMBER OF VARIABLES : N = 3

NUMBER OF CONSTRAINTS: M1 = 0 ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = \sum_{i=1}^{4} f_i^2(x) = F^T F$$

$$F = A + Gx + 0.5x^T BxD$$

A,B,D,G : cf. Appendix A

CONSTRAINTS :

LOWER BOUNDS : XL = (- , - , -)

UPPER BOUNDS : XU = (- , - , -)

START: X = (0.1, 0.1, 0.1)

F(X) = 0.939853 (FEASIBLE)

SOLUTION: X* = (0, 0, 0)

F(X*) = 0.7966

R(X*) = 0

PROBLEM:	244 (Bigg's Function)
CLASSIFICATION:	SUR-T1-7
SOURCE:	63/9
NUMBER OF VARIABLES :	N = 3
NUMBER OF CONSTRAINTS:	M1 = 0 , M-M1 = 0 , B = 0

OBJECTIVE FUNCTION :

$$f(x) = \sum_{i=1}^{10} [\exp(-x_1 z_i) - x_3 \exp(-x_2 z_i) - y_i]^2$$

$$y_i = \exp(-z_i) - 5\exp(-10 z_i)$$

$$z_i = 0.1 i$$

$$i = 1,..,10$$

CONSTRAINTS :

LOWER BOUNDS :	XL = (- , - , -)
UPPER BOUNDS :	XU = (- , - , -)
START:	X = (1, 2, 1)
	F(X) = 1.55347 (FEASIBLE)
SOLUTION:	X* = (1, 10, 5)
	F(X*) = 0
	R(X*) = 0

PROBLEM: 245

CLASSIFICATION: SUR-T1-8

SOURCE: 11/14

NUMBER OF VARIABLES : N = 3

NUMBER OF CONSTRAINTS: M1 = 0 ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = \sum_{i=1}^{10} \{[\exp(-ix_1/10) - \exp(-ix_2/10)] - x_3[\exp(-i/10) - \exp(-i)]\}^2$$

CONSTRAINTS :

LOWER BOUNDS : XL = (- , - , -)

UPPER BOUNDS : XU = (- , - , -)

START: X = (0, 10, 20)

F(X) = 1031.15 (FEASIBLE)

SOLUTION: X* = (1, 10, 1)

F(X*) = 0

R(X*) = 0

PROBLEM:	246
CLASSIFICATION:	SUR-T1-9
SOURCE:	41
NUMBER OF VARIABLES :	N = 3
NUMBER OF CONSTRAINTS:	M1 = 0 ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = 100[x_3 - ((x_1 + x_2)/2)^2]^2 + (1 - x_1)^2 + (1 - x_2)^2$$

CONSTRAINTS :

LOWER BOUNDS : XL = (- , - , -)

UPPER BOUNDS : XU = (- , - , -)

START:	X = (-1.2 , 2 , 0)	
	F(X) = 8.4	(FEASIBLE)
SOLUTION:	X* = (1, 1, 1)	
	F(X*) = 0	
	R(X*) = 0	

PROBLEM: 247 (Helical valley in x_3 direction)

CLASSIFICATION: GBR-T1-2

SOURCE: 37/32/41/14/83

NUMBER OF VARIABLES : N = 3

NUMBER OF CONSTRAINTS: M1 = 0 ,M-M1 = 0 ;B = 3

OBJECTIVE FUNCTION :

$$f(x) = 100\{[x_3 - 10\upsilon(x_1,x_2)]^2 + [(x_1^2 + x_2^2)^{1/2} - 1]^2\} + x_3^2$$

$$\upsilon(x_1,x_2) = \begin{cases} \frac{1}{2\pi} * \arctan(x_2/x_1) & , x_1 > 0 \\ \frac{1}{2} + \frac{1}{2\pi} * \arctan(x_2/x_1) & , x_1 < 0 \end{cases}$$

CONSTRAINTS :

LOWER BOUNDS : XL = (0.1, - , -2.5)

UPPER BOUNDS : XU = (- , - , 7.5)

START: X = (-1, 0, 0)

F(X) = 2500 (NOT FEASIBLE)

SOLUTION: X* = (1, 0, 0)

F(X*) = 0

R(X*) = 0

PROBLEM:	248 (Around-the-World-Problem)
CLASSIFICATION:	LQR-T1-4
SOURCE:	64/62/63
NUMBER OF VARIABLES :	N = 3
NUMBER OF CONSTRAINTS:	M1 = 1(1) ,M-M1 = 1 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = -x_2$$

CONSTRAINTS :

$$g_1(x) = 1 - 2x_2 + x_1 \geq 0$$

$$g_2(x) = x_1^2 + x_2^2 + x_3^2 - 1 = 0$$

LOWER BOUNDS : XL = (- , - , -)

UPPER BOUNDS : XU = (- , - , -)

START:	X	= (-0.1, -1 , 0.1)	
	F(X)	= 1	(FEASIBLE)
SOLUTION:	X*	= (0.6 ,0.8 , 0)	
	F(X*)	= -0.8	
	R(X*)	= 0	

PROBLEM:	249
CLASSIFICATION:	QQR-T1-5
SOURCE:	79
NUMBER OF VARIABLES :	N = 3
NUMBER OF CONSTRAINTS:	M1 = 1 ,M-M1 = 0 ,B = 1

OBJECTIVE FUNCTION :

$$f(x) = x_1^2 + x_2^2 + x_3^2$$

CONSTRAINTS :

$$g_1(x) = x_1^2 + x_2^2 - 1 \geq 0$$

LOWER BOUNDS : XL = (1 , - , -)

UPPER BOUNDS : XU = (- , - , -)

START:	X	= (1, 1, 1)	
	F(X)	= 3	(FEASIBLE)
SOLUTION:	X*	= (1, 0, 0)	
	F(X*)	= 1	
	R(X*)	= 0	

PROBLEM:	250 (Rosenbrock's-Post-Office-Prob.)
CLASSIFICATION:	PLR-T1-3
SOURCE:	35/71/27/26/73
NUMBER OF VARIABLES :	N = 3
NUMBER OF CONSTRAINTS:	M1 = 2(2) ,M-M1 = 0 ,B = 6

OBJECTIVE FUNCTION :

$$f(x) = -x_1 x_2 x_3$$

CONSTRAINTS :

$$g_1(x) = x_1 + 2x_2 + 2x_3 \geq 0$$

$$g_2(x) = 72 - x_1 - 2x_2 - 2x_3 \geq 0$$

LOWER BOUNDS :	XL = (0, 0, 0)	
UPPER BOUNDS :	XU = (20, 11, 42)	
START:	X = (10, 10, 10)	
	F(X) = -1000	(FEASIBLE)
SOLUTION:	X* = (20, 11, 15)	
	F(X*) = -3300	
	R(X*) = 0	

PROBLEM: 251

CLASSIFICATION: PLR-T1-4

SOURCE: 11/43

NUMBER OF VARIABLES : N = 3

NUMBER OF CONSTRAINTS: M1 = 1(1) ,M-M1 = 0 ,B = 6

OBJECTIVE FUNCTION :

$$f(x) = -x_1 x_2 x_3$$

CONSTRAINTS :

$$g_1(x) = 72 - x_1 - 2x_2 - 2x_3 \geq 0$$

LOWER BOUNDS : XL = (0, 0, 0)

UPPER BOUNDS : XU = (42, 42, 42)

START: X = (10, 10, 10)

F(X) = -1000 (FEASIBLE)

SOLUTION: X* = (24, 12, 12)

F(X*) = -3456

R(X*) = 0

PROBLEM:	252
CLASSIFICATION:	PQR-T1-5
SOURCE:	54
NUMBER OF VARIABLES :	N = 3
NUMBER OF CONSTRAINTS:	M1 = 0 ,M-M1 = 1 ,B = 1

OBJECTIVE FUNCTION :

$$f(x) = 0.01(x_1 - 1)^2 + (x_2 - x_1^2)^2$$

CONSTRAINTS :

$$g_1(x) = x_1 + x_3^2 + 1 = 0$$

LOWER BOUNDS : XL = (- , - , -)

UPPER BOUNDS : XU = (-1, - , -)

START:	X	= (-1, 2, 2)	
	F(X)	= 1.04	(NOT FEASIBLE)
SOLUTION:	X*	= (-1, 1, 0)	
	F(X*)	= 0.04	
	R(X*)	= 0	

PROBLEM: 253

CLASSIFICATION: GLR-T1-1

SOURCE: 41

NUMBER OF VARIABLES : N = 3

NUMBER OF CONSTRAINTS: M1 = 1(1) ,M-M1 = 0 ,B = 3

OBJECTIVE FUNCTION :

$$f(x) = \sum_{j=1}^{8} c_j[(a_{1j} - x_1)^2 + (a_{2j} - x_2)^2 + (a_{3j} - x_3)^2]^{1/2}$$

$a_{1j}, a_{2j}, a_{3j}, c_j$: cf. Appendix A

CONSTRAINTS :

$$g_1(x) = 30 - 3x_1 - 3x_3 \geq 0$$

LOWER BOUNDS : XL = (0 , 0 , 0)

UPPER BOUNDS : XU = (- , - , -)

START: X = (0, 2, 0)

F(X) = 86.5395 (FEASIBLE)

SOLUTION: X* = (0.3333 ,0.3333 ,0.3333)

F(X*) = 87.3794

R(X*) = 0

PROBLEM: 254

CLASSIFICATION: GQR-T1-5

SOURCE: 54

NUMBER OF VARIABLES : N = 3

NUMBER OF CONSTRAINTS: M1 = 0 ,M-M1 = 2 ,B = 1

OBJECTIVE FUNCTION :

$$f(x) = \log(x_3) - x_2$$

CONSTRAINTS :

$$g_1(x) = x_2^2 + x_3^2 - 4 = 0$$

$$g_2(x) = x_3 - 1 - x_1^2 = 0$$

LOWER BOUNDS : XL = (- , - , 1)

UPPER BOUNDS : XU = (- , - , -)

START: X = (1, 1, 1)

F(X) = -1 (NOT FEASIBLE)

SOLUTION: X* = (0 , 1.732 , 1) [= (0,√3,1)]

F(X*) = -1.73205 [= -√3]

R(X*) = 0

PROBLEM: 255

CLASSIFICATION: QUR-T1-3

SOURCE: 7

NUMBER OF VARIABLES : N = 4

NUMBER OF CONSTRAINTS: M1 = 0 ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = 100(x_2 - x_1^2) + (1 - x_1)^2 + 90(x_4 - x_3^2) + (1 - x_3)^2$$
$$+ 10.1[(x_2 - 1)^2 + (x_4 - 1)^2] + 19.8(x_2 - 1)(x_4 - 1)$$

CONSTRAINTS :

LOWER BOUNDS : XL = (- , - , - , -)

UPPER BOUNDS : XU = (- , - , - , -)

START: X = (-3, 1, -3, 1)

F(X) = -1488 (FEASIBLE)

SOLUTION: X* = (1, 1, 1, 1)

F(X*) = 0

R(X*) = 0

PROBLEM: 256 (Powell's Function)

CLASSIFICATION: PUR-T1-9

SOURCE: 65/41/67/14/83/63/37

NUMBER OF VARIABLES : N = 4

NUMBER OF CONSTRAINTS: M1 = 0 ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = (x_1 + 10x_2)^2 + 5(x_3 - x_4)^2 + (x_2 - 2x_3)^4 + 10(x_1 - x_4)^4$$

CONSTRAINTS :

LOWER BOUNDS : XL = (- , - , - , -)

UPPER BOUNDS : XU = (- , - , - , -)

START: X = (3, -1, 0, 1)

F(X) = 215 (FEASIBLE)

SOLUTION: X* = (0, 0, 0, 0)

F(X*) = 0

R(X*) = 0

PROBLEM: 257

CLASSIFICATION: PBR-T1-3

SOURCE: 79

NUMBER OF VARIABLES : N = 4

NUMBER OF CONSTRAINTS: M1 = 0 ,M-M1 = 0 ,B = 2

OBJECTIVE FUNCTION :

$$f(x) = 100(x_1^2 - x_2)^2 + (x_1 - 1)^2 + 90(x_3^2 - x_4)^2 + (x_3 - 1)^2 + 10.1[(x_2 - 1)^2 + (x_4 - 1)^2] + 19.8(x_1 - 1)(x_4 - 1)$$

CONSTRAINTS :

LOWER BOUNDS : XL = (0 , - , 0 , -)

UPPER BOUNDS : XU = (- , - , - , -)

START: X = (-3, -1, -3, -1)

F(X) = 19271.2 (NOT FEASIBLE)

SOLUTION: X* = (1, 1, 1, 1)

F(X*) = 0

R(X*) = 0

PROBLEM:	258 (Wood's Function)
CLASSIFICATION:	PUR-T1-10
SOURCE:	18/63/14/7
NUMBER OF VARIABLES :	N = 4
NUMBER OF CONSTRAINTS:	M1 = 0 ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2 + 90(x_4 - x_3^2)^2 + (1 - x_3)^2$$
$$+ 10.1[(x_2 - 1)^2 + (x_4 - 1)^2] + 19.8(x_2 - 1)(x_4 - 1)$$

CONSTRAINTS :

LOWER BOUNDS : XL = (- , - , - , -)

UPPER BOUNDS : XU = (- , - , - , -)

START:	X	= (-3, -1, -3, -1)	
	F(X)	= 19192	(FEASIBLE)
SOLUTION:	X*	= (1, 1, 1, 1)	
	F(X*)	= 0	
	R(X*)	= 0	

PROBLEM: 259

CLASSIFICATION: PBR-T1-3

SOURCE: 41

NUMBER OF VARIABLES : N = 4

NUMBER OF CONSTRAINTS: M1 = 0 ,M-M1 = 0 ,B = 1

OBJECTIVE FUNCTION :

$$f(x) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2 + 90(x_4 - x_3^2)^2 + (1 - x_3)^3$$
$$+ 10.1\ (x_2 - 1)^2 + (x_4 - 1)^2 + 19.8(x_2 - 1)(x_4 - 1)$$

CONSTRAINTS :

LOWER BOUNDS : XL = (- , - , - , -)

UPPER BOUNDS : XU = (- , - , - , 1)

START: X = (0, 0, 0, 0)

F(X) = 32.9 (FEASIBLE)

SOLUTION: X* = (1, 1, 1, 1)

F(X*) = 0

R(X*) = 0

PROBLEM: 260

CLASSIFICATION: PUR-T1-11

SOURCE: 37/18

NUMBER OF VARIABLES : N = 4

NUMBER OF CONSTRAINTS: M1 = 0 ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2 + 90(x_4 - x_3^2)^2 + (1 - x_3)^2$$
$$+ 9.9[(x_2 - 1) + (x_4 - 1)]^2 + 0.2[(x_2 - 1)^2 + (x_4 - 1)^2]$$

CONSTRAINTS :

LOWER BOUNDS : XL = (- , - , - , -)

UPPER BOUNDS : XU = (- , - , - , -)

START: X = (-3, -1, -3, -1)

F(X) = 19192 (FEASIBLE)

SOLUTION: X* = (1, 1, 1, 1)

F(X*) = 0

R(X*) = 0

PROBLEM: 261

CLASSIFICATION: SUI-T1-1

SOURCE: 20/21/41/37/79

NUMBER OF VARIABLES : N = 4

NUMBER OF CONSTRAINTS: M1 = 0 ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = (e^{x_1} - x_2)^4 + 100(x_2 - x_3)^6 + [\tan(x_3 - x_4)]^4 + x_1^8 + (x_4 - 1)^2$$

CONSTRAINTS :

LOWER BOUNDS : XL = (- , - , - , -)

UPPER BOUNDS : XU = (- , - , - , -)

START: X = (0, 0, 0, 0)

F(X) = 2 (FEASIBLE)

SOLUTION: X* = (0, 1, 1, 1)

F(X*) = 0

R(X*) = 0

PROBLEM: 262

CLASSIFICATION: LLR-T1-1

SOURCE: 63/62

NUMBER OF VARIABLES : N = 4

NUMBER OF CONSTRAINTS: M1 = 3(3) ,M-M1 = 1(1) ,B = 4

OBJECTIVE FUNCTION :

$$f(x) = -0.5x_1 - x_2 - 0.5x_3 - x_4$$

CONSTRAINTS :

$$g_1(x) = 10 - x_1 - x_2 - x_3 - x_4 \geq 0$$

$$g_2(x) = 10 - 0.2x_1 - 0.5x_2 - x_3 - 2x_4 \geq 0$$

$$g_3(x) = 10 - 2x_1 - x_2 - 0.5x_3 - 0.2x_4 \geq 0$$

$$g_4(x) = x_1 + x_2 + x_3 - 2x_4 - 6 = 0$$

LOWER BOUNDS : XL = (0 , 0 , 0 , 0)

UPPER BOUNDS : XU = (- , - , - , -)

START: X = (1, 1, 1, 1)

F(X) = -3 (FEASIBLE)

SOLUTION: X* = (0 , 8.667 , 0 , 1.333)

F(X*) = -10

R(X*) = 0

PROBLEM: 263

CLASSIFICATION: LPR-T1-6

SOURCE: 54

NUMBER OF VARIABLES : N = 4

NUMBER OF CONSTRAINTS: M1 = 2 ,M-M1 = 2 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = -x_1$$

CONSTRAINTS :

$$g_1(x) = x_2 - x_1^3 \geq 0$$
$$g_2(x) = x_1^2 - x_2 \geq 0$$
$$g_3(x) = x_2 - x_1^3 - x_3^2 = 0$$
$$g_4(x) = x_1^2 - x_2 - x_4^2 = 0$$

LOWER BOUNDS : XL = (- , - , - , -)

UPPER BOUNDS : XU = (- , - , - , -)

START: X = (10, 10, 10, 10)

F(X) = -10 (NOT FEASIBLE)

SOLUTION: X* = (1, 1, 0, 0)

F(X*) = -1

R(X*) = 0

PROBLEM:	264 (modified Rosen-Suzuki-Problem)
CLASSIFICATION:	QQR-T1-6
SOURCE:	70/44
NUMBER OF VARIABLES :	N = 4
NUMBER OF CONSTRAINTS:	M1 = 3 , M-M1 = 0 , B = 0

OBJECTIVE FUNCTION :

$$f(x) = x_1^2 + x_2^2 + 2x_3^2 + x_4^2 - 5x_1 - 5x_2 - 21x_3 + 7x_4$$

CONSTRAINTS :

$$g_1(x) = -x_1^2 - x_2^2 - x_3^2 - x_4^2 - x_1 + x_2 + x_3 + x_4 + 8 \geq 0$$

$$g_2(x) = -x_1^2 - 2x_2^2 - x_3^2 - 2x_4^2 + x_1 + x_4 + 9 \geq 0$$

$$g_3(x) = -2x_1^2 - x_2^2 - x_3^2 - 2x_1 + x_2 + x_4 + 5 \geq 0$$

LOWER BOUNDS :	XL = (- , - , - , -)	
UPPER BOUNDS :	XU = (- , - , - , -)	
START:	X = (0, 0, 0, 0)	
	F(X) = 0	(FEASIBLE)
SOLUTION:	X* = (0, 1, 2, -1)	
	F(X*) = -44	
	R(X*) = 0	

PROBLEM: 265

CLASSIFICATION: GLR-T1-2

SOURCE: 7/22

NUMBER OF VARIABLES : N = 4

NUMBER OF CONSTRAINTS: $M_1 = 0$, $M-M_1 = 2(2)$, $B = 4$

OBJECTIVE FUNCTION :

$$f(x) = \sum_{i=1}^{2} [1 - \exp(-10x_i \exp(-x_{i+2}))]$$

CONSTRAINTS :

$$g_1(x) = x_1 + x_2 - 1 = 0$$

$$g_2(x) = x_3 + x_4 - 1 = 0$$

LOWER BOUNDS : XL = (0, 0, 0, 0)

UPPER BOUNDS : XU = (1, 1, 1, 1)

START: X = (0, 0, 0, 0)

F(X) = 0 (NOT FEASIBLE)

SOLUTION: X* = (1, 0, 1, 0)

F(X*) = 0.974747 [= 1 - exp(-10exp(-1))]

R(X*) = 0

PROBLEM:	266
CLASSIFICATION:	SUR-T1-10
SOURCE:	46
NUMBER OF VARIABLES :	N = 5
NUMBER OF CONSTRAINTS:	M1 = 0 ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = \sum_{i=1}^{10} f_i^2(x) = F^T F$$

$$F = A + Cx + 0.5x^T BxD$$

A,B,C,D : cf. Appendix A

CONSTRAINTS :

LOWER BOUNDS : XL = (- , - , - , - , -)

UPPER BOUNDS : XU = (- , - , - , - , -)

START:	X	= (0.1, 0.1, 0.1, 0.1, 0.1)
	F(X)	= 1.47938 (FEASIBLE)
SOLUTION:	X*	= (0, 0, 0, 0, 0)
	F(X*)	= 1
	R(X*)	= 0

PROBLEM:	267 (Bigg's Function)
CLASSIFICATION:	SUR-T1-11
SOURCE:	63/9
NUMBER OF VARIABLES :	N = 5
NUMBER OF CONSTRAINTS:	M1 = 0 ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = \sum_{i=1}^{11} [x_3 \exp(-x_1 z_i) - x_4 \exp(-x_2 z_i) + 3\exp(-x_5 z_i) - y_i]^2$$

$$z_i = 0.1i$$

$$y_i = \exp(-z_i) - 5\exp(-10z_i) + 3\exp(-4z_i)$$

CONSTRAINTS :

LOWER BOUNDS : XL = (- , - , - , - , -)

UPPER BOUNDS : XU = (- , - , - , - , -)

START:	X	= (2, 2, 2, 2, 2)
	F(X)	= 2.29679 (FEASIBLE)
SOLUTION:	X*	= (1, 10, 1, 5, 4)
	F(X*)	= 0
	R(X*)	= 0

PROBLEM: 268

CLASSIFICATION: QLR-T1-2

SOURCE: 78/43

NUMBER OF VARIABLES : N = 5

NUMBER OF CONSTRAINTS: M1 = 5(5) ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = x^T D^T D x - 2d^T D x + d^T d$$

$$D = \begin{bmatrix} -74 & 80 & 18 & -11 & -4 \\ 14 & -69 & 21 & 28 & 0 \\ 66 & -72 & -5 & 7 & 1 \\ -12 & 66 & -30 & -23 & 3 \\ 3 & 8 & -7 & -4 & 1 \\ 4 & -12 & 4 & 4 & 0 \end{bmatrix} \qquad d = \begin{bmatrix} 51 \\ -61 \\ -56 \\ 69 \\ 10 \\ -12 \end{bmatrix}$$

CONSTRAINTS :

$$g_1(x) = -x_1 - x_2 - x_3 - x_4 - x_5 + 5 \geq 0$$

$$g_2(x) = 10x_1 + 10x_2 - 3x_3 + 5x_4 + 4x_5 - 20 \geq 0$$

$$g_3(x) = -8x_1 + x_2 - 2x_3 - 5x_4 + 3x_5 + 40 \geq 0$$

$$g_4(x) = 8x_1 - x_2 + 2x_3 + 5x_4 - 3x_5 - 11 \geq 0$$

$$g_5(x) = -4x_1 - 2x_2 + 3x_3 - 5x_4 + x_5 + 30 \geq 0$$

LOWER BOUNDS : XL = (- , - , - , - , -)

UPPER BOUNDS : XU = (- , - , - , - , -)

START: X = (1, 1, 1, 1, 1)

F(X) = 12048 (FEASIBLE)

SOLUTION: X* = (1, 2, -1, 3, -4)

F(X*) = 0

R(X*) = 0

PROBLEM: 269

CLASSIFICATION: SLR-T1-1

SOURCE: 49/79

NUMBER OF VARIABLES : N = 5

NUMBER OF CONSTRAINTS: M1 = 0 ,M-M1 = 3(3) ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = (x_1 - x_2)^2 + (x_2 + x_3 - 2)^2 + (x_4 - 1)^2 + (x_5 - 1)^2$$

CONSTRAINTS :

$$g_1(x) = x_1 + 3x_2 = 0$$
$$g_2(x) = x_3 + x_4 - 2x_5 = 0$$
$$g_3(x) = x_2 - x_5 = 0$$

LOWER BOUNDS : XL = (- , - , - , - , -)

UPPER BOUNDS : XU = (- , - , - , - , -)

START: X = (2, 2, 2, 2, 2)

F(X) = 6 (NOT FEASIBLE)

SOLUTION: X* = (-0.7674 ,0.2558 ,0.6279 ,-0.1163 ,0.2558)

F(X*) = 4.09302 [= (-33,11,27,-5,11)/43]

[= 176/43]

R(X*) = 0

PROBLEM: 270

CLASSIFICATION: PQR-T1-6

SOURCE: 19

NUMBER OF VARIABLES : N = 5

NUMBER OF CONSTRAINTS: $M_1 = 1$, $M-M_1 = 0$, B = 4

OBJECTIVE FUNCTION :

$$f(x) = x_1x_2x_3x_4 - 3x_1x_2x_4 - 4x_1x_2x_3 + 12x_1x_2 - x_2x_3x_4 + 3x_2x_4 + 4x_2x_3 - 12x_2 - 2x_1x_3x_4 + 6x_1x_4 + 8x_1x_3 - 24x_1 + 2x_3x_4 - 6x_4 - 8x_3 + 24 + 1.5x_5^4 - 5.75x_5^3 + 5.25x_5^2$$

CONSTRAINTS :

$$g_1(x) = 34 - x_1^2 - x_2^2 - x_3^2 - x_4^2 - x_5^2 \geq 0$$

LOWER BOUNDS : XL = (1 , 2 , 3 , 4 , -)

UPPER BOUNDS : XU = (- , - , - , - , -)

START: X = (1.1 , 2.1 , 3.1 , 4.1 , -1)

F(X) = 12.5001 (FEASIBLE)

SOLUTION: X* = (1, 2, 3, 4, 2)

F(X*) = -1

R(X*) = 0

PROBLEM: 271

CLASSIFICATION: SUR-T1-12

SOURCE: 79

NUMBER OF VARIABLES : N = 6

NUMBER OF CONSTRAINTS: M1 = 0 ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = 10 \sum_{i=1}^{6} (16 - i)(x_i - 1)^2$$

CONSTRAINTS :

LOWER BOUNDS : XL = (- , - , - , - , - , -)

UPPER BOUNDS : XU = (- , - , - , - , - , -)

START: X = (0, 0, 0, 0, 0, 0)

F(X) = 750 (FEASIBLE)

SOLUTION: X* = (1, 1, 1, 1, 1, 1)

F(X*) = 0

R(X*) = 0

PROBLEM: 272

CLASSIFICATION: SUR-T1-13

SOURCE: 79

NUMBER OF VARIABLES : N = 6

NUMBER OF CONSTRAINTS: M1 = 0 ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = \sum_{i=1}^{13} [x_4 \exp(-x_1 t_i) - x_5 \exp(-x_2 t_i) + x_6 \exp(-x_3 t_i) - y_i]^2$$

$$y_i = \exp(-t_i) - 5\exp(-10t_i) + 3\exp(-4t_i)$$

$$t_i = i/10$$

CONSTRAINTS :

LOWER BOUNDS : XL = (- , - , - , - , - , -)

UPPER BOUNDS : XU = (- , - , - , - , - , -)

START: X = (1, 2, 1, 1, 1, 1)

F(X) = 0.779070 (FEASIBLE)

SOLUTION: X* = (1, 10, 4, 1, 5, 3)

F(X*) = 0

R(X*) = 0

PROBLEM: 273

CLASSIFICATION: PUR-T1-12

SOURCE: 79

NUMBER OF VARIABLES : N = 6

NUMBER OF CONSTRAINTS: M1 = 0 ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = 10 \sum_{i=1}^{6} (16 - i)(x_i - 1)^2 + 10[\sum_{i=1}^{6} (16 - i)(x_i - 1)^2]^2$$

CONSTRAINTS :

LOWER BOUNDS : XL = (- , - , - , - , - , -)

UPPER BOUNDS : XU = (- , - , - , - , - , -)

START: X = (0, 0, 0, 0, 0, 0)

F(X) = 57000 (FEASIBLE)

SOLUTION: X* = (1, 1, 1, 1, 1, 1)

F(X*) = 0

R(X*) = 0

PROBLEM:	274
CLASSIFICATION:	QUR-T1-4
SOURCE:	57/43/14/42
NUMBER OF VARIABLES :	N = 2
NUMBER OF CONSTRAINTS:	M 1 = 0 ,M-M 1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = x^T A_2 x$$

$$A_2(i,j) = \frac{1}{i+j-1} \qquad \text{2x2 Hilbert Matrix}$$

CONSTRAINTS :

LOWER BOUNDS : XL = (- , -)

UPPER BOUNDS : XU = (- , -)

START:	X	= (-4, -2)	
	F(X)	= 25.3333	(FEASIBLE)
SOLUTION:	X*	= (0, 0)	
	F(X*)	= 0	
	R(X*)	= 0	

PROBLEM:	275
CLASSIFICATION:	QUR-T1-5
SOURCE:	57/43/14/42
NUMBER OF VARIABLES :	N = 4
NUMBER OF CONSTRAINTS:	M1 = 0 ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = x^T A_4 x$$

$$A_4(i,j) = \frac{1}{i+j-1} \qquad \text{4x4 Hilbert Matrix}$$

CONSTRAINTS :

LOWER BOUNDS : XL = (- , - , - , -)

UPPER BOUNDS : XU = (- , - , - , -)

START:	X	= (-4 , -2 , -1.333 , -1)	
	F(X)	= 33.9651	(FEASIBLE)
SOLUTION:	X*	= (0, 0, 0, 0)	
	F(X*)	= 0	
	R(X*)	= 0	

PROBLEM: 276

CLASSIFICATION: QUR-T1-6

SOURCE: 57/43/14/42

NUMBER OF VARIABLES : N = 6

NUMBER OF CONSTRAINTS: M 1 = 0 ,M-M 1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = x^T A_6 x$$

$$A_6(i,j) = \frac{1}{i+j-1} \qquad 6x6 \quad \text{Hilbert Matrix}$$

CONSTRAINTS :

LOWER BOUNDS : XL = (- , - , - , - , - , -)

UPPER BOUNDS : XU = (- , - , - , - , - , -)

START: X = (-4 ,-2 ,-1.333 ,-1 ,-0.8 ,-0.6667)

F(X) = 38.1416 (FEASIBLE)

SOLUTION: X* = (0, 0, 0, 0, 0, 0)

F(X*) = 0

R(X*) = 0

PROBLEM: 277

CLASSIFICATION: LLR-T1-2

SOURCE: 15

NUMBER OF VARIABLES : N = 4

NUMBER OF CONSTRAINTS: M1 = 4(4) ,M-M1 = 0 ,B = 4

OBJECTIVE FUNCTION :

$$f(x) = c^T x$$

$$c_i = \sum_{j=1}^{4} \frac{1}{i+j-1} \qquad c \in R^4$$

CONSTRAINTS :

$$Ax - b \geq 0 \qquad A: 4x4 \text{ Matrix}, \quad b \in R^4$$
$$x \geq 0$$

$$a_{ij} = \frac{1}{i+j-1} \qquad (i,j = 1,2,3,4)$$

$$b_i = \sum_{j=1}^{4} \frac{1}{i+j-1} \qquad (i = 1,2,3,4)$$

LOWER BOUNDS : XL = (0 , 0 , 0 , 0)

UPPER BOUNDS : XU = (- , - , - , -)

START: X = (0, 0, 0, 0)

F(X) = 0 (NOT FEASIBLE)

SOLUTION: X* = (1, 1, 1, 1)

F(X*) = 5.07619

R(X*) = 0

PROBLEM:	278
CLASSIFICATION:	LLR-T1-3
SOURCE:	15
NUMBER OF VARIABLES :	N = 6
NUMBER OF CONSTRAINTS:	$M_1 = 6(6)$, $M-M_1 = 0$, $B = 6$

OBJECTIVE FUNCTION :

$$f(x) = c^T x$$

$$c_i = \sum_{j=1}^{6} \frac{1}{i+j-1} \qquad c \in R^6$$

CONSTRAINTS :

$$Ax - b \geq 0 \qquad x \geq 0 \qquad A: 6x6 \text{ Matrix}, \quad b \in R^6$$

$$a_{ij} = \frac{1}{i+j-1} \qquad (i,j = 1,..,6)$$

$$b_i = \sum_{j=1}^{6} \frac{1}{i+j-1} \qquad (i = 1,..,6)$$

LOWER BOUNDS :	XL = (0 , 0 , 0 , 0 , 0 , 0)
UPPER BOUNDS :	XU = (- , - , - , - , - , -)
START: X	= (0, 0, 0, 0, 0, 0)
F(X)	= 0 (NOT FEASIBLE)
SOLUTION: X*	= (1, 1, 1, 1, 1, 1)
F(X*)	= 7.83853
R(X*)	= 0

PROBLEM:	279
CLASSIFICATION:	LLR-T1-4
SOURCE:	15
NUMBER OF VARIABLES :	N = 8
NUMBER OF CONSTRAINTS:	M1 = 8(8) ,M-M1 = 0 ,B = 8

OBJECTIVE FUNCTION :

$$f(x) = c^T x$$

$$c_i = \sum_{j=1}^{8} \frac{1}{i+j-1} \qquad c \in R^8$$

CONSTRAINTS :

$$Ax - b \geq 0$$
$$x \geq 0$$

A: 8x8 Matrix, $b \in R^8$

$$a_{ij} = \frac{1}{i+j-1} \qquad (i,j = 1,..,8)$$

$$b_i = \sum_{j=1}^{8} \frac{1}{i+j-1} \qquad (i = 1,..,8)$$

LOWER BOUNDS :	XL = (0 , 0 , 0 , 0 , 0 , 0 , 0 , 0)
UPPER BOUNDS :	XU = (- , - , - , - , - , - , - , -)
START: X	= (0 , 0 , 0 , 0 , 0 , 0 , 0 , 0)
F(X)	= 0 (NOT FEASIBLE)
SOLUTION: X*	= (1 , 1 , 1 , 1 , 1 , 1 , 1 , 1)
F(X*)	= 10.6059
R(X*)	= 0

PROBLEM:	280
CLASSIFICATION:	LLR-T1-5
SOURCE:	15
NUMBER OF VARIABLES :	N = 10
NUMBER OF CONSTRAINTS:	$M_1 = 10(10)$, $M-M_1 = 0$, $B = 10$

OBJECTIVE FUNCTION :

$$f(x) = c^T x$$

$$c_i = \sum_{j=1}^{10} \frac{1}{i+j-1} \qquad c \in R^{10}$$

CONSTRAINTS :

$$Ax - b \geq 0$$
$$x \geq 0$$

A: 10x10 Matrix, $b \in R^{10}$

$$a_{ij} = \frac{1}{i+j-1} \qquad (i,j = 1,..,10)$$

$$b_i = \sum_{j=1}^{10} \frac{1}{i+j-1} \qquad (i = 1,..,10)$$

LOWER BOUNDS :	XL = (0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0)
UPPER BOUNDS :	XU = (- , - , - , - , - , - , - , - , - , -)
START: X	= (0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0)
F(X)	= 0 (NOT FEASIBLE)
SOLUTION: X*	= (1 , 1 , 1 , 1 , 1 , 1 , 1 , 1 , 1 , 1)
F(X*)	= 13.3754
R(X*)	= 0

PROBLEM: 281

CLASSIFICATION: GUR-T1-2

SOURCE: 79

NUMBER OF VARIABLES : N = 10

NUMBER OF CONSTRAINTS: M1 = 0 ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = \left[\sum_{i=1}^{10} i^3 (x_i - 1)^2 \right]^{1/3}$$

CONSTRAINTS :

LOWER BOUNDS : XL = (- , - , - , - , - , - , - , - , - , -)

UPPER BOUNDS : XU = (- , - , - , - , - , - , - , - , - , -)

START: X = (0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0)

F(X) = 14.4624 (FEASIBLE)

SOLUTION: X* = (1 , 1 , 1 , 1 , 1 , 1 , 1 , 1 , 1 , 1)

F(X*) = 0

R(X*) = 0

PROBLEM: 282

CLASSIFICATION: SUR-T1-14

SOURCE: 79

NUMBER OF VARIABLES : N = 10

NUMBER OF CONSTRAINTS: M 1 = 0 ,M-M 1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = (x_1 - 1)^2 + (x_{10} - 1)^2 + \sum_{i=1}^{9} 10(10 - i)(x_i^2 - x_{i+1})^2$$

CONSTRAINTS :

LOWER BOUNDS : XL = (- , - , - , - , - , - , - , - , - , -)

UPPER BOUNDS : XU = (- , - , - , - , - , - , - , - , - , -)

START: X = (-1.2, 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0)

F(X) = 192.464 (FEASIBLE)

SOLUTION: X* = (1 , 1 , 1 , 1 , 1 , 1 , 1 , 1 , 1 , 1)

F(X*) = 0

R(X*) = 0

PROBLEM: 283

CLASSIFICATION: PUR-T1-13

SOURCE: 79

NUMBER OF VARIABLES : N = 10

NUMBER OF CONSTRAINTS: M1 = 0 ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = \left[\sum_{i=1}^{10} i^3 (x_i - 1)^2 \right]^3$$

CONSTRAINTS :

LOWER BOUNDS : XL = (- , - , - , - , - , - , - , - , - , -)

UPPER BOUNDS : XU = (- , - , - , - , - , - , - , - , - , -)

START: X = (0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0)

F(X) = 0.276806E+11 (FEASIBLE)

SOLUTION: X* = (1 , 1 , 1 , 1 , 1 , 1 , 1 , 1 , 1 , 1)

F(X*) = 0

R(X*) = 0

PROBLEM:	284
CLASSIFICATION:	LQR-T1-5
SOURCE:	69/44

NUMBER OF VARIABLES : N = 15

NUMBER OF CONSTRAINTS: M1 = 10 , M-M1 = 0 , B = 0

OBJECTIVE FUNCTION :

$$f(x) = -\sum_{i=1}^{15} c_i \, x_i$$

CONSTRAINTS :

$$g_i(x) = b_i - \sum_{j=1}^{15} a_{ij} \, x_j^{\,2} \geq 0 \qquad (i = 1,\cdots,10)$$

a_{ij}, b_i, c_i : cf. Appendix A

LOWER BOUNDS : XL = (-,-,-,-,-,-,-,-,-,-,-,-,-,-,-)

UPPER BOUNDS : XU = (-,-,-,-,-,-,-,-,-,-,-,-,-,-,-)

START:	X	= (0,0,0,0,0,0,0,0,0,0,0,0,0,0,0)	
	F(X)	= 0	(FEASIBLE)
SOLUTION:	X*	= (1,1,1,1,1,1,1,1,1,1,1,1,1,1,1)	
	F(X*)	= -1840	
	R(X*)	= 0	

PROBLEM: 285

CLASSIFICATION: LQR-T1-6

SOURCE: 19/69

NUMBER OF VARIABLES : N = 15

NUMBER OF CONSTRAINTS: M1 =10 ,M-M1 = 0 ,8 = 0

OBJECTIVE FUNCTION :

$$f(x) = -486x_1 - 640x_2 - 758x_3 - 776x_4 - 477x_5 - 707x_6 - 175x_7 - 619x_8 - 627x_9 - 614x_{10} - 475x_{11} - 377x_{12} - 524x_{13} - 468x_{14} - 529x_{15}$$

CONSTRAINTS :

$$g_i(x) = b_i - \sum_{j=1}^{15} a_{ij}\, x_j^2 \geq 0 \qquad (i = 1,\cdots,10)$$

a_{ij}, b_i : cf. Appendix A

LOWER BOUNDS : XL = (-,-,-,-,-,-,-,-,-,-,-,-,-,-,-)

UPPER BOUNDS : XU = (-,-,-,-,-,-,-,-,-,-,-,-,-,-,-)

START: X = (0,0,0,0,0,0,0,0,0,0,0,0,0,0,0)

F(X) = 0 (FEASIBLE)

SOLUTION: X* = (1,1,1,1,1,1,1,1,1,1,1,1,1,1,1)

F(X*) = -8252

R(X*) = 0

PROBLEM: 236

CLASSIFICATION: SUR-T1-15

SOURCE: 79

NUMBER OF VARIABLES : N = 20

NUMBER OF CONSTRAINTS: M1= 0 ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = \sum_{i=1}^{10} 100\,(x_i^{\,2} - x_{i+10})^2 + (x_i - 1)^2$$

CONSTRAINTS :

LOWER BOUNDS : XL = (-,-,-,-,-,-,-,-,-,-,-,-,-,-,-,-,-,-,-,-)

UPPER BOUNDS : XU = (-,-,-,-,-,-,-,-,-,-,-,-,-,-,-,-,-,-,-,-)

START: X = (-1.2,-1.2,-1.2,-1.2,-1.2,-1.2,-1.2,-1.2,
-1.2,-1.2,1,1,1,1,1,1,1,1,1,1)

F(X) = 242 (FEASIBLE)

SOLUTION: X* = (1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1)

F(X*) = 0

R(X*) = 0

PROBLEM: 287

CLASSIFICATION: PUR-T1-14

SOURCE: 79

NUMBER OF VARIABLES : N = 20

NUMBER OF CONSTRAINTS: M1 = 0 ,M-M1 = 0 ,S = 0

OBJECTIVE FUNCTION :

$$f(x) = \sum_{i=1}^{5} 100\,(x_i^2 + x_{i+5})^2 + (x_i - 1)^2 + 90\,(x_{i+10}^2 + x_{i+15})^2 + (x_{i+10} - 1)^2 + 10.1\,[(x_{i+5} - 1)^2 + (x_{i+15} - 1)^2] + 19.8\,(x_{i+5} - 1)\cdot(x_{i+15} - 1)$$

CONSTRAINTS :

LOWER BOUNDS : XL = (-,-,-,-,-,-,-,-,-,-,-,-,-,-,-,-,-,-,-,-)

UPPER BOUNDS : XU = (-,-,-,-,-,-,-,-,-,-,-,-,-,-,-,-,-,-,-,-)

START: X = (-3,-3,-3,-3,-3,-1,-1,-1,-1,-1,
-3,-3,-3,-3,-3,-1,-1,-1,-1,-1)

F(X) = 95960 (FEASIBLE)

SOLUTION: X* = (1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1)

F(X*) = 0

R(X*) = 0

PROBLEM: 288

CLASSIFICATION: SUR-T1-16

SOURCE: 79

NUMBER OF VARIABLES : N = 20

NUMBER OF CONSTRAINTS: M1 = 0 ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = \sum_{i=1}^{5} (x_i + 10x_{i+5})^2 + 5(x_{i+10} - x_{i+15})^2 + (x_{i+5} - 2x_{i+10})^4 + 10(x_i - x_{i+15})^4$$

CONSTRAINTS :

LOWER BOUNDS : XL = (-,-,-,-,-,-,-,-,-,-,-,-,-,-,-,-,-,-,-,-)

UPPER BOUNDS : XU = (-,-,-,-,-,-,-,-,-,-,-,-,-,-,-,-,-,-,-,-)

START: X = (3,3,3,3,3,-1,-1,-1,-1,-1,0,0,0,0,0,1,1,1,1,1)

F(X) = 1075 (FEASIBLE)

SOLUTION: X* = (0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0)

F(X*) = 0

R(X*) = 0

PROBLEM:	289
CLASSIFICATION:	GUR-T1-3
SOURCE:	79
NUMBER OF VARIABLES :	N = 30
NUMBER OF CONSTRAINTS:	M1 = 0 ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = 1 - \exp\left[-\frac{1}{60} \cdot \sum_{i=1}^{30} x_i^2 \right]$$

CONSTRAINTS :

LOWER BOUNDS : XL = (-,-,-,-,-,....,-,-,-,-)

UPPER BOUNDS : XU = (-,-,-,-,-,....,-,-,-,-)

START:	X	= (cf. Appendix A1)
	F(X)	= 0.696313 (FEASIBLE)
SOLUTION:	X*	= (0,0,...,0,0)
	F(X*)	= 0
	R(X*)	= 0

PROBLEM:	290 (Quartic)
CLASSIFICATION:	PUR-T1-15
SOURCE:	57/43/42/63/59
NUMBER OF VARIABLES :	N = 2
NUMBER OF CONSTRAINTS:	M1 = 0 ,M-M1 = 0 ,B = 0
OBJECTIVE FUNCTION :	
	$f(x) = (x^T A\ x)^2$, A = diag.(1,2)
CONSTRAINTS :	
LOWER BOUNDS :	XL = (- , -)
UPPER BOUNDS :	XU = (- , -)
START:	X = (1, 1)
	F(X) = 9 (FEASIBLE)
SOLUTION:	X* = (0, 0)
	F(X*) = 0
	R(X*) = 0

PROBLEM:	291 (Quartic)
CLASSIFICATION:	PUR-T1- 16
SOURCE:	57/43/42/63/59
NUMBER OF VARIABLES :	N = 10
NUMBER OF CONSTRAINTS:	M1 = 0 ,M-M1 = 0 ,S = 0
OBJECTIVE FUNCTION :	
	$f(x) = (x^T A\ x)^2$, A = diag.(1,2,3,...,10)
CONSTRAINTS :	
LOWER BOUNDS :	XL = (- , - , - , - , - , - , - , - , - , -)
UPPER BOUNDS :	XU = (- , - , - , - , - , - , - , - , - , -)
START: X	= (1 , 1 , 1 , 1 , 1 , 1 , 1 , 1 , 1 , 1)
F(X)	= 3025 (FEASIBLE)
SOLUTION: X*	= (0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0)
F(X*)	= 0
R(X*)	= 0

PROBLEM:	292 (Quartic)
CLASSIFICATION:	PUR-T1-17
SOURCE:	57/43/42/63/59
NUMBER OF VARIABLES :	N = 30
NUMBER OF CONSTRAINTS:	M1 = 0 ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = (x^T A\, x)^2 \qquad , A = \mathrm{diag.}(1,2,3,\ldots,30)$$

CONSTRAINTS :

LOWER BOUNDS : XL = (-,-,-,-,....,-,-,-,-)

UPPER BOUNDS : XU = (-,-,-,-,....,-,-,-,-)

START:	X	= (1,1,1,1,....,1,1,1,1)	
	F(X)	= 216225	(FEASIBLE)
SOLUTION:	X*	= (0,0,...,0,0)	
	F(X*)	= 0	
	R(X*)	= 0	

PROBLEM:	293 (Quartic)
CLASSIFICATION:	PUR-T1-18
SOURCE:	57/43/42/63/59
NUMBER OF VARIABLES :	N = 50
NUMBER OF CONSTRAINTS:	M1 = 0 ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = (x^T A\ x)^2 \qquad , \ A = \mathrm{diag.}(1,2,3,\ldots,50)$$

CONSTRAINTS :

LOWER BOUNDS : XL = (-,-,-,-,....,-,-,-,-)

UPPER BOUNDS : XU = (-,-,-,-,....,-,-,-,-)

START:	X	= (1,1,1,1,....,1,1,1,1)	
	F(X)	= 0.162563E+07	(FEASIBLE)
SOLUTION:	X*	= (0,0,...,0,0)	
	F(X*)	= 0	
	R(X*)	= 0	

PROBLEM: 294 (Banana Function)

CLASSIFICATION: SUR-T1- 17

SOURCE: 71/43/42/63/58

NUMBER OF VARIABLES : N = 6

NUMBER OF CONSTRAINTS: $M_1 = 0$, $M-M_1 = 0$, $B = 0$

OBJECTIVE FUNCTION :

$$f(x) = \sum_{k=1}^{5} [\ 100\ (x_{k+1} - x_k^2)^2 + (1 - x_k)^2\ ]$$

CONSTRAINTS :

LOWER BOUNDS : XL = (- , - , - , - , - , -)

UPPER BOUNDS : XU = (- , - , - , - , - , -)

START: X = (-1.2 , 1 ,-1.2 , 1 ,-1.2 , 1)

F(X) = 1040.60 (FEASIBLE)

SOLUTION: X* = (1, 1, 1, 1, 1, 1)

F(X*) = 0

R(X*) = 0

PROBLEM: 295 (Banana Function)

CLASSIFICATION: SUR-T1- 18

SOURCE: 71/43/42/63/58

NUMBER OF VARIABLES : N = 10

NUMBER OF CONSTRAINTS: M1 = 0 ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = \sum_{k=1}^{9} [\ 100\ (x_{k+1} - x_k^{\ 2})^2 + (1 - x_k)^2\ ]$$

CONSTRAINTS :

LOWER BOUNDS : XL = (- , - , - , - , - , - , - , - , - , -)

UPPER BOUNDS : XU = (- , - , - , - , - , - , - , - , - , -)

START: X = (-1.2, 1,-1.2, 1,-1.2, 1,-1.2, 1,-1.2, 1)

F(X) = 2057 (FEASIBLE)

SOLUTION: X* = (1 , 1 , 1 , 1 , 1 , 1 , 1 , 1 , 1 , 1)

F(X*) = 0

R(X*) = 0

PROBLEM: 296 (Banana Function)

CLASSIFICATION: SUR-T1-19

SOURCE: 71/43/42/63/58

NUMBER OF VARIABLES : N = 16

NUMBER OF CONSTRAINTS: M1 = 0 ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = \sum_{k=1}^{15} [\ 100\ (x_{k+1} - x_k^{\ 2})^2 + (1 - x_k)^2\ ]$$

CONSTRAINTS :

LOWER BOUNDS : XL = (-,-,-,-,-,-,-,-,-,-,-,-,-,-,-,-)

UPPER BOUNDS : XU = (-,-,-,-,-,-,-,-,-,-,-,-,-,-,-,-)

START: X = (-1.2, 1 ,-1.2, 1 ,-1.2, 1 ,-1.2, 1 ,-1.2, 1 ,
-1.2, 1 ,-1.2, 1 ,-1.2, 1)

F(X) = 3561.60 (FEASIBLE)

SOLUTION: X* = (1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1)

F(X*) = 0

R(X*) = 0

PROBLEM:	297 (Banana Function)
CLASSIFICATION:	SUR-T1-20
SOURCE:	71/43/42/63/58
NUMBER OF VARIABLES :	N = 30
NUMBER OF CONSTRAINTS:	$M_1 = 0$, $M-M_1 = 0$, $B = 0$

OBJECTIVE FUNCTION :

$$f(x) = \sum_{k=1}^{29} [\ 100\ (x_{k+1} - x_k^{\ 2})^2 + (1 - x_k)^2\ ]$$

CONSTRAINTS :

LOWER BOUNDS : XL = (-,-,-,-,....,-,-,-,-)

UPPER BOUNDS : XU = (-,-,-,-,....,-,-,-,-)

START:	X = (-1.2,....,-1.2)	
	F(X) = 7139	(FEASIBLE)
SOLUTION:	X* = (1,1,...,1,1)	
	F(X*) = 0	
	R(X*) = 0	

PROBLEM:	298 (Banana Function)
CLASSIFICATION:	SUR-T1-21
SOURCE:	71/43/42/63/58
NUMBER OF VARIABLES :	N = 50
NUMBER OF CONSTRAINTS:	M1 = 0 ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = \sum_{k=1}^{49} [\ 100\ (x_{k+1} - x_k^{\ 2})^2 + (1 - x_k)^2\ ]$$

CONSTRAINTS :

LOWER BOUNDS : XL = (-,-,-,-,....,-,-,-,-)

UPPER BOUNDS : XU = (-,-,-,-,....,-,-,-,-)

START:	X	= (-1.2,...,-1.2)
	F(X)	= 12221 (FEASIBLE)
SOLUTION:	X*	= (1,1,...,1,1)
	F(X*)	= 0
	R(X*)	= 0

PROBLEM:	299 (Banana Function)
CLASSIFICATION:	SUR-T1-22
SOURCE:	71/43/42/63/58
NUMBER OF VARIABLES :	N =100
NUMBER OF CONSTRAINTS:	M1 = 0 ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = \sum_{k=1}^{99} [\ 100\ (x_{k+1} - x_k^{\ 2})^2 + (1 - x_k)^2\ ]$$

CONSTRAINTS :

LOWER BOUNDS : XL = (-,-,-,-,....,-,-,-,-)

UPPER BOUNDS : XU = (-,-,-,-,....,-,-,-,-)

START:	X = (-1.2,...-1.2)	
	F(X) = 24926	(FEASIBLE)
SOLUTION:	X* = (1,1,...,1,1)	
	F(X*) = 0	
	R(X*) = 0	

PROBLEM: 300

CLASSIFICATION: QUR-T1-7

SOURCE: 39/14

NUMBER OF VARIABLES : N = 20

NUMBER OF CONSTRAINTS: M1 = 0 ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = x^T A\, x - 2x_1 \qquad , \; A = \begin{bmatrix} 1 & -1 & & & & 0 \\ -1 & 2 & -1 & & & \\ & -1 & 2 & -1 & & \\ & & \ddots & \ddots & \ddots & \\ & & & -1 & 2 & -1 \\ 0 & & & & -1 & 2 \end{bmatrix}$$

CONSTRAINTS :

LOWER BOUNDS : XL = (-,-,-,-,-,-,-,-,-,-,-,-,-,-,-,-,-,-,-,-)

UPPER BOUNDS : XU = (-,-,-,-,-,-,-,-,-,-,-,-,-,-,-,-,-,-,-,-)

START: X = (0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0)

F(X) = 0 (FEASIBLE)

SOLUTION: X* = (20,19,18,17,16,15,14,13,12,11,
10, 9, 8, 7, 6, 5, 4, 3, 2, 1)

F(X*) = -20

R(X*) = 0

PROBLEM: 301

CLASSIFICATION: QUR-T1-8

SOURCE: 39/14

NUMBER OF VARIABLES : N = 50

NUMBER OF CONSTRAINTS: M1 = 0 ,M-M 1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = x^T A\, x - 2x_1 \qquad , A = \begin{bmatrix} 1 & -1 & & & & 0 \\ -1 & 2 & -1 & & & \\ & -1 & 2 & -1 & & \\ & & \ddots & \ddots & \ddots & \\ & & & -1 & 2 & -1 \\ 0 & & & & -1 & 2 \end{bmatrix}$$

CONSTRAINTS :

LOWER BOUNDS : XL = (-,-,-,-,....,-,-,-,-)

UPPER BOUNDS : XU = (-,-,-,-,....,-,-,-,-)

START: X = (0,0,...,0,0)

F(X) = 0 (FEASIBLE)

SOLUTION: X* = (1,2,3,4,5,....,48,49,50)

F(X*) = -50

R(X*) = 0

PROBLEM: 302

CLASSIFICATION: QUR-T1-9

SOURCE: 39/14

NUMBER OF VARIABLES : N =100

NUMBER OF CONSTRAINTS: M 1 = 0 ,M-M 1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = x^T A\, x - 2x_1 \qquad , A = \begin{bmatrix} 1 & -1 & & & & & 0 \\ -1 & 2 & -1 & & & & \\ & -1 & 2 & -1 & & & \\ & & \ddots & \ddots & \ddots & & \\ & & & -1 & 2 & -1 & \\ 0 & & & & -1 & 2 & \end{bmatrix}$$

CONSTRAINTS :

LOWER BOUNDS : XL = (-,-,-,-,....,-,-,-,-)

UPPER BOUNDS : XU = (-,-,-,-,....,-,-,-,-)

START: X = (0,0,...,0,0)

F(X) = 0 (FEASIBLE)

SOLUTION: X* = (100,99,98,97,.....,4,3,2,1)

F(X*) = -100

R(X*) = 0

PROBLEM: 303

CLASSIFICATION: PUR-T1-19

SOURCE: 4/83

NUMBER OF VARIABLES : N = 20

NUMBER OF CONSTRAINTS: $M_1 = 0$, $M-M_1 = 0$, $B = 0$

OBJECTIVE FUNCTION :

$$f(x) = \sum_{i=1}^{20} x_i^2 + \left(\sum_{i=1}^{20} \frac{1}{2} i x_i\right)^2 + \left(\sum_{i=1}^{20} \frac{1}{2} i\, x_i\right)^4$$

CONSTRAINTS :

LOWER BOUNDS : XL = (-,-,-,-,-,-,-,-,-,-,-,-,-,-,-,-,-,-,-,-)

UPPER BOUNDS : XU = (-,-,-,-,-,-,-,-,-,-,-,-,-,-,-,-,-,-,-,-)

START: X = (0.1,0.1,0.1,0.1,0.1,0.1,0.1,0.1,0.1,0.1,
0.1,0.1,0.1,0.1,0.1,0.1,0.1,0.1,0.1,0.1)

F(X) = 12265.5 (FEASIBLE)

SOLUTION: X* = (0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0)

F(X*) = 0

R(X*) = 0

PROBLEM: 304

CLASSIFICATION: PUR-T1-20

SOURCE: 4/83

NUMBER OF VARIABLES : N = 50

NUMBER OF CONSTRAINTS: M1 = 0 ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = \sum_{i=1}^{50} x_i^2 + \left(\sum_{i=1}^{50} \frac{1}{2} i x_i\right)^2 + \left(\sum_{i=1}^{50} \frac{1}{2} i x_i\right)^4$$

CONSTRAINTS :

LOWER BOUNDS : XL = (-,-,-,-,....,-,-,-,-)

UPPER BOUNDS : XU = (-,-,-,-,....,-,-,-,-)

START: X = (0.1,...,0.1)

F(X) = 0.165207E+08 (FEASIBLE)

SOLUTION: X* = (0,0,...,0,0)

F(X*) = 0

R(X*) = 0

PROBLEM: 305

CLASSIFICATION: PUR-T1-21

SOURCE: 4/83

NUMBER OF VARIABLES : N =100

NUMBER OF CONSTRAINTS: $M_1 = 0$, $M-M_1 = 0$, $B = 0$

OBJECTIVE FUNCTION :

$$f(x) = \sum_{i=1}^{100} x_i^2 + \left(\sum_{i=1}^{100} \frac{1}{2} i x_i\right)^2 + \left(\sum_{i=1}^{100} \frac{1}{2} i x_i\right)^4$$

CONSTRAINTS :

LOWER BOUNDS : XL = (-,-,-,-,....,-,-,-,-)

UPPER BOUNDS : XU = (-,-,-,-,....,-,-,-,-)

START: X = (0.1,...,0.1)

F(X) = 0.406492E+10 (FEASIBLE)

SOLUTION: X* = (0,0,...,0,0)

F(X*) = 0

R(X*) = 0

PROBLEM: 307

CLASSIFICATION: SUR-P1-1

SOURCE: 37/45

NUMBER OF VARIABLES : N = 2

NUMBER OF CONSTRAINTS: M1 = 0 ,M-M1 = 0 ,B = 2

OBJECTIVE FUNCTION :

$$f(x) = \sum_{i=1}^{10} [y_i - (\exp(ix_1) + \exp(ix_2))]^2$$

$$y_i = 2 + 2i \qquad (i = 1,..,10)$$

CONSTRAINTS :

LOWER BOUNDS : XL = (0 , 0)

UPPER BOUNDS : XU = (- , -)

START: X = (0.3 , 0.4)

F(X) = 4171.31 (FEASIBLE)

SOLUTION: X* = (0.2578 ,0.2578)

F(X*) = 124.360

R(X*) = 0

PROBLEM: 308

CLASSIFICATION: SUR-P1-2

SOURCE: 37/45

NUMBER OF VARIABLES : N = 2

NUMBER OF CONSTRAINTS: M1 = 0 ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = [x_1^2 + x_2^2 + x_1 x_2]^2 + \sin^2 x_1 + \cos^2 x_2$$

CONSTRAINTS :

LOWER BOUNDS : XL = (- , -)

UPPER BOUNDS : XU = (- , -)

START: X = (3, 0.1)

F(X) = 87.686 (FEASIBLE)

SOLUTION: X* = (-0.1554 ,0.6946)

F(X*) = 0.773199

R(X*) = 0

PROBLEM: 309

CLASSIFICATION: PUR-P1-1

SOURCE: 7

NUMBER OF VARIABLES : N = 2

NUMBER OF CONSTRAINTS: M1 = 0 ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = 1.41x_1^4 - 12.76x_1^3 + 39.91x_1^2 - 51.93x_1 + 24.37 + (x_2 - 3.9)^2$$

CONSTRAINTS :

LOWER BOUNDS : XL = (- , -)

UPPER BOUNDS : XU = (- , -)

START: X = (0, 0)

F(X) = 39.58 (FEASIBLE)

SOLUTION: X* = (3.463 , 3.9)

F(X*) = -3.98717

R(X*) = 0

PROBLEM:	311
CLASSIFICATION:	PUR-P1-2
SOURCE:	41
NUMBER OF VARIABLES :	N = 2
NUMBER OF CONSTRAINTS:	M1 = 0 ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = (x_1^2 + x_2 - 11)^2 + (x_1 + x_2^2 - 7)^2$$

CONSTRAINTS :

LOWER BOUNDS : XL = (- , -)

UPPER BOUNDS : XU = (- , -)

START:	X = (1, 1)	
	F(X) = 106	(FEASIBLE)
SOLUTION:	X* = (3, 2)	
	F(X*) = 0	
	R(X*) = 0	

PROBLEM: 312

CLASSIFICATION: PUR-P1-3

SOURCE: 41

NUMBER OF VARIABLES : N = 2

NUMBER OF CONSTRAINTS: M1 = 0 ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = (x_1^2 + 12x_2 - 1)^2 + (49x_1^2 + 49x_2^2 + 84x_1 + 2324x_2 - 681)^2$$

CONSTRAINTS :

LOWER BOUNDS : XL = (- , -)

UPPER BOUNDS : XU = (- , -)

START: X = (1, 1)

F(X) = 0.333077E+07 (FEASIBLE)

SOLUTION: X* = (-21.03 ,-36.76)

F(X*) = 0

R(X*) = 0

PROBLEM: 314

CLASSIFICATION: GUR-P1-1

SOURCE: 12/41

NUMBER OF VARIABLES : N = 2

NUMBER OF CONSTRAINTS: M1 = 0 ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = (x_1 - 2)^2 + (x_2 - 1)^2 + 0.04/g_1(x) + h_1^2(x)/0.2$$

where $g_1(x) = -x_1^2/4 - x_2^2 + 1$

$h_1(x) = x_1 - 2x_2 + 1$

CONSTRAINTS :

LOWER BOUNDS : XL = (- , -)

UPPER BOUNDS : XU = (- , -)

START: X = (2, 2)

F(X) = 5.99 (FEASIBLE)

SOLUTION: X* = (1.789 , 1.374)

F(X*) = 0.169040

R(X*) = 0

PROBLEM:	315 (Around-the-World Problem)
CLASSIFICATION:	LQR-P1-1
SOURCE:	35/64
NUMBER OF VARIABLES :	N = 2
NUMBER OF CONSTRAINTS:	M1 = 3(1) ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = -x_2$$

CONSTRAINTS :

$$g_1(x) = 1 - 2x_2 + x_1 \geq 0$$
$$g_2(x) = x_1^2 + x_2^2 \geq 0$$
$$g_3(x) = 1 - x_1^2 - x_2^2 \geq 0$$

LOWER BOUNDS : XL = (- , -)

UPPER BOUNDS : XU = (- , -)

START:	X = (-0.1, -0.9)	
	F(X) = 0.9	(FEASIBLE)
SOLUTION:	X* = (0.6, 0.8)	
	F(X*) = -0.8	
	R(X*) = 0.8640E-11	

PROBLEM: 316

CLASSIFICATION: QQR-P1-1

SOURCE: 49

NUMBER OF VARIABLES : N = 2

NUMBER OF CONSTRAINTS: M1 = 0 ,M-M1 = 1 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = (x_1 - 20)^2 + (x_2 + 20)^2$$

CONSTRAINTS :

$$g_1(x) = x_1^2/100 + x_2^2/100 - 1 = 0$$

LOWER BOUNDS : XL = (- , -)

UPPER BOUNDS : XU = (- , -)

START: X = (0, 0)

F(X) = 800 (NOT FEASIBLE)

SOLUTION: X* = (7.071 ,-7.071)

F(X*) = 334.315

R(X*) = 0.1120E-10

PROBLEM: 317

CLASSIFICATION: QQR-P1-2

SOURCE: 49

NUMBER OF VARIABLES : N = 2

NUMBER OF CONSTRAINTS: M1 = 0 ,M-M1 = 1 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = (x_1 - 20)^2 + (x_2 + 20)^2$$

CONSTRAINTS :

$$g_1(x) = x_1^2/100 + x_2^2/64 - 1 = 0$$

LOWER BOUNDS : XL = (- , -)

UPPER BOUNDS : XU = (- , -)

START: X = (0, 0)

F(X) = 800 (NOT FEASIBLE)

SOLUTION: X* = (7.352 ,-5.423)

F(X*) = 372.467

R(X*) = 0.2480E-09

PROBLEM:	318
CLASSIFICATION:	QQR-P1-3
SOURCE:	49
NUMBER OF VARIABLES :	N = 2
NUMBER OF CONSTRAINTS:	M1 = 0 ,M-M1 = 1 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = (x_1 - 20)^2 + (x_2 + 20)^2$$

CONSTRAINTS :

$$g_1(x) = x_1^2/100 + x_2^2/36 - 1 = 0$$

LOWER BOUNDS : XL = (- , -)

UPPER BOUNDS : XU = (- , -)

START:	X = (0, 0)	
	F(X) = 800	(NOT FEASIBLE)
SOLUTION:	X* = (7.809 ,-3.746)	
	F(X*) = 412.750	
	R(X*) = 0.1620E-12	

PROBLEM: 319

CLASSIFICATION: QQR-P1-4

SOURCE: 49

NUMBER OF VARIABLES : N = 2

NUMBER OF CONSTRAINTS: M1 = 0 ,M-M1 = 1 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = (x_1 - 20)^2 + (x_2 + 20)^2$$

CONSTRAINTS :

$$g_1(x) = x_1^2/100 + x_2^2/16 - 1 = 0$$

LOWER BOUNDS : XL = (- , -)

UPPER BOUNDS : XU = (- , -)

START: X = (0, 0)

F(X) = 800 (NOT FEASIBLE)

SOLUTION: X* = (8.492 ,-2.112)

F(X*) = 452.404

R(X*) = 0.3710E-09

PROBLEM: 320

CLASSIFICATION: QQR-P1-5

SOURCE: 49

NUMBER OF VARIABLES : N = 2

NUMBER OF CONSTRAINTS: M1 = 0 ,M-M1 = 1 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = (x_1 - 20)^2 + (x_2 + 20)^2$$

CONSTRAINTS :

$$g_1(x) = x_1^2/100 + x_2^2/4 - 1 = 0$$

LOWER BOUNDS : XL = (- , -)

UPPER BOUNDS : XU = (- , -)

START: X = (0, 0)

F(X) = 800 (NOT FEASIBLE)

SOLUTION: X* = (9.395 ,-0.6646)

F(X*) = 485.531

R(X*) = 0.3290E-13

PROBLEM: 321

CLASSIFICATION: QQR-P1-6

SOURCE: 49

NUMBER OF VARIABLES : N = 2

NUMBER OF CONSTRAINTS: M1 = 0 ,M-M1 = 1 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = (x_1 - 20)^2 + (x_2 + 20)^2$$

CONSTRAINTS :

$$g_1(x) = x_1^2/100 + x_2^2/1 - 1 = 0$$

LOWER BOUNDS : XL = (- , -)

UPPER BOUNDS : XU = (- , -)

START: X = (0, 0)

F(X) = 800 (NOT FEASIBLE)

SOLUTION: X* = (9.816 ,-0.1909)

F(X*) = 496.112

R(X*) = 0.1680E-10

PROBLEM: 322

CLASSIFICATION: QQR-P1-7

SOURCE: 49

NUMBER OF VARIABLES : N = 2

NUMBER OF CONSTRAINTS: M1 = 0 ,M-M1 = 1 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = (x_1 - 20)^2 + (x_2 + 20)^2$$

CONSTRAINTS :

$$g_1(x) = x_1^2/100 + x_2^2/0.01 - 1 = 0$$

LOWER BOUNDS : XL = (- , -)

UPPER BOUNDS : XU = (- , -)

START: X = (0, 0)

F(X) = 800 (NOT FEASIBLE)

SOLUTION: X* = (9.998 ,-0.1999E-02)

F(X*) = 499.96

R(X*) = 0.98E-11

PROBLEM: 323

CLASSIFICATION: QQR-P1-8

SOURCE: 41

NUMBER OF VARIABLES : N = 2

NUMBER OF CONSTRAINTS: M1 = 2(1) ,M-M1 = 0 ,B = 2

OBJECTIVE FUNCTION :

$$f(x) = x_1^2 + x_2^2 - 4x_1 + 4$$

CONSTRAINTS :

$$g_1(x) = x_1 - x_2 + 2 \geq 0$$

$$g_2(x) = -x_1^2 + x_2 - 1 \geq 0$$

$$x_1 \geq 0$$

$$x_2 \geq 0$$

LOWER BOUNDS : XL = (0 , 0)

UPPER BOUNDS : XU = (- , -)

START: X = (0, 1)

F(X) = 5 (FEASIBLE)

SOLUTION: X* = (0.5536 , 1.306)

F(X*) = 3.79894

R(X*) = 0.3130E-08

PROBLEM: 324

CLASSIFICATION: QQR-P1-9

SOURCE: 79

NUMBER OF VARIABLES : N = 2

NUMBER OF CONSTRAINTS: M1 = 2 ,M-M1 = 0 ,B = 1

OBJECTIVE FUNCTION :

$$f(x) = 0.01x_1^2 + x_2^2$$

CONSTRAINTS :

$$g_1(x) = x_1x_2 - 25 \geq 0$$

$$g_2(x) = x_1^2 + x_2^2 - 25 \geq 0$$

$$x_1 \geq 2$$

LOWER BOUNDS : XL = (2 , -)

UPPER BOUNDS : XU = (- , -)

START: X = (2, 2)

F(X) = 4.04 (NOT FEASIBLE)

SOLUTION: X* = (15.81 , 1.581)

F(X*) = 5

R(X*) = 0.3880E-07

PROBLEM: 325

CLASSIFICATION: QQR-P1-10

SOURCE: 41

NUMBER OF VARIABLES : N = 2

NUMBER OF CONSTRAINTS: M1 = 2(1) ,M-M1 = 1 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = x_1^2 + x_2$$

CONSTRAINTS :

$$g_1(x) = -(x_1 + x_2) + 1 \geq 0$$

$$g_2(x) = -(x_1 + x_2^2) + 1 \geq 0$$

$$g_3(x) = x_1^2 + x_2^2 - 9 = 0$$

LOWER BOUNDS : XL = (- , -)

UPPER BOUNDS : XU = (- , -)

START: X = (-3, 0)

F(X) = 9 (FEASIBLE)

SOLUTION: X* = (-2.372 ,-1.836)

F(X*) = 3.79134

R(X*) = 0.312E-8

PROBLEM: 326

CLASSIFICATION: QGR-P1-1

SOURCE: 41

NUMBER OF VARIABLES : N = 2

NUMBER OF CONSTRAINTS: M1 = 2 ,M-M1 = 0 ,B = 2

OBJECTIVE FUNCTION :

$$f(x) = x_1^2 + x_2^2 - 16x_1 - 10x_2$$

CONSTRAINTS :

$$g_1(x) = 11 - x_1^2 + 6x_1 - 4x_2 \geq 0$$

$$g_2(x) = x_1x_2 - 3x_2 - e^{x_1-3} + 1 \geq 0$$

LOWER BOUNDS : XL = (0 , 0)

UPPER BOUNDS : XU = (- , -)

START: X = (4, 3)

F(X) = -69 (FEASIBLE)

SOLUTION: X* = (5.240 , 3.746)

F(X*) = -79.8078

R(X*) = 0.3980E-11

PROBLEM: 327

CLASSIFICATION: SQR-P1-1

SOURCE: 35/38/79

NUMBER OF VARIABLES : N = 2

NUMBER OF CONSTRAINTS: M1 = 1 ,M-M1 = 0 ,B = 2

OBJECTIVE FUNCTION :

$$f(x) = \sum_{i=1}^{44} \{y_i - x_1 - (0.49 - x_1)\exp[-x_2(z_i - 8)]\}^2$$

y_i, z_i: cf. Appendix A

CONSTRAINTS :

$$g_1(x) = -0.09 - x_1 x_2 + 0.49 x_2 \geq 0$$

LOWER BOUNDS : XL = (0.4, 0.4)

UPPER BOUNDS : XU = (- , -)

START: X = (0.42 , 5)

F(X) = 0.0307986 (FEASIBLE)

SOLUTION: X* = (0.4219 , 5)

F(X*) = 0.0306463

R(X*) = 0

PROBLEM: 328 (Gear train of minimum inertia)

CLASSIFICATION: PBR-P1-1

SOURCE: 27/55/26/73

NUMBER OF VARIABLES : N = 2

NUMBER OF CONSTRAINTS: M1 = 0 ,M-M1 = 0 ,B = 4

OBJECTIVE FUNCTION :

$$f(x) = 0.1[12 + x_1^2 + (1 + x_2^2)/x_1^2 + (x_1^2 x_2^2 + 100)/x_1^4 x_2^4]$$

CONSTRAINTS :

LOWER BOUNDS : XL = (1, 1)

UPPER BOUNDS : XU = (3, 3)

START: X = (0.5, 0.5)

F(X) = 2563.32 (NOT FEASIBLE)

SOLUTION: X* = (1.743 , 2.030)

F(X*) = 1.74415

R(X*) = 0

PROBLEM: 329

CLASSIFICATION: PQR-P1-1

SOURCE: 35/38

NUMBER OF VARIABLES : N = 2

NUMBER OF CONSTRAINTS: M1 = 3 ,M-M1 = 0 ,B = 4

OBJECTIVE FUNCTION :

$$f(x) = (x_1 - 10)^3 + (x_2 - 20)^3$$

CONSTRAINTS :

$$g_1(x) = (x_1 - 5)^2 + (x_2 - 5)^2 - 100 \geq 0$$

$$g_2(x) = (x_1 - 6)^2 + (x_2 - 5)^2 \geq 0$$

$$g_3(x) = 82.81 - (x_1 - 6)^2 - (x_2 - 5)^2 \geq 0$$

LOWER BOUNDS : XL = (13, 0)

UPPER BOUNDS : XU = (16, 15)

START: X = (14.35 , 8.6)

F(X) = -1399.23 (FEASIBLE)

SOLUTION: X* = (14.09 ,0.8430)

F(X*) = -6961.81

R(X*) = 0.6750E-13

PROBLEM:	330 (Journal bearing design,case 2)
CLASSIFICATION:	PPI-P1-1
SOURCE:	27/6/26/73
NUMBER OF VARIABLES :	N = 2
NUMBER OF CONSTRAINTS:	M1 = 1 ,M-M1 = 0 ,B = 4

OBJECTIVE FUNCTION :

$$f(x) = 0.1[0.44(x_1^3/x_2^2) + 10/x_1 + 0.592(x_1/x_2^3)]$$

CONSTRAINTS :

$$g_1(x) = 1 - 8.62(x_2^3/x_1) \geq 0$$

LOWER BOUNDS :	XL = (0, 0)	
UPPER BOUNDS :	XU = (5, 5)	
START:	X = (2.5, 2.5)	
	F(X) = 0.519472	(NOT FEASIBLE)
SOLUTION:	X* = (1.287 ,0.5305)	
	F(X*) = 1.62058	
	R(X*) = 0.3440E-09	

PROBLEM: 331

CLASSIFICATION: GLI-P1-1

SOURCE: 77

NUMBER OF VARIABLES : N = 2

NUMBER OF CONSTRAINTS: M_1 = 1(1) ,$M-M_1$ = 0 ,B = 3

OBJECTIVE FUNCTION :

$$f(x) = \frac{1}{x_1} \ln\left[\frac{2\ln(x_2)}{\ln(x_1+x_2)}\right]$$

CONSTRAINTS :

$$g_1(x) = 1 - x_1 - x_2 \geq 0$$

LOWER BOUNDS : XL = (0.0001 ,0.0001)

UPPER BOUNDS : XU = (- , 1)

START: X = (0.5, 0.1)

F(X) = 4.39781 (FEASIBLE)

SOLUTION: X* = (0.6175 ,0.1039)

F(X*) = 4.258

R(X*) = 0

PROBLEM: 332 (Cam Design Problem)

CLASSIFICATION: GGR-P1-1

SOURCE: 26/55/73

NUMBER OF VARIABLES : N = 2

NUMBER OF CONSTRAINTS: M1 = 2 ,M-M1 = 0 ,B = 4

OBJECTIVE FUNCTION :

$$f(x) = \frac{\pi}{3.6} \sum_{i=1}^{100} (\log(t_i) + x_2 \sin(t_i) + x_1 \cos(t_i))^2 + (\log(t_i) + x_2 \cos(t_i) - x_1 \sin(t_i))^2$$

$$t_i = \pi(1/3+(i-1)/180), \; i = 1,\ldots,100$$

CONSTRAINTS :

$$g_1(x) = 30 - pmax \geq 0$$

$$g_2(x) = pmax + 30 \geq 0$$

$$pmax = \max_{1 \leq i \leq 100} \left(\frac{180}{\pi} \arctan \left|\frac{1/t_i - x_1}{\log(t_i) + x_2}\right|\right)$$

$$t_i = \pi(1/3+(i-1)/180), \; i = 1,\ldots,100$$

LOWER BOUNDS : XL = (0 , 0)

UPPER BOUNDS : XU = (1.5, 1.5)

START: X = (0.75 ,0.75)

F(X) = 217.361 (FEASIBLE)

SOLUTION: X* = (0.9114 ,0.02928)

F(X*) = 114.95

R(X*) = 0.1530E-07

PROBLEM:	333
CLASSIFICATION:	SUR-P1-3
SOURCE:	29
NUMBER OF VARIABLES :	N = 3
NUMBER OF CONSTRAINTS:	$M_1 = 0$, $M-M_1 = 0$, $B = 0$

OBJECTIVE FUNCTION :

$$f(x) = \sum_{i=1}^{8} \left[\frac{y_i - x_1 \exp(-x_2 a_i) - x_3}{y_i} \right]^2$$

a_i, y_i : cf. Appendix A

CONSTRAINTS :

LOWER BOUNDS : XL = (- , - , -)

UPPER BOUNDS : XU = (- , - , -)

START:	X	= (30 ,0.04 , 3)	
	F(X)	= 34.0064	(FEASIBLE)
SOLUTION:	X*	= (89.9 ,0.06699 ,0.4781)	
	F(X*)	= 0.0432 704	
	R(X*)	= 0	

PROBLEM:	334
CLASSIFICATION:	SUR-P1-4
SOURCE:	37/3
NUMBER OF VARIABLES :	N = 3
NUMBER OF CONSTRAINTS:	$M_1 = 0$, $M-M_1 = 0$, $B = 0$

OBJECTIVE FUNCTION :

$$f(x) = \sum_{i=1}^{15} [y_i - (x_1 + u_i/(x_2 v_i + x_3 w_i))]^2$$

$u_i = i$, $v_i = 16 - i$, $w_i = \min(u_i, v_i)$,

y_i: cf. Appendix A

CONSTRAINTS :

LOWER BOUNDS : XL = (- , - , -)

UPPER BOUNDS : XU = (- , - , -)

START:	X = (1, 1, 1)	
	F(X) = 41.6817	(FEASIBLE)
SOLUTION:	X* = (0.08248 , 1.135 , 2.34)	
	F(X*) = 0.821492E-02	
	R(X*) = 0	

PROBLEM: 335

CLASSIFICATION: LQR-P1-2

SOURCE: 79

NUMBER OF VARIABLES : N = 3

NUMBER OF CONSTRAINTS: M1 = 0 ,M-M1 = 2 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = -(0.001x_1 + x_2)$$

CONSTRAINTS :

$$g_1(x) = 1000x_1^2 + 100x_2^2 - x_3 = 0$$

$$g_2(x) = 100x_1^2 + 400x_2^2 + x_3 - 0.01 = 0$$

LOWER BOUNDS : XL = (- , - , -)

UPPER BOUNDS : XU = (- , - , -)

START: X = (1, 1, 1)

F(X) = -1.001 (NOT FEASIBLE)

SOLUTION: X* = (0.2031E-05,0.4472E-02,0.2000E-02)

F(X*) = -0.447214E-02

R(X*) = 0.8720E-08

PROBLEM: 336

CLASSIFICATION: LQR-P1-3

SOURCE: 41

NUMBER OF VARIABLES : N = 3

NUMBER OF CONSTRAINTS: $M_1 = 0$, $M-M_1 = 2(1)$, B = 0

OBJECTIVE FUNCTION :

$$f(x) = 7x_1 - 6x_2 + 4x_3$$

CONSTRAINTS :

$$g_1(x) = 5x_1 + 5x_2 - 3x_3 - 6 = 0$$

$$g_2(x) = x_1^2 + 2x_2^2 + 3x_3^2 - 1 = 0$$

LOWER BOUNDS : XL = (- , - , -)

UPPER BOUNDS : XU = (- , - , -)

START: X = (0, 0, 0)

F(X) = 0 (NOT FEASIBLE)

SOLUTION: X* = (0.5346 ,0.534 ,-0.2191)

F(X*) = -0.337896

R(X*) = 0.2850E-08

PROBLEM: 337

CLASSIFICATION: QQR-P1-11

SOURCE: 79

NUMBER OF VARIABLES : N = 3

NUMBER OF CONSTRAINTS: M1 = 1 ,M-M1 = 0 ,B = 2

OBJECTIVE FUNCTION :

$$f(x) = 9x_1^2 + x_2^2 + 9x_3^2$$

CONSTRAINTS :

$$g_1(x) = x_1 x_2 - 1 \geq 0$$

LOWER BOUNDS : XL = (- , 1 , -)

UPPER BOUNDS : XU = (- , - , 1)

START: X = (1, 1, 1)

F(X) = 19 (FEASIBLE)

SOLUTION: X* = (0.5774 , 1.732 ,-0.2026E-05)

F(X*) = 6

R(X*) = 0.1520E-09

PROBLEM: 338

CLASSIFICATION: QQR-P1-12

SOURCE: 49

NUMBER OF VARIABLES : N = 3

NUMBER OF CONSTRAINTS: M1 = 0 ,M-M1 = 2(1) ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = -(x_1^2 + x_2^2 + x_3^2)$$

CONSTRAINTS :

$$g_1(x) = 0.5x_1 + x_2 + x_3 - 1 = 0$$

$$g_2(x) = x_1^2 + \frac{2}{3}x_2^2 + \frac{1}{4}x_3^2 - 4 = 0$$

LOWER BOUNDS : XL = (- , - , -)

UPPER BOUNDS : XU = (- , - , -)

START: X = (0, 0, 0)

F(X) = 0 (NOT FEASIBLE)

SOLUTION: X* = (0.3669 , 2.244 ,-1.427)

F(X*) = -7.20570

R(X*) = 0.7760E-11

PROBLEM: 339 (Geometric Container Problem)

CLASSIFICATION: PQR-P1-2

SOURCE: 63/25/84

NUMBER OF VARIABLES : N = 3

NUMBER OF CONSTRAINTS: M1 = 1 ,M-M1 = 0 ,B = 3

OBJECTIVE FUNCTION :

$$f(x) = 0.2/(x_1 x_2 x_3) + 4/x_1 + 3/x_3$$

CONSTRAINTS :

$$g_1(x) = 10 - 2x_1 x_3 - x_1 x_2 \geq 0$$

LOWER BOUNDS : XL = (0 , 0 , 0)

UPPER BOUNDS : XU = (- , - , -)

START: X = (1, 1, 1)

F(X) = 7.2 (FEASIBLE)

SOLUTION: X* = (2.380 ,0.3162 , 1.943)

F(X*) = 3.36168

R(X*) = 0

PROBLEM:	340 (Pascal Problem)
CLASSIFICATION:	PLR-P1-1
SOURCE:	63
NUMBER OF VARIABLES :	N = 3
NUMBER OF CONSTRAINTS:	M1 = 1(1) ,M-M1 = 0 ,B = 1

OBJECTIVE FUNCTION :

$$f(x) = -x_1 x_2 x_3$$

CONSTRAINTS :

$$g_1(x) = 1.8 - x_1 - 2x_2 - 2x_3 \geq 0$$

LOWER BOUNDS :	XL = (- , - , -)	
UPPER BOUNDS :	XU = (1 , - , -)	
START:	X = (1, 1, 1)	
	F(X) = -1	(NOT FEASIBLE)
SOLUTION:	X* = (0.6 ,0.3 ,0.3)	
	F(X*) = -0.054	
	R(X*) = 0	

PROBLEM: 341

CLASSIFICATION: PQR-P1-3

SOURCE: 2/24/43/79

NUMBER OF VARIABLES : N = 3

NUMBER OF CONSTRAINTS: M1 = 1 ,M-M1 = 0 ,B = 3

OBJECTIVE FUNCTION :

$$f(x) = -x_1 x_2 x_3$$

CONSTRAINTS :

$$g_1(x) = -x_1^2 - 2x_2^2 - 4x_3^2 + 48 \geq 0$$

LOWER BOUNDS : XL = (0 , 0 , 0)

UPPER BOUNDS : XU = (- , - , -)

START: X = (1, 1, 1)

F(X) = -1 (FEASIBLE)

SOLUTION: X* = (4 , 2.828 , 2)

F(X*) = -22.6274

R(X*) = 0.3270E-07

PROBLEM:	342
CLASSIFICATION:	PQR-P1-4
SOURCE:	24/43
NUMBER OF VARIABLES :	N = 3
NUMBER OF CONSTRAINTS:	M1 = 1 ,M-M1 = 0 ,B = 3

OBJECTIVE FUNCTION :

$$f(x) = -x_1 x_2 x_3$$

CONSTRAINTS :

$$g_1(x) = 48 - x_1^2 - 2x_2^2 - 4x_3^2 \geq 0$$

LOWER BOUNDS : XL = (0 , 0 , 0)

UPPER BOUNDS : XU = (- , - , -)

START:	X	= (100 , 100 , 100)	
	F(X)	= -1	(FEASIBLE)
SOLUTION:	X*	= (4 , 2.828 , 2)	
	F(X*)	= -22.6274	
	R(X*)	= 0.175E-7	

PROBLEM:	343 (Flywheel Design)
CLASSIFICATION:	PPR-P1-1
SOURCE:	27/76/26/73
NUMBER OF VARIABLES :	N = 3
NUMBER OF CONSTRAINTS:	M1 = 2 ,M-M1 = 0 ,B = 6

OBJECTIVE FUNCTION :

$$f(x) = (-0.0201x_1^4x_2x_3^2)*10^{-7}$$

CONSTRAINTS :

$$g_1(x) = 675 - x_1^2x_2 \geq 0$$

$$g_2(x) = 0.419 - 10^{-7}x_1^2x_3^2 \geq 0$$

LOWER BOUNDS :	XL = (0, 0, 0)
UPPER BOUNDS :	XU = (36, 5,125)
START:	X = (22.3 ,0.5 , 125)
	F(X) = -3.88334 (NOT FEASIBLE)
SOLUTION:	X* = (16.51 , 2.477 , 124)
	F(X*) = -5.68478
	R(X*) = 0.28E-12

PROBLEM: 344

CLASSIFICATION: PPR-P1-2

SOURCE: 79

NUMBER OF VARIABLES : N = 3

NUMBER OF CONSTRAINTS: M1 = 0 ,M-M1 = 1 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = (x_1 - 1)^2 + (x_1 - x_2)^2 + (x_2 - x_3)^4$$

CONSTRAINTS :

$$g_1(x) = x_1(1 + x_2^2) + x_3^4 - 4 - 3\sqrt{2} = 0$$

LOWER BOUNDS : XL = (- , - , -)

UPPER BOUNDS : XU = (- , - , -)

START: X = (2, 2, 2)

F(X) = 1 (NOT FEASIBLE)

SOLUTION: X* = (1.105 , 1.197 , 1.535)

F(X*) = 0.0325682

R(X*) = 0.6670E-08

PROBLEM: 345

CLASSIFICATION: PPR-P1-3

SOURCE: 49

NUMBER OF VARIABLES : N = 3

NUMBER OF CONSTRAINTS: M1 = 0 ,M-M1 = 1 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = (x_1 - 1)^2 + (x_1 - x_2)^2 + (x_2 - x_3)^4$$

CONSTRAINTS :

$$g_1(x) = x_1(1 + x_2^2) + x_3^4 - 4 - 3\sqrt{2} = 0$$

LOWER BOUNDS : XL = (- , - , -)

UPPER BOUNDS : XU = (- , - , -)

START: X = (0, 0, 0)

F(X) = 1 (NOT FEASIBLE)

SOLUTION: X* = (1.105 , 1.197 , 1.535)

F(X*) = 0.0325682

R(X*) = 0.77E-08

PROBLEM:	346 (Design of disc flywheel)
CLASSIFICATION:	PPR-P1-4
SOURCE:	26/76
NUMBER OF VARIABLES :	N = 3
NUMBER OF CONSTRAINTS:	M1 = 2 ,M-M1 = 0 ,B = 6

OBJECTIVE FUNCTION :

$$f(x) = -(0.0201/10^7)x_1^4 x_2 x_3^2$$

CONSTRAINTS :

$$g_1(x) = 675 - x_1^2 x_2 \geq 0$$

$$g_2(x) = 0.419 - (x_1^2 x_3^2)/10^7 \geq 0$$

LOWER BOUNDS :	XL = (0, 0, 0)	
UPPER BOUNDS :	XU = (36, 5,125)	
START:	X = (22.3 ,0.5 , 125)	
	F(X) = -3.88334	(NOT FEASIBLE)
SOLUTION:	X* = (16.51 , 2.477 , 124)	
	F(X*) = -5.68478	
	R(X*) = 0.23E-12	

PROBLEM: 347

CLASSIFICATION: GLR-P1-1

SOURCE: 79

NUMBER OF VARIABLES : N = 3

NUMBER OF CONSTRAINTS: M1 = 1 ,M-M1 = 0 ,B = 6

OBJECTIVE FUNCTION :

$$f(x) = \sum_{i=1}^{3} A_i * \log(w_i)$$

A_i, w_i : cf. Appendix A

CONSTRAINTS :

$$g_1(x) = x_1 + x_2 + x_3 - 1 = 0$$

LOWER BOUNDS : XL = (0, 0, 0)

UPPER BOUNDS : XU = (1, 1, 1)

START: X = (0.7 ,0.2 ,0.1)

F(X) = 25698.3 (FEASIBLE)

SOLUTION: X* = (0, 0, 1)

F(X*) = 17374.6

R(X*) = 0

PROBLEM:	348 (Whirlpool Design Problem)
CLASSIFICATION:	GGR-PO-2
SOURCE:	73
NUMBER OF VARIABLES :	N = 3
NUMBER OF CONSTRAINTS:	M1 = 0 ,M-M1 = 1 ,B = 4

OBJECTIVE FUNCTION :

$$f(x) = FX$$

FX : cf. Appendix A

CONSTRAINTS :

$$g_1(x) = G(1) = 0$$

G : cf. Appendix A

LOWER BOUNDS :	XL = (- , 13.13 , -)
UPPER BOUNDS :	XU = (0.044 , 24 , 600)
START: X	= (0.1, 18 ,144)
F(X)	= 30.9861 (NOT FEASIBLE)
SOLUTION: X*	= (0.044 , 24 , 85.61)
F(X*)	= 36.9708
R(X*)	= 0.480E-9

PROBLEM: 350

CLASSIFICATION: SUR-P1-5

SOURCE: 37/47

NUMBER OF VARIABLES : N = 4

NUMBER OF CONSTRAINTS: M1 = 0 ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = \sum_{i=1}^{11} \left[y_i - \frac{x_1 (u_i^2 + x_2 u_i)}{(u_i^2 + x_3 u_i + x_4)} \right]^2$$

y_i, u_i: cf. Appendix A

CONSTRAINTS :

LOWER BOUNDS : XL = (- , - , - , -)

UPPER BOUNDS : XU = (- , - , - , -)

START: X = (0.25 ,0.39 ,0.415 ,0.39)

F(X) = 0.531317E-02 (FEASIBLE)

SOLUTION: X* = (0.1928 ,0.1913 ,0.1231 ,0.1361)

F(X*) = 0.307506E-03

R(X*) = 0

PROBLEM:	351
CLASSIFICATION:	SUR-P1-6
SOURCE:	41
NUMBER OF VARIABLES :	N = 4
NUMBER OF CONSTRAINTS:	M1 = 0 ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = 10^4 \sum_{i=1}^{7} \left(\frac{(x_1^2 + x_2^2 a_i + x_3^2 a_i^2)/(1 + x_4^2 a_i) - b_i}{b_i} \right)^2$$

a_i, b_i: cf. Appendix A

CONSTRAINTS :

LOWER BOUNDS : XL = (- , - , - , -)

UPPER BOUNDS : XU = (- , - , - , -)

START:	X = (2.7 , 90 , 1500 , 10)	
	F(X) = 29053	(FEASIBLE)
SOLUTION:	X* = (2.714 , 140.4 , 1707 , 31.51)	
	F(X*) = 318.581	
	R(X*) = 0	

PROBLEM: 352

CLASSIFICATION: SUR-P1-7

SOURCE: 37/23

NUMBER OF VARIABLES : N = 4

NUMBER OF CONSTRAINTS: M1 = 0 ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = \sum_{i=1}^{20} \left[(x_1 + x_2 t_i - \exp(t_i))^2 + (x_3 + x_4 \sin(t_i) - \cos(t_i))^2 \right]$$

$$t_i = 0.2\,i \qquad i = 1,\ldots,20$$

CONSTRAINTS :

LOWER BOUNDS : XL = (- , - , - , -)

UPPER BOUNDS : XU = (- , - , - , -)

START: X = (25, 5, -5, -1)

F(X) = 11303.8 (FEASIBLE)

SOLUTION: X* = (-10.22 , 11.91 ,-0.4580 ,0.5803)

F(X*) = 903.234

R(X*) = 0

PROBLEM:	353
CLASSIFICATION:	LGR-P1-1
SOURCE:	79
NUMBER OF VARIABLES :	N = 4
NUMBER OF CONSTRAINTS:	M1 = 2(1) ,M-M1 = 1(1) ,B = 4

OBJECTIVE FUNCTION :

$$f(x) = -(24.55x_1 + 26.75x_2 + 39.00x_3 + 40.50x_4)$$

CONSTRAINTS :

$$g_1(x) = 2.3x_1 + 5.6x_2 + 11.1x_3 + 1.3x_4 - 5 \geq 0$$

$$g_2(x) = 12.0x_1 + 11.9x_2 + 41.8x_3 + 52.1x_4 - 1.645\sqrt{Q(x)} - 12 \geq 0$$

$$g_3(x) = x_1 + x_2 + x_3 + x_4 - 1 = 0$$

$$Q(x) = (0.53x_1)^2 + (0.44x_2)^2 + (4.50x_3)^2 + (0.79x_4)^2$$

LOWER BOUNDS :	XL = (0 , 0 , 0 , 0)	
UPPER BOUNDS :	XU = (- , - , - , -)	
START:	X = (0 , 0 ,0.4 ,0.6)	
	F(X) = -39.9	(FEASIBLE)
SOLUTION:	X* = (0 , 0 ,0.3776 ,0.6224)	
	F(X*) = -39.9337	
	R(X*) = 0	

PROBLEM: 354

CLASSIFICATION: PLR-P1-2

SOURCE: 79

NUMBER OF VARIABLES : N = 4

NUMBER OF CONSTRAINTS: M1 = 1(1) ,M-M1 = 0 ,B = 4

OBJECTIVE FUNCTION :

$$f(x) = (x_1 + 10x_2)^2 + 5(x_3 - x_4)^2 + (x_2 - 2x_3)^4 + 10(x_1 - x_4)^4$$

CONSTRAINTS :

$$g_1(x) = x_1 + x_2 + x_3 + x_4 - 1 \geq 0$$

LOWER BOUNDS : XL = (- , - , - , -)

UPPER BOUNDS : XU = (20, 20, 20, 20)

START: X = (3, -1, 0, 1)

F(X) = 215 (FEASIBLE)

SOLUTION: X* = (0.5034 ,-0.4557 ,0.2358 ,0.3064)

F(X*) = 0.113784

R(X*) = 0

PROBLEM:	355 (Circuits Problem)
CLASSIFICATION:	PPR-P1-5
SOURCE:	63/61
NUMBER OF VARIABLES :	N = 4
NUMBER OF CONSTRAINTS:	M1 = 0 ,M-M1 = 1 ,B = 4

OBJECTIVE FUNCTION :

$$f(x) = r_1^2 + r_2^2 \; : \quad \begin{aligned} r_1 &= 11 - x_1x_4 - x_2x_4 + x_3x_4 \\ r_2 &= x_1 + 10x_2 - x_3 + x_4 + x_2x_4(x_3 - x_1) \\ r_3 &= 11 - 4x_1x_4 - 4x_2x_4 + x_3x_4 \\ r_4 &= 2x_1 + 20x_2 - 0.5x_3 + 2x_4 + 2x_2x_4(x_3 - 4x_1) \end{aligned}$$

CONSTRAINTS :

$$g_1(x) = r_1^2 + r_2^2 - r_3^2 - r_4^2 = 0$$

LOWER BOUNDS : XL = (0.1, 0.1, 0 , 0)

UPPER BOUNDS : XU = (- , - , - , -)

START:	X	= (0, 0, 0, 0)
	F(X)	= 121 (NOT FEASIBLE)
SOLUTION:	X*	= (1.917 ,0.1 , 0 , 1.972)
	F(X*)	= 69.6755
	R(X*)	= 0.209E-12

PROBLEM:	356 (Welded Beam Problem)
CLASSIFICATION:	PGR-PO-1
SOURCE:	73
NUMBER OF VARIABLES :	N = 4
NUMBER OF CONSTRAINTS:	M1 = 5(1) ,M-M1 = 0 ,B = 3

OBJECTIVE FUNCTION :

$$f(x) = 1.10471x_1^2x_2 + .04811x_3x_4(14 + x_2)$$

CONSTRAINTS :

$$g_1(x) = x_4 - x_1 \geq 0$$

$$g_2(x) = 10^{-6}(13600-(t_1^2+2t_1t_2x_2/(x_2^2+(x_1+x_3)^2)^{1/2}+t_2^2)^{1/2}) \geq 0$$

$$g_3(x) = 3 - 5.04/(x_4x_3^2) \geq 0$$

$$g_4(x) = 10^{-6}(4.013\sqrt{EG}(1-x_3\sqrt{E/G}/28)/196-6000) \geq 0$$

$$g_5(x) = .25-2.1952/(x_4x_3^3) \geq 0$$

$$t_1 = 6000/(1.414x_1x_2)$$

$$t_2 = 3000(14+x_2/2)(x_2^2+(x_1+x_3)^2)^{1/2}/J$$

$$J = .707x_1x_2(x_2^2/6+(x_1+x_3)^2/2),\ E = x_3x_4^3 10^7/4,\ G = 4x_3x_4^3 10^6$$

LOWER BOUNDS : XL = (0.125 , 0 , 0 , -)

UPPER BOUNDS : XU = (- , - , - , -)

START:	X	= (1, 7, 8, 1)	
	F(X)	= 15.8154	(FEASIBLE)
SOLUTION:	X*	= (0.2444 , 6.219 , 8.291 ,0.2444)	
	F(X*)	= 2.38116	
	R(X*)	= 0.8190E-15	

PROBLEM:	357
CLASSIFICATION:	GBR-PO-1
SOURCE:	26/73
NUMBER OF VARIABLES :	N = 4
NUMBER OF CONSTRAINTS:	M1 = 0 ,M-M1 = 0 ,B = 8

OBJECTIVE FUNCTION :

$$f(x) = FX$$

FX : cf. Appendix A

CONSTRAINTS :

LOWER BOUNDS : XL = (0, 0, 0, 0)

UPPER BOUNDS : XU = (150, 50,100,100)

START:	X	= (136 , 0 , 74.8 , 75.7)	
	F(X)	= 0.367334	(FEASIBLE)
SOLUTION:	X*	= (136 ,0.03137 , 73.59 , 72.19)	
	F(X*)	= 0.358457	
	R(X*)	= 0	

PROBLEM:	358
CLASSIFICATION:	SBR-P1-1
SOURCE:	37/60
NUMBER OF VARIABLES :	N = 5
NUMBER OF CONSTRAINTS:	M1 = 0 ,M-M1 = 0 ,B = 10

OBJECTIVE FUNCTION :

$$f(x) = \sum_{i=1}^{33} [y_i - (x_1 + x_2 \exp(-x_4 t_i) + x_3 \exp(-x_3 t_i))]^2$$

$$t_i = 10(i - 1) \qquad i = 1,..,33$$

y_i: cf. Appendix A

CONSTRAINTS :

LOWER BOUNDS : XL = (-0.5 , 1.5 ,-2 , 0.001 , 0.001)

UPPER BOUNDS : XU = (0.5, 2.5, -1.0, 0.1, 0.1)

START: X = (0.5, 1.5, -1 , 0.01, 0.02)

F(X) = 0.879026 (FEASIBLE)

SOLUTION: X* = (0.3754 , 1.936 ,-1.465 ,0.01287 , 0.02212)

F(X*) = 0.546E-04

R(X*) = 0

PROBLEM:	359
CLASSIFICATION:	LLR-P1-1
SOURCE:	79

NUMBER OF VARIABLES : N = 5

NUMBER OF CONSTRAINTS: M1=14(14) ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = -\left(\sum_{i=1}^{5} A(i)x_i - 24345\right)$$

A(i): cf. Appendix A

CONSTRAINTS :

$$g_1(x) = 2.4x_1 - x_2 \geq 0$$
$$g_2(x) = -1.2x_1 + x_2 \geq 0$$
$$g_3(x) = 60x_1 - x_3 \geq 0$$
$$g_4(x) = -20x_1 + x_3 \geq 0$$
$$g_5(x) = 9.3x_1 - x_4 \geq 0$$
$$g_6(x) = -9.0x_1 + x_4 \geq 0$$
$$g_7(x) = 7.0x_1 - x_5 \geq 0$$
$$g_8(x) = -6.5x_1 + x_5 \geq 0$$
$$g_9(x) = \sum_{i=1}^{5} B(i)x_i \geq 0$$
$$g_{10}(x) = \sum_{i=1}^{5} C(i)x_i \geq 0$$
$$g_{11}(x) = \sum_{i=1}^{5} D(i)x_i \geq 0$$
$$g_{12}(x) = -\sum_{i=1}^{5} B(i)x_i + 294000 \geq 0$$
$$g_{13}(x) = -\sum_{i=1}^{5} C(i)x_i + 294000 \geq 0$$
$$g_{14}(x) = -\sum_{i=1}^{5} D(i)x_i + 294000 \geq 0$$

B(i),C(i),D(i): cf. Appendix A

LOWER BOUNDS : XL = (- , - , - , - , -)

UPPER BOUNDS : XU = (- , - , - , - , -)

START: X = (2.52 , 5.04 , 94.5 , 23.31 , 17.14)

F(X) = -0.235124E+07 (FEASIBLE)

SOLUTION: X* = (4.537 , 10.89 , 272.2 , 42.2 , 31.76)

F(X*) = -0.528042E+07

R(X*) = 0

PROBLEM: 360

CLASSIFICATION: QQR-P1-13

SOURCE: 10/43

NUMBER OF VARIABLES : N = 5

NUMBER OF CONSTRAINTS: M1 = 2 ,M-M1 = 0 ,B = 9

OBJECTIVE FUNCTION :

$$f(x) = (-c_1 - c_2x_2 - c_3x_3 - c_4x_4 - c_5x_5)x_1 + 24345$$

c_i: cf. Appendix A

CONSTRAINTS :

$$g_1(x) = (c_6 + c_7x_2 + c_8x_3 + c_9x_4 + c_{10}x_5)x_1 \geq 0$$

$$g_2(x) = 277200 - g_1(x) \geq 0$$

LOWER BOUNDS : XL = (0 , 1.2 , 20 , 9 , 6.5)

UPPER BOUNDS : XU = (- , 2.4 , 60 , 9.3 , 7)

START: X = (2.52 , 2 , 37.5 , 9.25 , 6.8)

F(X) = -0.235124E+07 (FEASIBLE)

SOLUTION: X* = (4.537 , 2.4 , 60 , 9.3 , 7)

F(X*) = -0.528034E+07

R(X*) = 0

PROBLEM: 361

CLASSIFICATION: QQR-P1-14

SOURCE: 79

NUMBER OF VARIABLES : N = 5

NUMBER OF CONSTRAINTS: M1 = 6 ,M-M1 = 0 ,B = 8

OBJECTIVE FUNCTION :

$$f(x) = -(x_1[\ A(1) + \sum_{i=1}^{5} A(i)x_i\ ] - 24345)$$

A(i): cf. Appendix A

CONSTRAINTS :

$$g_1(x) = x_1(\ B(1) + \sum_{i=2}^{5} B(i)x_i) \geq 0$$

$$g_2(x) = x_1(\ C(1) + \sum_{i=2}^{5} C(i)x_i) \geq 0$$

$$g_3(x) = x_1(\ D(1) + \sum_{i=2}^{5} D(i)x_i) \geq 0$$

$$g_4(x) = 29400 - g_1(x) \geq 0$$

$$g_5(x) = 29400 - g_2(x) \geq 0$$

$$g_6(x) = 277200 - g_3(x) \geq 0$$

B(i),C(i),D(i): cf.Appendix A

LOWER BOUNDS : XL = (0 , 1.2 , 20 , 9 , -)

UPPER BOUNDS : XU = (- , 2.4 , 60 , 9.3 , 7)

START: X = (2.52 , 2 , 37.5 , 9.25 , 6.8)

F(X) = -0.235124E+07 (NOT FEASIBLE)

SOLUTION: X* = (0.5813 ,0.24 , 20 , 9.3 , 7)

F(X*) = -776412.12

R(X*) = 0

PROBLEM:	362(Opt. Gear Ratios for an Autom. Transm. of a car)
CLASSIFICATION:	GLR-PO-2
SOURCE:	73
NUMBER OF VARIABLES :	N = 5
NUMBER OF CONSTRAINTS:	M1 = 4(4) ,M-M1 = 0 ,B = 3

OBJECTIVE FUNCTION :

$$f(x) = FX$$

FX : cf. Appendix A

CONSTRAINTS :

$$g_1(x) = x_1 - x_2 \geq 0$$
$$g_2(x) = x_2 - x_3 \geq 0$$
$$g_3(x) = x_3 - x_4 \geq 0$$
$$g_4(x) = x_4 - x_5 \geq 0$$

LOWER BOUNDS :	XL = (15 , - , - , - , 2)	
UPPER BOUNDS :	XU = (20 , - , - , - , -)	
START:	X = (15 , 9.05 , 6.14 , 4.55 , 3.61)	
	F(X) = 0.2708	(FEASIBLE)
SOLUTION:	X* = (15.05 , 8.875 , 5.909 , 4.86 , 4.399)	
	F(X*) = 0.2623	
	R(X*) = 0	

PROBLEM:	364 (Coupler Curve Problem)
CLASSIFICATION:	GGR-PO-4
SOURCE:	73
NUMBER OF VARIABLES :	N = 6
NUMBER OF CONSTRAINTS:	M1 = 4(2) ,M-M1 = 0 ,8 = 6

OBJECTIVE FUNCTION :

$$f(x) = FX$$

FX : cf. Appendix A

CONSTRAINTS :

$$g_i(x) = G(i) \geq 0 \quad (i = 1,..,4)$$

G : cf. Appendix A

LOWER BOUNDS :	XL = (0.5, 0 , 0 , 2 , - , -)
UPPER BOUNDS :	XU = (3 , - , - , 10, - , -)
START: X	= (1 , 4.5, 4 , 5 , 3 , 3)
F(X)	= 2.30888 (FEASIBLE)
SOLUTION: X*	= (.9967, 4.197, 2.981, 3.964, 1.654, 1.253)
F(X*)	= 0.0606002
R(X*)	= 0.2090E-04

PROBLEM: 365

CLASSIFICATION: QGR-PO-2

SOURCE: 79

NUMBER OF VARIABLES : N = 7

NUMBER OF CONSTRAINTS: M 1 = 5 ,M-M 1 = 0 ,B = 4

OBJECTIVE FUNCTION :

$$f(x) = x_1 x_3$$

CONSTRAINTS :

$$g_1(x) = (x_4 - x_6)^2 + (x_5 - x_7)^2 - 4 \geq 0$$

$$g_2(x) = (x_3 x_4 - x_2 x_5)/P(x) - 1 \geq 0$$

$$g_3(x) = (x_3 x_6 - x_2 x_7)/P(x) - 1 \geq 0$$

$$g_4(x) = (x_1 x_3 + (x_2 - x_1)x_5 - x_3 x_4)/Q(x) - 1 \geq 0$$

$$g_5(x) = (x_1 x_3 + (x_2 - x_1)x_7 - x_3 x_6)/Q(x) - 1 \geq 0$$

$$P(x) = \sqrt{x_2^2} + x_3^2$$

$$Q(x) = \sqrt{x_3^2} + (x_2 - x_1)^2$$

LOWER BOUNDS : XL = (0 , - , 0 , - , 1 , - , 1)

UPPER BOUNDS : XU = (- , - , - , - , - , - , -)

START: X = (3 , 0 , 2 ,-1.5, 1.5, 5 , 0)

F(X) = 6 (NOT FEASIBLE)

SOLUTION: X* = (4.83, 0 , 4.83, 1 , 2.41, 2.41, 1)

F(X*) = 23.3137

R(X*) = 0.2470E-08

PROBLEM:	356 (Alkylation Process Optimization)
CLASSIFICATION:	PPR-PO-6
SOURCE:	73
NUMBER OF VARIABLES :	N = 7
NUMBER OF CONSTRAINTS:	M 14 ,M-M = 0 ,δ = 14

OBJECTIVE FUNCTION :

$$f(x) = 1.715x_1 + .035x_1x_6 + 4.0565x_3 + 10x_2 - .063x_3x_5 + 3000$$

$$g_1(x) = 1-c_1x_6^2-c_2x_3/x_1-c_3x_6 \geq 0$$

$$g_2(x) = 1-c_4x_1/x_3-c_5x_1x_6/x_3-c_6x_1x_6^2/x_3 \geq 0$$

$$g_3(x) = 1-c_7x_6^2-c_8x_5-c_9x_4-c_{10}x_6 \geq 0$$

$$g_4(x) = 1-c_{11}/x_5-c_{12}x_6/x_5-c_{13}x_4/x_5-c_{14}x_6^2/x_5 \geq 0$$

$$g_5(x) = 1-c_{15}x_7-c_{16}x_2/(x_3x_4)-c_{17}x_2/x_3 \geq 0$$

$$g_6(x) = 1-c_{18}/x_7-c_{19}x_2/(x_3x_7)-c_{20}x_2/(x_3x_4x_7) \geq 0$$

$$g_7(x) = 1-c_{21}/x_5-c_{22}x_7/x_5 \geq 0$$

$$g_8(x) = 1-c_{23}x_5-c_{24}x_7 \geq 0$$

$$g_9(x) = 1-c_{25}x_3-c_{26}x_1 \geq 0$$

$$g_{10}(x) = 1-c_{27}x_1/x_3-c_{28}/x_3 \geq 0$$

$$g_{11}(x) = 1-c_{29}x_2/(x_3x_4)-c_{30}x_2/x_3 \geq 0$$

$$g_{12}(x) = 1-c_{31}x_4-c_{32}x_3x_4/x_2 \geq 0$$

$$g_{13}(x) = 1-c_{33}x_1x_6-c_{34}x_1-c_{35}x_3 \geq 0$$

$$g_{14}(x) = 1-c_{36}x_3/x_1-c_{37}/x_1-c_{38}x_6 \geq 0$$

c_i: cf. Appendix A

LOWER BOUNDS :	XL = (1 , 1 , 1 , 85, 90, 3 , 145)
UPPER BOUNDS :	XU = (2000, 120, 5000, 93, 95, 12, 162)
START: X	= (1745, 110, 3048, 89, 92.8, 8 , 145)
F(X)	= 2125.66 (FEASIBLE)
SOLUTION: X*	= (905.4, 36.39, 2381.48, 88.99, 95, 12, 153.54)
F(X*)	= 704.306
R(X*)	= 0.7770E-13

PROBLEM: 367

CLASSIFICATION: GQR-P1-1

SOURCE: 63/82

NUMBER OF VARIABLES : N = 7

NUMBER OF CONSTRAINTS: M1= 3(2) ,M-M1 = 2(1) ,B = 7

OBJECTIVE FUNCTION :

$$f(x) = -5x_1 - 5x_2 - 4x_3 - x_1x_3 - 6x_4 - (5x_5)/(1 + x_5)$$
$$- (8x_6)/(1 + x_6) - 10(1 - 2\exp(-x_7) + \exp(-2x_7))$$

CONSTRAINTS :

$$g_1(x) = 10 - \sum_{i=1}^{7} x_i \geq 0$$

$$g_2(x) = 5 - \sum_{i=1}^{4} x_i \geq 0$$

$$g_3(x) = 5 - x_1 - x_3 - x_5 - x_6^2 - x_7^2 \geq 0$$

$$g_4(x) = 2x_4 + x_5 + 0.8x_6 + x_7 - 5 = 0$$

$$g_5(x) = x_2^2 + x_3^2 + x_5^2 + x_6^2 - 5 = 0$$

LOWER BOUNDS : XL = (0 , 0 , 0 , 0 , 0 , 0 , 0)

UPPER BOUNDS : XU = (- , - , - , - , - , - , -)

START: X = (0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1)

F(X) = -3.28238 (FEASIBLE)

SOLUTION: X* = (1.47, 1.98, 0.35, 1.2, 0.57, 0.78, 1.41)

F(X*) = -37.4130

R(X*) = 0.126E-9

PROBLEM:	368
CLASSIFICATION:	PBR-P1-2
SOURCE:	63/82
NUMBER OF VARIABLES :	N = 8
NUMBER OF CONSTRAINTS:	M 1 = 0 ,M-M 1 = 0 ,B = 16

OBJECTIVE FUNCTION :

$$f(x) = -[\sum_{i=1}^{8} x_i^2]*[\sum_{i=1}^{8} x_i^4] + [\sum_{i=1}^{8} x_i^3]^2$$

CONSTRAINTS :

LOWER BOUNDS : XL = (0 , 0 , 0 , 0 , 0 , 0 , 0 , 0)

UPPER BOUNDS : XU = (1 , 1 , 1 , 1 , 1 , 1 , 1 , 1)

START: X = (1 , 1 , 1 , 1 , 1 , 1 , 1 , 1)

F(X) = 0 (FEASIBLE)

SOLUTION: X* = (0.498,0.499,0.502,0.504,0.503,0.502, 1 , 1)

F(X*) = -0.749976

R(X*) = 0

PROBLEM: 369

CLASSIFICATION: LPR-P-1

SOURCE: 73

NUMBER OF VARIABLES : N = 8

NUMBER OF CONSTRAINTS: M1= 6(3) ,M-M1 = 0 ,B = 16

OBJECTIVE FUNCTION :

$$f(x) = x_1 + x_2 + x_3$$

CONSTRAINTS :

$$g_1(x) = 1-c_{10}x_4-c_{11}x_6 \geq 0$$

$$g_2(x) = 1-c_{12}x_5-c_{13}x_7-c_{14}x_4 \geq 0$$

$$g_3(x) = 1-c_{15}x_8-c_{16}x_5 \geq 0$$

$$g_4(x) = 1-c_1x_4/(x_1x_6)-c_2/x_6-c_3/(x_1x_6) \geq 0$$

$$g_5(x) = 1-c_4x_5/(x_2x_7)-c_5x_4/x_7-c_6x_4/(x_2x_7) \geq 0$$

$$g_6(x) = 1-c_7/(x_3x_8)-c_8x_5/x_8-c_9x_5/(x_3x_8) \geq 0$$

C: cf. Appendix A

LOWER BOUNDS : XL = (100, 1000, 1000, 10, 10, 10, 10, 10)

UPPER BOUNDS : XU = (10000,10000,10000,1000,1000,1000,1000,1000)

START: X = (5000,5000,5000, 200, 350, 150, 225, 425)

F(X) = 15000 (NOT FEASIBLE)

SOLUTION: X* = (579.3,1360,5101,182,295.6,217.9,286.4,395.6)

F(X*) = 7049.24

R(X*) = 0.234E-11

PROBLEM: 370

CLASSIFICATION: SUR-PO-8

SOURCE: 47/14

NUMBER OF VARIABLES : N = 6

NUMBER OF CONSTRAINTS: M1 = 0 ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = x_1^2 + (x_2 - x_1^2 - 1)^2 +$$

$$+ \sum_{i=2}^{30} \{ \sum_{j=2}^{6} (j - 1) x_j (\frac{i-1}{29})^{j-2} - [\sum_{j=1}^{6} x_j (\frac{i-1}{29})^{j-1}]^2 - 1 \}^2$$

CONSTRAINTS :

LOWER BOUNDS : XL = (- , - , - , - , - , -)

UPPER BOUNDS : XU = (- , - , - , - , - , -)

START: X = (0, 0, 0, 0, 0, 0)

F(X) = 30 (FEASIBLE)

SOLUTION: X* = (-0.01572,1.012,-0.2329,-1.26,-1.514,0.993)

F(X*) = 0.228767E-02

R(X*) = 0

PROBLEM: 371

CLASSIFICATION: SUR-PO-9

SOURCE: 47/14

NUMBER OF VARIABLES : N = 9

NUMBER OF CONSTRAINTS: M1 = 0 ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = x_1^2 + (x_2 - x_1^2 - 1)^2 +$$

$$+ \sum_{i=2}^{30} \{ \sum_{j=2}^{9} (j-1) x_j \left(\frac{i-1}{29}\right)^{j-2} - [\sum_{j=1}^{9} x_j \left(\frac{i-1}{29}\right)^{j-1}]^2 - 1 \}^2$$

CONSTRAINTS :

LOWER BOUNDS : XL = (- , - , - , - , - , - , - , - , -)

UPPER BOUNDS : XU = (- , - , - , - , - , - , - , - , -)

START: X = (0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0)

F(X) = 30 (FEASIBLE)

SOLUTION: X* = (0.15E-04,0.1,-0.0147,-0.146,1,-2.62,4.1,
-2.14,1.05)

F(X*) = 0.139977E-05

R(X*) = 0

PROBLEM: 372

CLASSIFICATION: SGR-P1-1

SOURCE: 2/43

NUMBER OF VARIABLES : N = 9

NUMBER OF CONSTRAINTS: M1=12 ,M-M1 = 0 ,B = 6

OBJECTIVE FUNCTION :

$$f(x) = \sum_{i=4}^{9} x_i^2$$

CONSTRAINTS :

$g_1(x) = x_1+x_2\exp(-5x_3)+x_4-127 \geq 0$

$g_2(x) = x_1+x_2\exp(-3x_3)+x_5-151 \geq 0$

$g_3(x) = x_1+x_2\exp(-x_3)+x_6-379 \geq 0$

$g_4(x) = x_1+x_2\exp(x_3)+x_7-421 \geq 0$

$g_5(x) = x_1+x_2\exp(3x_3)+x_8-460 \geq 0$

$g_6(x) = x_1+x_2\exp(5x_3)+x_9-426 \geq 0$

$g_7(x) = -x_1-x_2\exp(-5x_3)+x_4+127 \geq 0$

$g_8(x) = -x_1-x_2\exp(-3x_3)+x_5+151 \geq 0$

$g_9(x) = -x_1-x_2\exp(-x_3)+x_6+379 \geq 0$

$g_{10}(x) = -x_1-x_2\exp(x_3)+x_7+421 \geq 0$

$g_{11}(x) = -x_1-x_2\exp(3x_3)+x_8+460 \geq 0$

$g_{12}(x) = -x_1-x_2\exp(5x_3)+x_9+426 \geq 0$

LOWER BOUNDS : XL = (- , - , - , 0 , 0 , 0 , 0 , 0 , 0)

UPPER BOUNDS : XU = (- , - , - , - , - , - , - , - , -)

START: X = (300,-100,-0.1997,-127,-151,379,421,460,426)

F(X) = 752888 (NOT FEASIBLE)

SOLUTION: X* = (523.31,-156.95,-0.2,29.61,86.62,47.33,26.24,22.92,39.47)

F(X*) = 13390.1

R(X*) = 0.108E-10

PROBLEM: 373

CLASSIFICATION: SGR-P1-2

SOURCE: 2/43

NUMBER OF VARIABLES : N = 9

NUMBER OF CONSTRAINTS: M 1= 0 ,M-M 1 = 6 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = \sum_{i=4}^{9} x_i^2$$

CONSTRAINTS :

$$g_1(x) = x_1 + x_2 \exp(-5x_3) + x_4 - 127 = 0$$

$$g_2(x) = x_1 + x_2 \exp(-3x_3) + x_5 - 151 = 0$$

$$g_3(x) = x_1 + x_2 \exp(-x_3) + x_6 - 379 = 0$$

$$g_4(x) = x_1 + x_2 \exp(x_3) + x_7 - 421 = 0$$

$$g_5(x) = x_1 + x_2 \exp(3x_3) + x_8 - 460 = 0$$

$$g_6(x) = x_1 + x_2 \exp(5x_3) + x_9 - 426 = 0$$

LOWER BOUNDS : XL = (- , - , - , - , - , - , - , - , -)

UPPER BOUNDS : XU = (- , - , - , - , - , - , - , - , -)

START: X = (300,-100,-0.1997,-127,-151,379,421,460,426)

F(X) = 752888 (NOT FEASIBLE)

SOLUTION: X* = (523.31,-156.95,-0.2,29.61,-86.62,47.33,26.24,22.92,-39.47)

F(X*) = 13390.1

R(X*) = 0.218E-8

PROBLEM:	374
CLASSIFICATION:	LPR-P1-2
SOURCE:	64
NUMBER OF VARIABLES :	N = 10
NUMBER OF CONSTRAINTS:	M 1 =35 ,M-M 1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = x_{10}$$

CONSTRAINTS :

$$z(\Theta) = [\sum_{k=1}^{9} x_k \cos(k\Theta)]^2 + [\sum_{k=1}^{9} x_k \sin(k\Theta)]^2$$

$g_i(x) = z(\Theta_i) - (1 - x_{10})^2 \geq 0 \quad \Theta_i = \pi/4[(i-1)*0.1]\ , \quad i = 1,..,10$

$g_i(x) = (1 + x_{10})^2 - z(\Theta_i) \geq 0 \quad \Theta_i = \pi/4[(i-11)*0.1], \quad i = 11,..,20$

$g_i(x) = x_{10}^2 - z(\Theta_i) \geq 0 \quad \Theta_i = \pi/4[1.2+(i-21)*0.2], \quad i=21,..,35$

LOWER BOUNDS : XL = (- , - , - , - , - , - , - , - , - , -)

UPPER BOUNDS : XU = (- , - , - , - , - , - , - , - , - , -)

START:	X	= (0.1,0.1,0.1,0.1,0.1,0.1,0.1,0.1,0.1,0.1)
	F(X)	= 0.1 (NOT FEASIBLE)
SOLUTION:	X*	= (0.2182,0.2326,0.2785,0.2681,0.212,0.1259, 0.0341,-0.0261,-0.1422,0.2333)
	F(X*)	= 0.233264
	R(X*)	= 0.6610E-07

PROBLEM: 375

CLASSIFICATION: SQR-P1-2

SOURCE: 49

NUMBER OF VARIABLES : N = 10

NUMBER OF CONSTRAINTS: M1 = 0 ,M-M1 = 9(8) ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = -\sum_{i=1}^{10} x_i^2$$

CONSTRAINTS :

$$g_j(x) = [\sum_{i=1}^{10} x_i/e_{ij}] - 1 \qquad (j = 1,2,..,8)$$

$$g_9(x) = [\sum_{j=1}^{10} x_i^2/a_i^2] - 4$$

$$e_{ij} = \begin{cases} 1 & i \neq j \\ 2 & i = j \end{cases} \qquad \begin{matrix} (i = 1,..,10) \\ (j = 1,..,8) \end{matrix}$$

$$a_i^2 = 1 + (i - 1)/3 \qquad (i = 1,..,10)$$

LOWER BOUNDS : XL = (- , - , - , - , - , - , - , - , - , -)

UPPER BOUNDS : XU = (- , - , - , - , - , - , - , - , - , -)

START: X = (1 , 1 , 1 , 1 , 1 , 1 , 1 , 1 , 1 , 1)

F(X) = -10 (NOT FEASIBLE)

SOLUTION: X* = (0.1064,0.1064,0.1064,0.1064,0.1064,0.1064,
0.1064,0.1064,2.843,-2.642)

F(X*) = -15.16

R(X*) = 0.6760E-08

PROBLEM: 376 (Opt. of a multi-spindle autom. lathe)

CLASSIFICATION: GPR-PO-1

SOURCE: 73

NUMBER OF VARIABLES : N = 10

NUMBER OF CONSTRAINTS: $M_1 = 14$, M-M1 = 1(1) , B = 20

OBJECTIVE FUNCTION :

$$f(x) = -20000(.15x_1+14x_2-.06)/(.002+x_1+60x_2)$$

CONSTRAINTS :

$$g_3(x) = x_1-x_{10}/(x_6x_4)-10/x_4 \geq 0 \qquad g_1(x) = x_1-.75/(x_3x_4) \geq 0$$

$$g_4(x) = x_1-.19/(x_4x_7)-10/x_4 \geq 0 \qquad g_2(x) = x_1-x_9/(x_5x_4) \geq 0$$

$$g_5(x) = x_1-.125/(x_4x_8) \geq 0$$

$$g_6(x) = 10000x_2-.00131x_9x_5^{.666}x_4^{1.5} \geq 0$$

$$g_7(x) = 10000x_2-.001038x_{10}x_6^{1.6}x_4^{3} \geq 0$$

$$g_8(x) = 10000x_2-.000223x_7^{.666}x_4^{1.5} \geq 0$$

$$g_9(x) = 10000x_2-.000076x_8^{3.55}x_4^{5.66} \geq 0$$

$$g_{10}(x) = 10000x_2-.000698x_3^{1.2}x_4^{2} \geq 0$$

$$g_{11}(x) = 10000x_2-.00005x_3^{1.6}x_4^{3} \geq 0$$

$$g_{12}(x) = 10000x_2-.00000654x_3^{2.42}x_4^{4.17} \geq 0$$

$$g_{13}(x) = 10000x_2-.000257x_3^{.666}x_4^{1.5} \geq 0$$

$$g_{14}(x) = 30-2.003x_4x_5-1.885x_6x_4-.184x_8x_4-2x_3^{.803}x_4 \geq 0$$

$$g_{15}(x) = x_9+x_{10}-.255 = 0$$

LOWER BOUNDS : XL = (0,0,0.5E-04,10,XL(I)= 0.5E-04 FOR I= 5 TO 10)

UPPER BOUNDS : XU = (10,0.1,0.0081,1000,0.0017,0.0013,0.0027,
0.002, 1 , 1)

START: X = (10,0.005,0.0081,100,0.0017,0.0013,0.0027,
0.002,0.15,0.105)
F(X) = -2931.47 (FEASIBLE)

SOLUTION: X* = (.14727,.1,.0081,628.72,.0017,.001182,.0027,
.00135,.15741,.097593)
F(X*) = -4430.88
R(X*) = 0.60E-13

PROBLEM: 377

CLASSIFICATION: GLR-PO-3

SOURCE: 79

NUMBER OF VARIABLES : N = 10

NUMBER OF CONSTRAINTS: M1 = 0 ,M-M1 = 3(3) ,B = 20

OBJECTIVE FUNCTION :

$$f(x) = \sum_{i=1}^{10} x_i(A_i + \log(x_i/(\sum_{j=1}^{10} x_j)))$$

A_i: cf. Appendix A

CONSTRAINTS :

$$g_1(x) = x_1 - 2x_2 + 2x_3 + x_6 + x_{10} - 2 = 0$$

$$g_2(x) = x_4 - 2x_5 + x_6 + x_7 - 1 = 0$$

$$g_3(x) = x_3 + x_7 + x_8 + 2x_9 + x_{10} - 1 = 0$$

LOWER BOUNDS : XL = (XL(I) = 0.1E-04 FOR I = 1 TO 10)

UPPER BOUNDS : XU = (10,10,10,10,10,10,10,10,10,10)

START: X = (0.1,0.1,0.1,0.1,0.1,0.1,0.1,0.1,0.1,0.1)

F(X) = -20.9603 (NOT FEASIBLE)

SOLUTION: X* = (10,10,1,10,9.5,10,.1E-4,...,.1E-4)

F(X*) = -795.001

R(X*) = 0.5530E-08

PROBLEM: 378

CLASSIFICATION: GGR-PO-5

SOURCE: 79

NUMBER OF VARIABLES : N = 10

NUMBER OF CONSTRAINTS: M1 = 0 ,M-M1 = 3 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = \sum_{i=1}^{10} \exp(x_i) * [A_i + x_i - \log(\sum_{j=1}^{10} \exp(x_j))]$$

A_i: cf. Appendix A

CONSTRAINTS :

$$g_1(x) = \exp(x_1) + 2\exp(x_2) + 2\exp(x_3) + \exp(x_6) + \exp(x_{10}) - 2 = 0$$

$$g_2(x) = \exp(x_4) + 2\exp(x_5) + \exp(x_6) + \exp(x_7) - 1 = 0$$

$$g_3(x) = \exp(x_3) + \exp(x_7) + \exp(x_8) + 2\exp(x_9) + \exp(x_{10}) - 1 = 0$$

LOWER BOUNDS : XL = (- , - , - , - , - , - , - , - , - , -)

UPPER BOUNDS : XU = (- , - , - , - , - , - , - , - , - , -)

START: X = (X(I) = -2.3 FOR I = 1 TO 10)

F(X) = -21.0145 (NOT FEASIBLE)

SOLUTION: X* = (-3.2024,-1.9123,-0.2444,-15.67,-0.7217,
-7.2736,-3.5965,-4.0206,-3.2885,-2.3344)

F(X*) = -47.7610

R(X*) = 0.3390E-09

PROBLEM: 379

CLASSIFICATION: SUR-P1-10

SOURCE: 37/60

NUMBER OF VARIABLES : N = 11

NUMBER OF CONSTRAINTS: M1= 0 ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = \sum_{i=1}^{65} [y_i - (x_1 \exp(-x_5 t_i) + x_2 \exp(-x_6 (t_i - x_9)^2) + x_3 \exp(-x_7 (t_i - x_{10})^2) + x_4 \exp(-x_8 (t_i - x_{11})^2))]^2$$

$$t_i = 0.1(i - 1) \ , \ (i=1,..,65)$$

y_i: cf. Appendix A

CONSTRAINTS :

LOWER BOUNDS : XL = (- , - , - , - , - , - , - , - , - , - , -)

UPPER BOUNDS : XU = (- , - , - , - , - , - , - , - , - , - , -)

START: X = (1.3,0.65,0.65,0.7,0.6, 3, 5, 7, 2, 4.5, 5.5)

F(X) = 2.09342 (FEASIBLE)

SOLUTION: X* = (1.3099,0.4315,0.6336,0.5994,0.7542,0.9042, 1.3658,4.8237,2.3987,4.5689,5.6753)

F(X*) = 0.0401377

R(X*) = 0

PROBLEM: 380 (Chemical equilibrium Calculation)

CLASSIFICATION: PPR-PO-7

SOURCE: 73

NUMBER OF VARIABLES : N = 12

NUMBER OF CONSTRAINTS: M1 = 3 ,M-M1 = 0 ,B = 24

OBJECTIVE FUNCTION :

$$f(x) = 100000 \prod_{i=1}^{11} x_i^{a_i}$$

a_i : cf. Appendix A

CONSTRAINTS :

$$g_1(x) = 1-c_1x_1-c_2x_2-c_3x_3-c_4x_4x_5 \geq 0$$

$$g_2(x) = 1-c_5x_1-c_6x_2-c_7x_3-c_8x_4x_{12}-c_9x_5/x_{12} -c_{10}x_6/x_{12}-c_{11}x_7x_{12}-c_{12}x_4x_5-c_{13}x_2x_5/x_{12} -c_{14}x_2x_4x_5-c_{15}x_2x_5/(x_4x_{12}^2)-c_{16}x_{10}/x_{12} \geq 0$$

$$g_3(x) = 1-c_{17}x_1-c_{18}x_2-c_{19}x_3-c_{20}x_4-c_{21}x_5-c_{22}x_6 -c_{23}x_8-c_{24}x_4x_5-c_{25}x_2x_5-c_{26}x_2x_4x_5-c_{27}x_2x_5/x_4 -c_{28}x_9-c_{29}x_1x_9-c_{30}x_{11} \geq 0$$

c_i: cf. Appendix A

LOWER BOUNDS : XL = (XL(I) = 0.1 FOR I = 1 TO 12)

UPPER BOUNDS : XU = (XU(I) = 100 FOR I = 1 TO 12)

START: X = (X (I) = 4 FOR I = 1 TO 12)

F(X) = 0.227683 (NOT FEASIBLE)

SOLUTION: X* = (2.6632,4.5173,7.1338,2.2373,4.0784,1.3183, 4.1252,2.8562,1.6756,2.1789,5.1234,6.6593)

F(X*) = 3.16859

R(X*) = 0

PROBLEM: 381 (Linear Optimal Hog Ration Problem)

CLASSIFICATION: LLR-P1-2

SOURCE: 63/51/12

NUMBER OF VARIABLES : N = 13

NUMBER OF CONSTRAINTS: M1 = 3(3) ,M-M1 = 1(1) ,B = 13

OBJECTIVE FUNCTION :

$$f(x) = \sum_{i=1}^{13} r_i x_i$$

CONSTRAINTS :

$$g_1(x) = -18 + \sum_{i=1}^{13} \bar{s}_i x_i \geq 0$$

$$g_2(x) = -1 + \sum_{i=1}^{13} \bar{u}_i x_i \geq 0$$

$$g_3(x) = -0.9 + \sum_{i=1}^{13} \bar{v}_i x_i \geq 0$$

$$g_4(x) = \sum_{i=1}^{13} x_i - 1 = 0$$

r_i, s_i, u_i, v_i: cf. Appendix A

LOWER BOUNDS : XL = (0,0,0,0,0,0,0,0,0,0,0,0,0)

UPPER BOUNDS : XU = (-,-,-,-,-,-,-,-,-,-,-,-,-)

START: X = (X(I) = 0.1 FOR I = 1 TO 13)

F(X) = 2.067 (FEASIBLE)

SOLUTION: X* = (0.7856,0,0,0,0,0.1739,0,0,0.0206,0,0,0,0.02)

F(X*) = 1.01490

R(X*) = 0.6940E-16

PROBLEM: 382 (Nonlinear Optimal Hog Ration Problem)

CLASSIFICATION: LGI-P1-1

SOURCE: 63/17

NUMBER OF VARIABLES : N = 13

NUMBER OF CONSTRAINTS: M1 = 3 ,M-M1 = 1(1) ,8 = 13

OBJECTIVE FUNCTION :

$$f(x) = \sum_{i=1}^{13} r_i x_i$$

CONSTRAINTS :

$$g_1(x) = -18 + \sum_{j=1}^{13} \bar{s}_j x_j + (-1.645)[\sum_{j=1}^{13} \sigma_{s_j}^2 x_j^2]^{1/2} \geq 0$$

$$g_2(x) = -1 + \sum_{j=1}^{13} \bar{u}_j x_j + (-1.645)[\sum_{j=1}^{13} \sigma_{u_j}^2 x_j^2]^{1/2} \geq 0$$

$$g_3(x) = -0.9 + \sum_{j=1}^{13} \bar{v}_j x_j + (-1.645)[\sum_{j=1}^{13} \sigma_{v_j}^2 x_j^2]^{1/2} \geq 0$$

$$g_4(x) = \sum_{j=1}^{13} x_j - 1 = 0$$

r_i, s_j, u_j, v_j: cf. Appendix A

LOWER BOUNDS : XL = (0,0,0,0,0,0,0,0,0,0,0,0,0)

UPPER BOUNDS : XU = (-,-,-,-,-,-,-,-,-,-,-,-,-)

START: X = (X(I) = 0.1 FOR I = 1 TO 13)

F(X) = 2.067 (FEASIBLE)

SOLUTION: X* = (0.132,0,0,0,0,0.3253,0,0,0.5167,0,0,0,0.025)

F(X*) = 1.03831

R(X*) = 0.8330E-05

PROBLEM: 383

CLASSIFICATION: PLI-P1-1

SOURCE: 77

NUMBER OF VARIABLES : N = 14

NUMBER OF CONSTRAINTS: M1 = 0 ,M-M1 = 1(1) ,B = 28

OBJECTIVE FUNCTION :

$$f(x) = \sum_{i=1}^{14} a_i/x_i$$

a_i: cf. Appendix A

CONSTRAINTS :

$$g_1(x) = \sum_{i=1}^{14} c_i x_i - 1 = 0$$

c_i: cf. Appendix A

LOWER BOUNDS : XL = (0,0,0,0,0,0,0,0,0,0,0,0,0,0)

UPPER BOUNDS : XU = (XU(I) = 0.04 FOR I = 1 TO 5)
(XU(I) = 0.03 FOR I = 6 TO 14)

START: X = (X (I) = 0.01 FOR I = 1 TO 14)

F(X) = 0.231752E+07 (NOT FEASIBLE)

SOLUTION: X* = (0.04,0.0382,0.0358,0.033,0.0303,0.0279,0.0265
0.0249,0.023,0.0216,0.0202,0.0192,0.0203,0.0253)

F(X*) = 728566

R(X*) = 0.1390E-16

PROBLEM: 384

CLASSIFICATION: LQR-P1-4

SOURCE: 19/69

NUMBER OF VARIABLES : N = 15

NUMBER OF CONSTRAINTS: $M_1 = 10$, $M-M_1 = 0$, $B = 0$

OBJECTIVE FUNCTION :

$$f(x) = -486x_1 - 640x_2 - 758x_3 - 776x_4 - 477x_5 - 707x_6 - 175x_7 - 619x_8 - 627x_9 - 614x_{10} - 475x_{11} - 377x_{12} - 524x_{13} - 468x_{14} - 529x_{15}$$

CONSTRAINTS :

$$g_i(x) = b_i - \sum_{j=1}^{15} a_{ij}x_j^2 \geq 0$$

$$(i = 1,..,10)$$

a_{ij}, b_i: cf. Appendix A

LOWER BOUNDS : XL = (-,-,-,-,-,-,-,-,-,-,-,-,-,-,-)

UPPER BOUNDS : XU = (-,-,-,-,-,-,-,-,-,-,-,-,-,-,-)

START: X = (0,0,0,0,0,0,0,0,0,0,0,0,0,0,0)

F(X) = 0 (FEASIBLE)

SOLUTION: X* = (0.861,0.917,0.92,0.896,1.037,0.973,0.822,1.2,1.156,1.144,1.03,0.909,1.082,0.847,1.172)

F(X*) = -8310.26

R(X*) = 0.3930E-08

PROBLEM:	385
CLASSIFICATION:	LQR-P1-5
SOURCE:	19/69
NUMBER OF VARIABLES :	N = 15
NUMBER OF CONSTRAINTS:	M1=10 ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = -486x_1 - 640x_2 - 758x_3 - 776x_4 - 477x_5 - 707x_6 - 175x_7 - 619x_8 - 627x_9 - 614x_{10} - 475x_{11} - 377x_{12} - 524x_{13} - 468x_{14} - 529x_{15}$$

CONSTRAINTS :

$$g_i(x) = b_i - \sum_{j=1}^{15} a_{ij}x_j^2 \geq 0$$

$$(i = 1,..,10)$$

a_{ij}, b_i: cf. Appendix A

LOWER BOUNDS :	XL = (-,-,-,-,-,-,-,-,-,-,-,-,-,-,-)
UPPER BOUNDS :	XU = (-,-,-,-,-,-,-,-,-,-,-,-,-,-,-)
START: X	= (0,0,0,0,0,0,0,0,0,0,0,0,0,0,0)
F(X)	= 0 (FEASIBLE)
SOLUTION: X*	= (0.813,0.113,1.086,0.998,1.075,1.069,0.628, 1.093,0.914,0.862,1.005,0.877,0.987,1.041,1.186)
F(X*)	= -8315.29
R(X*)	= 0.6320E-09

PROBLEM: 386

CLASSIFICATION: LQR-P1-6

SOURCE: 19/69

NUMBER OF VARIABLES : N = 15

NUMBER OF CONSTRAINTS: $M_1 = 11$, $M-M_1 = 0$, $B = 0$

OBJECTIVE FUNCTION :

$$f(x) = -486x_1 - 640x_2 - 758x_3 - 776x_4 - 477x_5 - 707x_6 - 175x_7 - 619x_8 - 627x_9 - 614x_{10} - 475x_{11} - 377x_{12} - 524x_{13} - 468x_{14} - 529x_{15}$$

CONSTRAINTS :

$$g_i(x) = b_i - \sum_{j=1}^{15} a_{ij}x_j^2 \geq 0 \qquad (i = 1,..,10)$$

$$g_{11}(x) = 0.5 \sum_{j=1}^{15} j(x_j - 2)^2 - 70 \geq 0$$

a_{ij}, b_j: cf. Appendix A

LOWER BOUNDS : XL = (-,-,-,-,-,-,-,-,-,-,-,-,-,-,-)

UPPER BOUNDS : XU = (-,-,-,-,-,-,-,-,-,-,-,-,-,-,-)

START: X = (0,0,0,0,0,0,0,0,0,0,0,0,0,0,0)

F(X) = 0 (FEASIBLE)

SOLUTION: X* = (1.004,1.087,1.103,1.031,0.929,1.257,0.761, 0.857,1.09,0.981,0.851,0.966,0.9060.838,0.809)

F(X*) = -8154.37

R(X*) = 0.9790E-08

PROBLEM: 387

CLASSIFICATION: LQR-P1-7

SOURCE: 19/69

NUMBER OF VARIABLES : N = 15

NUMBER OF CONSTRAINTS: M1=11 ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = -486x_1 - 640x_2 - 758x_3 - 776x_4 - 477x_5 - 707x_6 - 175x_7 - 619x_8 - 627x_9 - 614x_{10} - 475x_{11} - 377x_{12} - 524x_{13} - 468x_{14} - 529x_{15}$$

CONSTRAINTS :

$$g_i(x) = b_i - \sum_{j=1}^{15} a_{ij}x_j^2 \geq 0 \qquad (i = 1,..,10)$$

$$g_{11}(x) = 0.5 \sum_{j=1}^{15} j(x_j - 2)^2 - 61 \geq 0$$

a_{ij}, b_i: cf. Appendix A

LOWER BOUNDS : XL = (-,-,-,-,-,-,-,-,-,-,-,-,-,-,-)

UPPER BOUNDS : XU = (-,-,-,-,-,-,-,-,-,-,-,-,-,-,-)

START: X = (0,0,0,0,0,0,0,0,0,0,0,0,0,0,0)

F(X) = 0 (FEASIBLE)

SOLUTION: X* = (1.013,1.016,1.031,0.997,0.985,1.037,0.993,
0.972,0.999,0.995,0.959,1.008,0.982,0.991,0.978)

F(X*) = -8250.14

R(X*) = 0.1010E-07

PROBLEM: 388

CLASSIFICATION: LQR-P1-8

SOURCE: 19/69

NUMBER OF VARIABLES : N = 15

NUMBER OF CONSTRAINTS: M1 = 15(4) ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = -486x_1 - 640x_2 - 758x_3 - 776x_4 - 477x_5 - 707x_6 - 175x_7 - 619x_8 - 627x_9 - 614x_{10} - 475x_{11} - 377x_{12} - 524x_{13} - 468x_{14} - 529x_{15}$$

CONSTRAINTS :

$$g_{i-11}(x) = b_i - \sum_{j=1}^{15} a_{ij}x_j \geq 0 \qquad (i = 12,13,14,15)$$

$$g_{4+i}(x) = b_i - \sum_{j=1}^{15} a_{ij}x_j^2 \geq 0 \qquad (i = 1,..,10)$$

$$g_{15}(x) = 0.5 \sum_{j=1}^{15} j(x_j - 2)^2 - 193.121 \geq 0$$

a_{ij}, b_i: cf. Appendix A

LOWER BOUNDS : XL = (-,-,-,-,-,-,-,-,-,-,-,-,-,-,-)

UPPER BOUNDS : XU = (-,-,-,-,-,-,-,-,-,-,-,-,-,-,-)

START: X = (0,0,0,0,0,0,0,0,0,0,0,0,0,0,0)

F(X) = 0 (FEASIBLE)

SOLUTION: X* = (0.627,1.433,1.463,0.731,0.786,1.205,-1.143, 1.061,-0.134,1.182,0.969,-0.845,0.481,-0.34,0.686)

F(X*) = -5821.08

R(X*) = 0.6890E-07

PROBLEM: 389

CLASSIFICATION: LQR-P1-9

SOURCE: 19/69

NUMBER OF VARIABLES : N = 15

NUMBER OF CONSTRAINTS: M1=15(4) ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = -486x_1 - 640x_2 - 758x_3 - 776x_4 - 707x_6 - 175x_7 - 619x_8 - 627x_9 - 614x_{10} - 475x_{11} - 377x_{12} - 524x_{13} - 468x_{14} - 529x_{15}$$

CONSTRAINTS :

$$g_{i-11}(x) = b_i - \sum_{j=1}^{15} a_{ij}x_j \geq 0 \qquad (i = 12,13,14,15)$$

$$g_{i+4}(x) = b_i - \sum_{j=1}^{15} a_{ij}x_j^2 \geq 0 \qquad (i = 1,..,10)$$

$$g_{15}(x) = 0.5 \sum_{j=1}^{15} j(x_j - 2)^2 - 200 \geq 0$$

a_{ij}, b_i: cf. Appendix A

LOWER BOUNDS : XL = (-,-,-,-,-,-,-,-,-,-,-,-,-,-,-)

UPPER BOUNDS : XU = (-,-,-,-,-,-,-,-,-,-,-,-,-,-,-)

START: X = (0,0,0,0,0,0,0,0,0,0,0,0,0,0,0)

F(X) = 0 (FEASIBLE)

SOLUTION: X* = (0.671,1.388,1.468,0.76,0.829,1.164,-1.258, 0.982,0.068,1.147,0.986,-0.888,0.565,-0.581,0.721)

F(X*) = -5809.72

R(X*) = 0.8390E-09

PROBLEM:	391
CLASSIFICATION:	GUR-PO-2
SOURCE:	79
NUMBER OF VARIABLES :	N = 30
NUMBER OF CONSTRAINTS:	M1 = 0 ,M-M1 = 0 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = \sum_{i=1}^{30} \alpha_i(x)$$

$$\alpha_i(x) = 420x_i + (i - 15)^3 + \sum_{j=1}^{30} v_{ij}[(\sin(\log(v_{ij})))^5 + (\cos(\log(v_{ij})))^5]$$

$$v_{ij} = \sqrt{x_j^2 + i/j}$$

CONSTRAINTS :

LOWER BOUNDS : XL = (-,-,-,-,....,-,-,-,-)

UPPER BOUNDS : XU = (-,-,-,-,....,-,-,-,-)

START: X_i = $-2.8742711\alpha_i(0)$

F(X) = -0.495817E+07 (FEASIBLE)

SOLUTION: X* = (cf. Appendix A1)

F(X*) = 0

R(X*) = 0

PROBLEM:	392
CLASSIFICATION:	QLR-PO-1
SOURCE:	77
NUMBER OF VARIABLES :	N = 30
NUMBER OF CONSTRAINTS:	M1 = 45(45) ,M-M1 = 0 ,B = 30

OBJECTIVE FUNCTION :

$$f(x) = -\sum_{t=1}^{5} \sum_{j=1}^{3} \Big[r_{1jt}\, x_{Ajt} - r_{2jt}\, x_{Ajt}^2 - k_{Ajt}\, x_{Ajt} - k_{1jt}\, x_{pjt} - k_{pjt}\, x_{pjt} - k_{zjt}(x_{pjt} - x_{Ajt-1})^2 - k_{Ljt}(x_{pjt} - x_{Ajt})^2 - k_{Ljt} \sum_{s=1}^{t} (x_{pjs} - x_{Ajs}) \Big]$$

CONSTRAINTS :

$$g_1 \cdots g_{15} : \quad h_{jt} - x_{Ajt} \geqq 0 \qquad j = 1,\ldots,3 \; ; \; t = 1,\ldots,5$$

$$g_{16} \cdots g_{30} : \quad b_{it} - \sum_{j=1}^{3} t_{ij}\, x_{pjt} \geqq 0 \qquad i = 1,..,3 \; ; \; t = 1..5$$

$$g_{31} \cdots g_{45} : \quad \sum_{s=1}^{t} (x_{pjs} - x_{Ajs}) \geqq 0 \qquad j = 1,..,3 \; ; \; t = 1,..,5$$

$$x_{Ajt} \geqq 0 \qquad j = 1,\ldots,3 \; ; \; t = 1,\ldots,5$$

$$x_{Pjt} \geqq 0 \qquad j = 1,\ldots,3 \; ; \; t = 1,\ldots,5$$

Notation:

$$x_1 = x_{A11} \; ; \; x_2 = x_{A21} \; ; \; x_3 = x_{A31} \; ; \; x_4 = x_{A12} \; ; \ldots$$

$$x_{15} = x_{A35} \; ; \; x_{16} = x_{P11} \; ; \; \ldots \; x_{30} = x_{P35} \; ;$$

LOWER BOUNDS :	XL = (0,0,0,0,....,0,0,0,0)	
UPPER BOUNDS :	XU = (-,-,-,-,....,-,-,-,-)	
START:	X = (cf. Appendix A1)	
	F(X) = -845999	(NOT FEASIBLE)
SOLUTION:	X* = (cf. Appendix A1)	
	F(X*) = -0.169888E+07	
	R(X*) = 0.2840E-13	

PROBLEM:	393(Synthetic Natural Gas Produktion Probl.)
CLASSIFICATION:	PPR-PO- 8
SOURCE:	73
NUMBER OF VARIABLES :	N = 48
NUMBER OF CONSTRAINTS:	M1= 1 ,M-M1 = 2(2) ,B = 72

OBJECTIVE FUNCTION :

$$f(x) = FX$$

FX : cf. Appendix A

CONSTRAINTS :

$$g_1(x) = G(2) = 0$$
$$g_2(x) = G(3) = 0$$
$$g_3(x) = PHI(1) \geqq 0$$

PHI,G :cf. Appendix A

LOWER BOUNDS :	XL = (0.002,....,0.002)		
UPPER BOUNDS :	XU = (cf. Appendix A1)		
START:	X	= (cf. Appendix A1)	
	F(X)	= 1.86230	(FEASIBLE)
SOLUTION:	X*	= (cf. Appendix A1)	
	F(X*)	= 0.863380	
	R(X*)	= 0.1140E-06	

PROBLEM: 394

CLASSIFICATION: PQR-P1- 4

SOURCE: 77

NUMBER OF VARIABLES : N = 20

NUMBER OF CONSTRAINTS: M1= 0 ,M-M1 = 1 ,B = 0

OBJECTIVE FUNCTION :

$$f(x) = \sum_{i=1}^{20} i(x_i^2 + x_i^4)$$

CONSTRAINTS :

$$g_1(x) = \sum_{i=1}^{20} x_i^2 = 1$$

LOWER BOUNDS : XL = (-,-,-,-,-,-,-,-,-,-,-,-,-,-,-,-,-,-,-,-)

UPPER BOUNDS : XU = (-,-,-,-,-,-,-,-,-,-,-,-,-,-,-,-,-,-,-,-)

START: X = (2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2)

F(X) = 4200 (NOT FEASIBLE)

SOLUTION: X* = (cf. Appendix A1)

F(X*) = 1.91667

R(X*) = 0.7370E-08

PROBLEM: 395

CLASSIFICATION: PQR-P1-5

SOURCE: 77

NUMBER OF VARIABLES : N = 50

NUMBER OF CONSTRAINTS: $M_1 = 0$, $M-M_1 = 1$, $B = 0$

OBJECTIVE FUNCTION :

$$f(x) = \sum_{i=1}^{50} i(x_i^2 + x_i^4)$$

CONSTRAINTS :

$$g_1(x) = \sum_{i=1}^{50} x_i^2 = 1$$

LOWER BOUNDS : XL = (-,-,-,-,....,-,-,-,-)

UPPER BOUNDS : XU = (-,-,-,-,....,-,-,-,-)

START: X = (2,2,....,2,2)

F(X) = 25500 (NOT FEASIBLE)

SOLUTION: X* = (cf. Appendix A1)

F(X*) = 1.91667

R(X*) = 0.6560E-08

Appendix A

ADDITIONAL PROBLEM DATA

This appendix summarizes constant data, extensive definitions of problem functions, and solution results which would break the documentation scheme used in Chapter IV to describe the test problems. The corresponding abbreviations are explained in the test problem documentations. If problem functions are presented in form of FORTRAN-statements instead of analytical expressions, the complete subroutine TPno(MODE) is listed for the matter of completeness.

TP 204:

$$A = (0.13294 \quad -0.244378 \quad 0.325895)^T$$

$$D = (2.5074 \quad -1.36401 \quad 1.02282)^T$$

$$H = \begin{bmatrix} -0.564255 & 0.392417 \\ -0.404979 & 0.927589 \\ -0.0735084 & 0.535493 \end{bmatrix}$$

$$B = \begin{bmatrix} 5.66598 & 2.77141 \\ 2.77141 & 2.12413 \end{bmatrix} = (H^T H)^{-1}$$

TP 236 - 239:

$B_1 = 75.1963666677$
$B_2 = -3.8112755343$
$B_3 = 0.1269366345$
$B_4 = -0.0020567665$
$B_5 = 0.0000103450$
$B_6 = -6.8306567613$
$B_7 = 0.0302344793$
$B_8 = -0.0012813448$
$B_9 = 0.0000352559$
$B_{10} = -0.0000002266$
$B_{11} = 0.2564581253$
$B_{12} = -0.0034604030$
$B_{13} = 0.0000135139$
$B_{14} = -28.1064434908$
$B_{15} = -0.0000052375$
$B_{16} = -0.0000000063$
$B_{17} = 0.0000000007$
$B_{18} = 0.0003405462$
$B_{19} = -0.0000016638$
$B_{20} = -2.8673112392$

TP 243:

$$A = (0.14272 \quad -0.184918 \quad -0.521869 \quad -0.685306)^T$$

$$D = (1.75168 \quad -1.35195 \quad -0.479048 \quad -0.3648 \quad)^T$$

$$G = \begin{bmatrix} -0.564255 & 0.392417 & -0.404979 \\ 0.927589 & -0.0735083 & 0.535493 \\ 0.658799 & -0.636666 & -0.681091 \\ -0.869487 & 0.586387 & 0.289826 \end{bmatrix}$$

$$B = \begin{bmatrix} 2.95137 & 4.87407 & -2.0506 \\ 4.87407 & 9.39321 & -3.93181 \\ -2.0506 & -3.93189 & 2.64745 \end{bmatrix}$$

TP 253:

j	c_j	a_{1j}	a_{2j}	a_{3j}
1	1	0	0	0
2	1	10	0	0
3	1	10	10	0
4	1	0	10	0
5	1	0	0	10
6	1	10	0	10
7	1	10	10	10
8	1	0	10	10

TP 266:

$$A = \begin{pmatrix} 0.0426149 & 0.0352053 & 0.0878058 & 0.0330812 & 0.0580924 \\ 0.649704 & 0.344144 & -0.627443 & 0.001828 & -0.224783 \end{pmatrix}^T$$

$$D = \begin{pmatrix} 2.34659 & 2.84048 & 1.13888 & 3.02286 & 1.72139 \\ 0.153917 & 0.290577 & -0.159378 & 54.6910 & -0.444873 \end{pmatrix}^T$$

$$C = \begin{bmatrix} -0.564255 & 0.392417 & -0.404979 & 0.927589 & -0.0735083 \\ 0.535493 & 0.658799 & -0.636666 & -0.681091 & -0.869487 \\ 0.586387 & 0.289826 & 0.854402 & 0.789312 & 0.949721 \\ 0.608734 & 0.984915 & 0.375699 & 0.239547 & 0.463136 \\ 0.774227 & 0.325421 & -0.151719 & 0.448051 & 0.149926 \\ -0.435033 & -0.688583 & 0.222278 & -0.524653 & 0.413248 \\ 0.759468 & -0.627795 & 0.0403142 & 0.724666 & -0.0182537 \\ -0.152448 & -0.546437 & 0.484134 & 0.353951 & 0.887866 \\ -0.821772 & -0.53412 & -0.798498 & -0.658572 & 0.662362 \\ 0.819831 & -0.910632 & -0.480344 & -0.871758 & -0.978666 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.354033 & -0.0230349 & -0.211938 & -0.0554288 & 0.220429 \\ -0.0230349 & 0.29135 & -0.00180333 & -0.111141 & 0.0485461 \\ -0.211938 & -0.00180333 & 0.815808 & -0.133538 & -0.38067 \\ -0.0554288 & -0.111141 & -0.133538 & 0.389198 & -0.131586 \\ 0.220429 & 0.0485461 & -0.38067 & -0.131586 & 0.534706 \end{bmatrix}$$

$i \backslash j$		1	2	3	4	5	6	7	8	9	10	11	12	13
c_j		20	40	400	20	80	20	40	140	380	280	80	40	140
	1	100	100	10	5	10	0	0	25	0	10	55	5	45
	2	90	100	10	35	20	5	0	35	55	25	20	0	40
	3	70	50	0	55	25	100	40	50	0	30	60	10	30
	4	50	0	0	65	35	100	35	60	0	15	0	75	35
a_{ij}	5	50	10	70	60	45	45	0	35	65	5	75	100	75
	6	40	0	50	95	50	35	10	60	0	45	15	20	0
	7	30	60	30	90	0	30	5	25	0	70	20	25	70
	8	20	30	40	25	40	25	15	10	80	20	30	30	5
	9	10	70	10	35	25	65	0	30	0	0	25	0	15
	10	5	10	100	5	20	5	10	35	95	70	20	10	35

TP 327:

i	y_i	z_i	i	y_i	z_i
1	0.49	8	23	0.41	22
2	0.49	8	24	0.40	22
3	0.48	10	25	0.42	24
4	0.47	10	26	0.40	24
5	0.48	10	27	0.40	24
6	0.47	10	28	0.41	26
7	0.46	12	29	0.40	26
8	0.46	12	30	0.41	26
9	0.45	12	31	0.41	28
10	0.43	12	32	0.40	28
11	0.45	14	33	0.40	30
12	0.43	14	34	0.40	30
13	0.43	14	35	0.38	30
14	0.44	16	36	0.41	32
15	0.43	16	37	0.40	32
16	0.43	16	38	0.40	34
17	0.46	18	39	0.41	36
18	0.45	18	40	0.38	36
19	0.42	20	41	0.40	38
20	0.42	20	42	0.40	38
21	0.43	20	43	0.39	40
22	0.41	22	44	0.39	42

TP 285:

j	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	b_i
a_{1j}	100	100	10	5	10	0	0	25	0	10	55	5	45	20	0	385
a_{2j}	90	100	10	35	20	5	0	35	55	25	20	0	40	25	10	470
a_{3j}	70	50	0	55	25	100	40	50	0	30	60	10	30	0	40	560
a_{4j}	50	0	0	65	35	100	35	60	0	15	0	75	35	30	65	565
a_{5j}	50	10	70	60	45	45	0	35	65	5	75	100	75	10	0	645
a_{6j}	40	0	50	95	50	35	10	60	0	45	15	20	0	5	5	430
a_{7j}	30	60	30	90	0	30	5	25	0	70	20	25	70	15	15	485
a_{8j}	20	30	40	25	40	25	15	10	80	20	30	30	5	65	20	455
a_{9j}	10	70	10	35	25	65	0	30	0	0	25	0	15	50	55	390
a_{10j}	5	10	100	5	20	5	10	35	95	70	20	10	35	10	30	460

TP 289:

X = (-1.03, 1.07, -1.10, 1.13, -1.17, 1.20, -1.23, 1.27, -1.30, 1.33,
-1.37, 1.40, -1.43, 1.47, -1.50, 1.53, -1.57, 1.60, -1.63, 1.67,
-1.70, 1.73, -1.77, 1.80, -1.83, 1.87, -1.90, 1.93, -1.97, 2.00)

TP 333:

i	a_i	y_i
1	4	72.1
2	5.75	65.6
3	7.5	55.9
4	24	17.1
5	32	9.8
6	48	4.5
7	72	1.3
8	96	0.6

TP 334:

i	y_i
1	0.14
2	0.18
3	0.22
4	0.25
5	0.29
6	0.32
7	0.35
8	0.39

i	y_i
9	0.37
10	0.58
11	0.73
12	0.96
13	1.34
14	2.10
15	4.39

TP 347:

$$w_1 = (x_1 + x_2 + x_3 + 0.03)/(0.09x_1 + x_2 + x_3 + 0.03)$$

$$w_2 = (x_2 + x_3 + 0.03)/(0.07x_2 + x_3 + 0.03)$$

$$w_3 = (x_3 + 0.03)/(0.13x_3 + 0.03)$$

$A_1 = 8204.37$; $A_2 = 9008.72$; $A_3 = 9330.46$

TP 348:

```
      SUBROUTINE TP348 (MODE)
      COMMON/L1/N,NILI,NINL,NELI,NENL
      COMMON/L2/X(3)
      COMMON/L3/G(1)
      COMMON/L4/GF(3)
      COMMON/L5/GG(1,3)
      COMMON/L6/FX
      COMMON/L9/INDEX1
      COMMON/L10/INDEX2
      COMMON/L11/LXL
      COMMON/L12/LXU
      COMMON/L13/XL(3)
      COMMON/L14/XU(3)
      COMMON/L20/LEX,NEX,FEX,XEX(3)
      LOGICAL LEX,LXL(3),LXU(3),INDEX1(1),INDEX2(1)
      REAL*8 X,GG,GF,G,FX,FEX,XEX,XL,XU,RHO,XMU,CP,PR,PI,D,TIN,TSURF,
     F H,W,RHOC,RHOA,AF,AT,AC,GI,RE,XMDOT,DELP,H1,COSTM,COSTT,COSTF,
     F HO,XVAL,ETAF,ETAS,HEF,Q,DSQRT,DTANH,DEXP
      DATA RHO,XMU,CP,PR,PI,D,TIN,TSURF,H,W,RHOC,RHOA/.747D-1,
     F .443D-1,.240D+0,.709D+0,3.14159D+0,.525D+0,75.D+0,45.D+0,
     F 13.13D+0,3.166D+0,559.D+0,169.D+0/
      GOTO (1,2,3,4,5),MODE
1     N=3
      NILI=0
      NINL=0
      NELI=0
      NENL=1
      DO 6 I=1,3
      LXL(I)=.FALSE.
6     LXU(I)=.TRUE.
      LXL(2)=.TRUE.
      XL(2)=.1313D+2
      XU(1)=.44D-1
      XU(2)=.24D+2
      XU(3)=.6D+3
      X(1)=.1D+0
      X(2)=.18D+2
      X(3)=.144D+3
      LEX=.FALSE.
      NEX=1
      XEX(1)=.44000000D-01
      XEX(2)=.14717224D+02
      XEX(3)=.13855457D+03
      FEX=.72202560D+02
      RETURN
```

```
2     AF=X(2)/X(1)*.2D+1*(W*H-.3D+2*PI*D**2/.4D+1)/.144D+3
      AT=.3D+2*PI*D*X(2)/.144D+3
      AC=(H*X(2)-.1D+2*D*X(2)-X(2)/X(1)*.6D-2*H)/.144D+3
      GI=(RHO*X(3)*(H*X(2))/(AC*.144D+3))*.6D+2
      RE=GI*.1083D+1/(.12D+2*XMU)
      IF (RE.LT..1D-9) RE=.1D-9
      HO=(.195D+0*GI*CP)/(PR**.67*RE**.35)
      XMDOT=RHO*X(3)*H*X(2)/.144D+3*.6D+2
      DELP=.1833D-5/RHO*GI**2*.3D+1*(AF/AC*RE**-.5+.1D+0*AT/AC)
      IF(HO.LT..1D-9) HO=.1D-9
      XVAL=.732D-1*DSQRT(HO)
      ETAF=DTANH(XVAL)/XVAL
      ETAS=.1D+1-AF/(AF+AT)*(.1D+1-ETAF)
      HEF=.1D+1-DEXP(-ETAS*HO*(AF+AT)/(XMDOT*CP))
      Q=HEF*(TIN-TSURF)*XMDOT*CP
      IF (MODE .EQ. 4) GOTO 7
      H1=DELP/RHO*XMDOT/.198D+7
      IF (H1.LT..1D-9) H1=.1D-9
      COSTM=DSQRT(H1)/.718D-1+.4D+1
      COSTT=.101D+1*.3D+2*X(2)*PI/.4D+1*(D**2-(D-.36D-1)**2)
      COSTF=.47D+0*H*W*.6D-2*RHOA/.1728D+4*X(2)/X(1)
      COSTT=COSTT*RHOC/.1728D+4
      FX=COSTM+COSTT+COSTF
3     RETURN
4     IF (.NOT.INDEX1(1)) GOTO 5
      GOTO 2
7     G(1)=6.D+3-Q
5     RETURN
      END
```

TP 350:

i	y_i	u_i
1	0.1957	4.0000
2	0.1947	2.0000
3	0.1735	1.0000
4	0.1600	0.5000
5	0.0844	0.2500
6	0.0627	0.1670
7	0.0456	0.1250
8	0.0342	0.1000
9	0.0323	0.0833
10	0.0235	0.0714
11	0.0246	0.0625

TP 351:

i	a_i	b_i
1	0.0	7.391
2	0.000428	11.18
3	0.00100	16.44
4	0.00161	16.20
5	0.00209	22.20
6	0.00348	24.02
7	0.00525	31.32

TP 357:

```
      SUBROUTINE TP357(MODE)
      COMMON/L1/N,NILI,NINL,NELI,NENL
     F       /L2/X(4)
     F       /L6/FX
     F       /L11/LXL(4)
     F       /L12/LXU(4)
     F       /L13/XL(4)
     F       /L14/XU(4)
     F       /L20/LEX,NEX,FEX,XEX(4)
      LOGICAL LEX,LXL,LXU
      REAL*8 X,FX,XL,XU,FEX,XEX,XPT(36),YPT(36),P0,Q0,R0,S0,
     F        DALPHA,SUM,P1,Q1,R1,S1,ALPHA,CA,SA,PI,QI,A,B,C,AABB,
     F        TEST,J,PH,SP,CP,RI,SI,TEST1,CALCX,CALCY,SQL,
     F        DSQRT,DABS,DCOS,DSIN,DASIN,DATAN
      DATA(XPT(KI),KI=1,36)/0.113D+3,0.1101D+3,0.1062D+3,0.1013D+3,
     F              0.954D+2,0.888D+2,0.816D+2,0.74D+2,0.661D+2,
     F              0.584D+2,0.51D+2,0.443D+2,0.387D+2,0.345D+2,
     F              0.324D+2,0.329D+2,0.364D+2,0.428D+2,0.509D+2,
     F              0.59D+2,0.658D+2,0.715D+2,0.765D+2,0.811D+2,
     F              0.856D+2,0.902D+2,0.946D+2,0.989D+2,0.103D+3,
     F              0.1067D+3,0.1099D+3,0.1125D+3,0.1144D+3,
     F              0.1155D+3,0.1157D+3,0.1149D+3/,
     F     (YPT(KJ),KJ=1,36)/0.402D+2,0.468D+2,0.533D+2,0.594D+2,
     F              0.65D+2,0.699D+2,0.739D+2,0.769D+2,0.789D+2,
     F              0.798D+2,0.797D+2,0.785D+2,0.765D+2,0.736D+2,
     F              0.702D+2,0.66D+2,0.609D+2,0.543D+2,0.458D+2,
     F              0.361D+2,0.265D+2,0.181D+2,0.114D+2,0.62D+1,
     F              0.26D+1,0.3D+0,-0.7D+0,-0.6D+0,0.7D+0,0.31D+1,
     F              0.64D+1,0.105D+2,0.155D+2,0.21D+2,0.271D+2,
     F              0.336D+2/
      DATA P0,Q0,R0,S0/0.9D+2,0.0D+0,0.0D+0,0.0D+0/
      GOTO(1,2,3,3,3),MODE
    1 N=4
      NILI=0
      NINL=0
      NENL=0
      NENL=0
      X(1)=0.136D+3
      X(2)=0.0D+0
      X(3)=0.748D+2
      X(4)=0.757D+2
      DO 6 I=1,4
      LXL(I)=.TRUE.
    6 LXU(I)=.TRUE.
      LEX=.FALSE.
      NEX=1
      FEX=0.35845660D+0
      XEX(1)=0.13600762D+3
      XEX(2)=0.31371415D-1
      XEX(3)=0.73594390D+2
      XEX(4)=0.72187426D+2
      DO 7 K=1,4
    7 XL(K)=0.0D+0
      XU(1)=0.150D+3
      XU(2)=0.5D+2
      XU(3)=0.1D+3
      XU(4)=0.1D+3
      RETURN
```

```
 2 DALPHA=0.3141527D+1/0.18D+2
   SUM=0.0D+0
   P1=X(1)
   Q1=X(2)
   R1=X(3)
   S1=X(4)
   DO 54 I=2,36
   ALPHA=DALPHA*(I-1)
   CA=DCOS(ALPHA)
   SA=DSIN(ALPHA)
   PI=P1*CA-Q1*SA+P0*(0.1D+1-CA)+Q0*SA
   QI=P1*SA+Q1*CA+Q0*(0.1D+1-CA)-P0*SA
   A=R0*S1-S0*R1-Q1*R0+P1*S0+PI*Q1-P1*QI+QI*R1-PI*S1
   B=-R0*R1-S0*S1+P1*R0+Q1*S0-P1*PI-Q1*QI+PI*R1+QI*S1
   C=-R1*R0-S1*S0+PI*R0+QI*S0+P1*R1+Q1*S1-(P1*P1+Q1*Q1+PI*PI+QI
  F*QI)/0.2D+1
   AABB=A*A+B*B
   IF(AABB.LT.0.1D-29) GOTO 50
   TEST=C/DSQRT(AABB)
   IF(DABS(TEST).GT.0.1D+1) GOTO 51
   J=0.1D+1
52 PH=DASIN(TEST)-DATAN(B/A)
55 SP=DSIN(PH)
   CP=DCOS(PH)
   RI=R1*CP-S1*SP+PI-P1*CP+Q1*SP
   SI=R1*SP+S1*CP+QI-P1*SP-Q1*CP
   TEST1=(R1-R0)**2+(S1-S0)**2
   IF(TEST1.LT.0.1D-9) TEST1=0.1D-9
   IF(DABS((TEST1-(RI-R0)**2-(SI-S0)**2)/TEST1).LT.0.1D-2)GOTO53
   IF(J.EQ.0.2D+1) GOTO 51
   TEST=-TEST
   J=0.2D+1
   GOTO 52
50 PH=-DATAN(B/A)
   GOTO 55
51 FX=0.1D+21
   RETURN
53 CALCX=XPT(1)*CP-YPT(1)*SP+PI-P1*CP+Q1*SP
   CALCY=XPT(1)*SP+YPT(1)*CP+QI-P1*SP-Q1*CP
54 SUM=SUM+(CALCX-XPT(I))**2+(CALCY-YPT(I))**2
   SQL=(R1-R0)**2+(S1-S0)**2+(R1-P1)**2+(S1-Q1)**2+(P1-P0)**2+
  F(Q1-Q0)**2
   FX=SUM/0.1D+3+SQL/0.625D+5
 3 RETURN
   END
```

TP 358:

i	y_i	i	y_i
1	0.844	18	0.558
2	0.908	19	0.538
3	0.932	20	0.522
4	0.936	21	0.506
5	0.925	22	0.490
6	0.908	23	0.478
7	0.881	24	0.467
8	0.850	25	0.457
9	0.818	26	0.448
10	0.784	27	0.438
11	0.751	28	0.431
12	0.718	29	0.424
13	0.685	30	0.420
14	0.658	31	0.414
15	0.628	32	0.411
16	0.603	33	0.406
17	0.580		

TP 359:

i	A(i)	B(i)	C(i)	D(i)
1	-8720288.849	-145421.402	-155011.1084	-326669.5104
2	150512.5253	2931.1506	4360.53352	7390.68412
3	-156.6950325	-40.427932	12.9492344	-27.8986976
4	476470.3222	5106.192	10236.884	16643.076
5	729482.8271	15711.36	13176.786	30988.146

TP 360:

i	c_i	i	c_i
1	-8720288.849	6	-326669.5104
2	150512.5253	7	7390.68412
3	-156.6950325	8	-27.8986976
4	476470.3222	9	16643.076
5	729482.8271	10	30988.146

TP 361:

i	A(i)	B(i)	C(i)	D(i)
1	-8720288.849	-145421.402	-155011.1084	-326669.5104
2	150512.5253	2931.1506	4360.53352	7390.68412
3	-156.6950325	-40.427932	12.9492344	-27.8986976
4	476470.3222	5106.192	10236.884	16643.076
5	729482.8271	15711.36	13176.786	30988.146

TP 362:

```
      SUBROUTINE TP362(MODE)
      COMMON/L1/N,NILI,NINL,NELI,NENL
     F       /L2/X(5)
     F       /L3/G(4)
     F       /L6/FX
     F       /L9/INDEX1(4)
     F       /L11/LXL(5)
     F       /L12/LXU(5)
     F       /L13/XL(5)
     F       /L14/XU(5)
     F       /L20/LEX,NEX,FEX,XEX(5)
      LOGICAL INDEX1,LXL,LXU,LEX
      REAL*8 X,G,FX,XL,XU,FEX,XEX,TP362A
      GOTO (1,2,3,4,5),MODE
    1 N=5
      NILI=4
      NINL=0
      NELI=0
      NENL=0
      X(1)=15.D+0
      X(2)=9.05D+0
      X(3)=6.14D+0
      X(4)=4.55D+0
      X(5)=3.61D+0
      DO 14 I=2,4
      LXL(I)=.FALSE.
   14 LXU(I)=.FALSE.
      LXL(1)=.TRUE.
      LXU(1)=.TRUE.
      LXL(5)=.TRUE.
      LXU(5)=.FALSE.
      XL(1)=15.D+0
      XU(1)=20.D+0
      XL(5)=2.D+0
      LEX=.FALSE.
      NEX=1
      FEX=.26229998D+0
      XEX(1)=.15050962D+2
      XEX(2)=.88751199D+1
      XEX(3)=.59088230D+1
      XEX(4)=.48604810D+1
      XEX(5)=.43992690D+1
      RETURN
```

```
  2 FX=TP362A(X)
  3 RETURN
  4 DO 41 I=1,4
 41 IF (INDEX1(I)) G(I)=X(I)-X(I+1)
  5 RETURN
    END
    DOUBLE PRECISION FUNCTION TP362A(X)
    REAL*8 X(5),RPM,TORQUE,RAD,CON1,CON2,RPMIN,RPMAX,EI,VI,DT,VMAX,
   F V0,TSHIFT,TMAX,ACC,FORCE,V,ACC0,T,TT
    DATA RAD,CON1,CON2,RPMIN,RPMAX,EI,VI,DT,VMAX,V0,TSHIFT,TMAX/
   F 1.085D+0,1.466667D+0,12.90842D+0,6.D+2,5.7D+3,.6D+0,98.D+0,
   F .1D-1,1.D+2,5.D+0,.25D+0,1.D+2/
 13 IT=0
    ACC=0.D+0
    V=V0
    I=1
302 FORCE=.0239D+0*V**2+31.2D+0
301 RPM=V*CON2*X(I)
    IF (RPM.LT.RPMIN) GOTO 300
    IF (RPM.GE.RPMAX) GOTO 305
    IF (RPM.GE.6.D+2.AND.RPM.LT.1.9D+3)
   F TORQUE=.3846154D-7*RPM**3-.2108974359D-3*RPM**2
   F +.42455128205133D+0*RPM-1.8711538461540295D+2
    IF (RPM.GE.1.9D+3.AND.RPM.LT.3.D+3)
   F TORQUE=-.492424D-8*RPM**3+.1867424242D-4*RPM**2
   F +.1229545454547D-1*RPM+64.999999999986D+0
    IF (RPM.GE.3.D+3.AND.RPM.LT.4.5D+3)
   F TORQUE=-.26667D-9*RPM**3+.3D-5*RPM**2
   F -.1263333333336D-1*RPM+1.5510000000002947D+2
    IF (RPM.GE.4.5D+3.AND.RPM.LT.5.6D+3)
   F TORQUE=-.664141D-8*RPM**3+.8337626263D-4*RPM**2
   F -.34351868688129D+0*RPM+5.973636363847145D+2
    IF (RPM.GE.5.6D+3.AND.RPM.LT.6.D+3)
   F TORQUE=-.2539683D-7*RPM**3+.38158730157D-3*RPM**2
   F -1.9223492062348D+0*RPM+3.3806666664571530 4D+3
    ACC0=ACC
    ACC=RAD*(X(I)*TORQUE-FORCE*RAD)/(EI*X(I)**2+VI)
    IT=IT+1
    T=DT*IT
    V=V+(ACC0+ACC)/2.D+0*DT/CON1
    IF(T.GT.TMAX) GOTO 311
    IF (V.GE.VMAX) GOTO 311
    GOTO302
300 TP362A=TMAX
    RETURN
305 I=I+1
    IF (T.EQ.0.D+0) GOTO 301
    TT=T+TSHIFT
306 ACC=-FORCE*RAD**2/VI
    IT=IT+1
    T=DT*IT
    V=V+ACC*DT/CON1
    IF (T.LT.TT)GOTO 307
    GOTO 302
307 FORCE=.0293D+0*V**2+31.2D+0
    GOTO 306
311 TP362A=T/100.D+0
    RETURN
    END
```

TP 364:

```
      SUBROUTINE TP364(MODE)
      COMMON/L1/N,NILI,NINL,NELI,NENL
     F       /L2/X(6)
     F       /L3/G(4)
     F       /L6/FX
     F       /L9/INDEX1(4)
     F       /L11/LXL(6)
     F       /L12/LXU(6)
     F       /L13/XL(6)
     F       /L14/XU(6)
     F       /L20/LEX,NEX,FEX,XEX(6)
      LOGICAL INDEX1,LXL,LXU,LEX
      REAL*8 X,G,FX,XL,XU,FEX,XEX,XMU1,XMU2,DCOS,TP364A
      GOTO(1,2,3,4,5),MODE
    1 N=6
      NILI=2
      NINL=2
      NELI=0
      NENL=0
      X(1)=1.D+0
      X(2)=4.5D+0
      X(3)=4.D+0
      X(4)=5.D+0
      X(5)=3.D+0
      X(6)=3.D+0
      DO 11 I=1,4
   11 LXL(I)=.TRUE.
      LXL(5)=.FALSE.
      LXL(6)=.FALSE.
      LXU(5)=.FALSE.
      LXU(6)=.FALSE.
      LXU(1)=.TRUE.
      LXU(4)=.TRUE.
      LXU(2)=.FALSE.
      LXU(3)=.FALSE.
      XL(1)=.5D+0
      XL(2)=0.D+0
      XL(3)=0.D+0
      XL(4)=2.D+0
      XU(1)=3.D+0
      XU(4)=10.D+0
```

```
      LEX=.FALSE.
      NEX=1
      FEX=0.95426035D+0
      XEX(1)=0.50000000D+0
      XEX(2)=0.20689165D+1
      XEX(3)=0.36715256D+1
      XEX(4)=0.31491593D+1
      XEX(5)=0.20407360D+1
      XEX(6)=-0.38370290D+0
      RETURN
    2 FX=TP364A(X)
    3 RETURN
    4 XMU1=.7853981633D+0
      XMU2=2.356194491D+0
      IF (INDEX1(1)) G(1)=-X(1)+X(2)+X(3)-X(4)
      IF (INDEX1(2)) G(2)=-X(1)-X(2)+X(3)+X(4)
      IF (INDEX1(3)) G(3)=-X(2)*X(2)-X(3)*X(3)+(X(4)-X(1))*
     F (X(4)-X(1))+2.D+0*X(2)*X(3)*DCOS(XMU1)
      IF (INDEX1(4)) G(4)=X(2)*X(2)+X(3)*X(3)-(X(4)+X(1))*
     F (X(4)+X(1))-2.D+0*X(2)*X(3)*DCOS(XMU2)
    5 RETURN
      END
      DOUBLE PRECISION FUNCTION TP364A(X)
      REAL*8 X(6),X1A(31),Y1A(31),PHI(31),X1(31),Y1(31),
     F  COSS,COSY,SINS,SINY,XINC,PI,DCOS,DSIN,DSQRT,DABS,DFLOAT
      PI=3.141592654D+0
      XINC=2.D+0*PI/30.D+0
      DO 1 I=1,31
    1 PHI(I)=XINC*DFLOAT(I-1)
      CALL TP364B(PHI,X1,Y1)
      TP364A=0.D+0
      DO 2 I=1,31
      CALL TP364C(X,PHI(I),COSS)
      SINS=DSQRT(DABS(1.D+0-COSS*COSS))
      COSY=(X(4)+X(3)*COSS-X(1)*DCOS(PHI(I)))/X(2)
      SINY=(X(3)*SINS-X(1)*DSIN(PHI(I)))/X(2)
      X1A(I)=X(1)*DCOS(PHI(I))+X(5)*COSY-X(6)*SINY
      Y1A(I)=X(1)*DSIN(PHI(I))+X(5)*SINY+X(6)*COSY
    2 TP364A=TP364A+(X1A(I)-X1(I))**2+(Y1A(I)-Y1(I))**2
      TP364A=DSQRT(TP364A/31.D+0)
      RETURN
      END
      SUBROUTINE TP364B(PHI,X1,Y1)
      REAL*8 PHI(31),X1(31),Y1(31),PI,DSIN
      PI=3.141592654D+0
      DO 1 I=1,31
      X1(I)=.4D+0+DSIN((2.D+0*PI)*((PI-PHI(I))/
     F (2.D+0*PI)-.16D+0))
    1 Y1(I)=2.D+0+.9D+0*DSIN(PI-PHI(I))
      RETURN
      END
      SUBROUTINE TP364C(X,PHI,W)
      REAL*8 X(6),K,L,M,W,PHI,PI,A,B,C,TERM,DSIN,DCOS,DSQRT,DABS
      PI=3.141592654D+0
      M=2.D+0*X(1)*X(3)*DSIN(PHI)
      L=2.D+0*X(3)*X(4)-2.D+0*X(1)*X(3)*DCOS(PHI)
      K=X(1)*X(1)-X(2)*X(2)+X(3)*X(3)+X(4)*X(4)-
     F 2.D+0*X(4)*X(1)*DCOS(PHI)
      A=L*L+M*M
      B=2.D+0*K*L
      C=K*K-M*M
      TERM=DSQRT(DABS(B*B-4.D+0*A*C))
      IF ((PI-PHI).LT.0.D+0) TERM=-TERM
      W=(-B+TERM)/(2.D+0*A)
      RETURN
      END
```

TP 366:

i	C_i
1	.0059553571
2	.88392857
3	-.1175625
4	1.1088
5	.1303533
6	-.0066033
7	.00066173269
8	.017239878
9	-.0056595559
10	-.019120592
11	56.85075
12	1.08702
13	.32175
14	-.03762
15	.006198
16	2462.3121
17	-25.125634
18	161.18996
19	5000

i	C_i
20	-489510
21	44.333333
22	.33
23	.022556
24	-.007595
25	.00061
26	-.0005
27	.819672
28	.819672
29	24500
30	-250
31	.010204082
32	.000012244898
33	.0000625
34	.0000625
35	-.00007625
36	1.22
37	1
38	-1

TP 369:

i	C_i
1	833.33252
2	100
3	-83333.333
4	1250
5	1
6	-1250
7	1250000
8	1
9	-2500
10	.0025
11	.0025
12	.0025
13	.0025
14	-.0025
15	.01
16	-.01

TP 377 - 378:

i	A_i
1	-6.089
2	-17.164
3	-34.054
4	-5.914
5	-24.721
6	-14.986
7	-24.100
8	-10.708
9	-26.662
10	-22.179

TP 379:

i	y_i	i	y_i
1	1.366	34	0.375
2	1.191	35	0.372
3	1.112	36	0.391
4	1.013	37	0.396
5	0.991	38	0.405
6	0.885	39	0.428
7	0.831	40	0.429
8	0.847	41	0.523
9	0.786	42	0.562
10	0.725	43	0.607
11	0.746	44	0.653
12	0.679	45	0.672
13	0.608	46	0.708
14	0.655	47	0.633
15	0.616	48	0.668
16	0.606	49	0.645
17	0.602	50	0.632
18	0.626	51	0.591
19	0.651	52	0.559
20	0.724	53	0.597
21	0.649	54	0.625
22	0.649	55	0.739
23	0.694	56	0.710
24	0.644	57	0.729
25	0.624	58	0.720
26	0.661	59	0.636
27	0.612	60	0.581
28	0.558	61	0.428
29	0.533	62	0.292
30	0.495	63	0.162
31	0.500	64	0.098
32	0.423	65	0.054
33	0.395		

TP 380:

i	a_i
1	-.00133172
2	-.002270927
3	-.00248546
4	-4.67
5	-4.671973
6	-.00814
7	-.008092
8	-.005
9	-.000909
10	-.00088
11	-.00119

i	c_i
1	.05367373
2	.021863746
3	.097733533
4	.0066940803
5	10^{-6}
6	10^{-5}
7	10^{-6}
8	10^{-10}
9	10^{-8}
10	10^{-2}
11	10^{-4}
12	.10898645
13	$1.6108052 \cdot 10^{-4}$
14	10^{-23}
15	$1.9304541 \cdot 10^{-6}$
16	10^{-3}
17	10^{-6}
18	10^{-5}
19	10^{-6}
20	10^{-9}
21	10^{-9}
22	10^{-3}
23	10^{-3}
24	.10898645
25	$1.6108052 \cdot 10^{-5}$
26	10^{-23}
27	$1.9304541 \cdot 10^{-8}$
28	10^{-5}
29	$1.1184059 \cdot 10^{-4}$
30	10^{-4}

TP 381 - 382:

Data for least-cost hog ration problem

Fraction x_i $i=1,2,\ldots,13$	Constituent	Cost/ton ($100) r_i	% Protein mean $\bar{s}_i$	% Protein var. $\sigma^2_{s_i}$	% Calcium mean $\bar{u}_i$	% Calcium var. $\sigma^2_{u_i}$	% Phosphorus mean $\bar{v}_i$	% Phosphorus var. $\sigma^2_{v_i}$
x_1	barley	0.80	11.6	0.4844	0.05	0.0001	0.35	0.0010
x_2	wheat	1.10	13.7	0.3003	0.07	0	0.37	0.0009
x_3	corn	0.85	9.5	0.1444	0	0	0.10	0.0001
x_4	soy	3.45	48.5	0.0588	0.33	0	0.62	0.0005
x_5	mustard	2.00	31.9	4.9863	0	0	0	0
x_6	dried milk	2.10	51.1	0.0653	1.27	0.0040	1.03	0.0021
x_7	fish solubles	3.00	65.5	21.0222	1.27	0.1404	1.69	0.0825
x_8	d_i-cal.phos.	0.80	0	0	23.35	1.3631	18.21	0.2073
x_9	limestone	0.45	0	0	35.84	0.5138	0.01	0
x_{10}	molasses	0.72	0	0	0.81	0.0289	0.08	0.0004
x_{11}	dehy. alfalfa	1.80	21.8	0.2970	1.79	0.0097	0.31	0.0005
x_{12}	shrimp meal	3.00	46.9	9.2933	7.34	0.3893	1.59	0.0107
x_{13}	mono-sodium-phosphate	0.60	0	0	0	0	22.45	1.0206

TP 383:

i	a_i	b_i	c_i
1	12842.275	25	5.47954
2	634.25	26	.83234
3	634.25	26	.94749
4	634.125	27	1.11082
5	1268.	28	2.64824
6	633.875	29	1.55868
7	633.75	30	1.75215
8	1267.	32	3.90896
9	760.05	33	2.74284
10	633.25	34	2.60541
11	1266.25	35	5.96184
12	632.875	37	3.29522
13	394.46	38	1.83517
14	940.838	36	2.81372

TP 385:

j	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	b_i
a_{1j}	100	100	10	5	10	0	0	25	0	10	55	5	45	20	0	385
a_{2j}	90	100	10	35	20	5	0	35	55	25	20	0	40	25	10	470
a_{3j}	70	50	0	55	25	100	40	50	0	30	60	10	30	0	40	560
a_{4j}	50	0	0	65	35	100	35	60	0	15	0	75	35	30	65	565
a_{5j}	50	10	70	60	45	45	0	35	65	5	75	100	75	10	0	645
a_{6j}	40	0	50	95	50	35	10	60	0	45	15	20	0	5	5	430
a_{7j}	30	60	30	90	0	30	5	25	0	70	20	25	70	15	15	485
a_{8j}	20	30	40	25	40	25	15	10	80	20	30	30	5	65	20	455
a_{9j}	10	70	10	35	25	65	0	30	0	0	25	0	15	50	55	390
a_{10j}	5	10	500	5	20	5	10	35	95	70	20	10	35	10	30	860

TP 335:

j	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	b_i
a_{1j}	100	100	10	5	10	0	0	25	0	10	55	5	45	20	0	385
a_{2j}	90	100	10	35	20	5	0	35	55	25	20	0	40	25	10	470
a_{3j}	70	50	0	55	25	100	40	50	0	30	60	10	30	0	40	560
a_{4j}	50	0	0	65	35	100	35	60	0	15	0	75	35	30	65	565
a_{5j}	50	10	70	60	45	45	0	35	65	5	75	100	75	10	0	645
a_{6j}	40	0	50	95	50	35	10	60	0	45	15	20	0	5	5	430
a_{7j}	30	60	30	90	0	30	5	25	0	70	20	25	70	15	15	485
a_{8j}	20	30	40	25	40	25	15	10	80	20	30	30	5	65	20	455
a_{9j}	10	70	10	35	25	65	0	30	500	0	25	0	15	50	55	890
a_{10j}	5	10	100	5	20	5	10	35	95	70	20	10	35	10	30	460

TP 386:

j	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	b_j
a_{1j}	100	100	10	5	10	0	0	25	0	10	55	5	45	20	0	385
a_{2j}	90	100	10	35	20	5	0	35	55	25	20	0	40	25	10	470
a_{3j}	70	50	0	55	25	100	40	50	0	30	60	10	30	0	40	560
a_{4j}	50	0	0	65	35	100	35	60	0	15	0	75	35	30	65	565
a_{5j}	50	10	70	60	45	45	0	35	65	5	75	100	75	10	0	645
a_{6j}	40	0	50	95	50	35	10	60	0	45	15	20	0	5	5	430
a_{7j}	30	60	30	90	0	30	5	25	0	70	20	25	70	15	15	485
a_{8j}	20	30	40	25	40	25	15	10	80	20	30	30	5	65	20	455
a_{9j}	10	70	10	35	25	65	0	30	0	0	25	0	15	50	55	390
a_{10j}	5	10	100	5	20	5	10	35	95	70	20	10	35	10	30	460

TP 387:

j	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	b_j
a_{1j}	100	100	10	5	10	0	0	25	0	10	55	5	45	20	0	385
a_{2j}	90	100	10	35	20	5	0	35	55	25	20	0	40	25	10	470
a_{3j}	70	50	0	55	25	100	40	50	0	30	60	10	30	0	40	560
a_{4j}	50	0	0	65	35	100	35	60	0	15	0	75	35	30	65	565
a_{5j}	50	10	70	60	45	45	0	35	65	5	75	100	75	10	0	645
a_{6j}	40	0	50	95	50	35	10	60	0	45	15	20	0	5	5	430
a_{7j}	30	60	30	90	0	30	5	25	0	70	20	25	70	15	15	485
a_{8j}	20	30	40	25	40	25	15	10	80	20	30	30	5	65	20	455
a_{9j}	10	70	10	35	25	65	0	30	0	0	25	0	15	50	55	390
a_{10j}	5	10	100	5	20	5	10	35	95	70	20	10	35	10	30	460

TP 388:

j	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	b_i
a_{1j}	100	100	10	5	10	0	0	25	0	10	55	5	45	20	0	385
a_{2j}	90	100	10	35	20	5	0	35	55	25	20	0	40	25	10	470
a_{3j}	70	50	0	55	25	100	40	50	0	30	60	10	30	0	40	560
a_{4j}	50	0	0	65	35	100	35	60	0	15	0	75	35	30	65	565
a_{5j}	50	10	70	60	45	45	0	35	65	5	75	100	75	10	0	645
a_{6j}	40	0	50	95	50	35	10	60	0	45	15	20	0	5	5	430
a_{7j}	30	60	30	90	0	30	5	25	0	70	20	25	70	15	15	485
a_{8j}	20	30	40	25	40	25	15	10	80	20	30	30	5	65	20	455
a_{9j}	10	70	10	35	25	65	0	30	0	0	25	0	15	50	55	390
a_{10j}	5	10	100	5	20	5	10	35	95	70	20	10	35	10	30	460

j	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	b_i
a_{12j}	1	2	3	4	5	6	7	8	9	10	15	16	17	18	19	70
a_{13j}	45	25	35	85	40	73	17	52	86	14	30	50	40	70	60	361
a_{14j}	53	74	26	17	25	25	26	24	85	35	14	23	37	56	10	265
a_{15j}	12	43	51	39	58	42	60	20	40	80	75	85	95	23	67	395

TP 389:

j	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	b_i
a_{1j}	100	100	10	5	10	0	0	25	0	10	55	5	45	20	0	385
a_{2j}	90	100	10	35	20	5	0	35	55	25	20	0	40	25	10	470
a_{3j}	70	50	0	55	25	100	40	50	0	30	60	10	30	0	40	560
a_{4j}	50	0	0	65	35	100	35	60	0	15	0	75	35	30	65	565
a_{5j}	50	10	70	60	45	45	0	35	65	5	75	100	75	10	0	645
a_{6j}	40	0	50	95	50	35	10	60	0	45	15	20	0	5	5	430
a_{7j}	30	60	30	90	0	30	5	25	0	70	20	25	70	15	15	485
a_{8j}	20	30	40	25	40	25	15	10	80	20	30	30	5	65	20	455
a_{9j}	10	70	10	35	25	65	0	30	0	0	25	0	15	50	55	390
a_{10j}	5	10	100	5	20	5	10	35	95	70	20	10	35	10	30	460

j	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	b_i
a_{12j}	1	2	3	4	5	6	7	8	9	10	15	16	17	18	19	70
a_{13j}	45	25	35	85	40	73	17	52	86	14	30	50	40	70	60	361
a_{14j}	53	74	26	17	25	25	26	24	85	35	14	23	37	56	10	265
a_{15j}	12	43	51	39	58	42	60	20	40	80	75	85	95	23	67	395

TP 391:

X* = (7898.84, 6316.09, 4957.31, 3806.63, 2846.71, 2060.11, 1429.46, 937.42, 566.67, 299.90, 119.83, 9.16, -49.40, -73.12, -79.28, -85.16, -108.03, -165.16, -273.82, -451.27, -714.77, -1081.60, -1569, -2194.25, -2974.59, -3927.29, -5069.59, -6418.77, -7992.06, -9806.72)

TP 392:

X = (80, 100, 400, 100, 200, 200, 100, 250, 400, 50, 200, 500, 50, 200, 500, 100, 120, 410, 120, 250, 250, 150, 300, 410, 600, 250, 510, 100, 250, 510)

X* = (99.99, 142.22, 519.88, 136.74, 103.47, 399.99, 191.70, 1.56, 500, 143.43, 82.39, 629.82, 99.92, 125.22, 600, 101.85, 142.25, 519.88, 144.58, 105.73, 409.59, 182.01, 29.34, 490.52, 143.43, 52.43, 629.70, 99.92, 125.12, 600)

TP 392:

	Product I					Product II					Product III				
	Period					Period					Period				
	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
r_{1jt}	1000			1100		520			600		910		1000		
r_{2jt}	0.3					0.1					0.2				
k_{Ajt}	120	150	150	170	170	65		80			105		120		
k_{1jt}	150			170		75			90		140			150	
k_{Pjt}	160			180		75			90		140			150	
k_{3jt}	.02	.2	.25			.01	.1	.1	.15		.015	.15			
k_{Ljt}	.005	.05	.06			.005	.05	.06			.005	.05	.06		
k_{Ljt}	80		100			45			50		75		90		
h_{jt}	100	180	220	150	100	280	400	450	450	400	520	400	500	630	600

	Product I			Product II			Product III		
	Machine			Machine			Machine		
	1	2	3	1	2	3	1	2	3
t_{ij}	.6	.3	.36	.4	.1	.08	.1	.12	.06

	Machine I					Machine II					Machine III				
	Period					Period					Period				
	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
b_{it}	170					170					180				

TP 393:

X = (1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
1, 1, 1, 1, 1.3, 1.3, 1.3, 1.3, 1.3, 1.3, 1, 1, 1, 1, 1, 1,
1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)

X* = (2, 0.002, 2, 0.034, 0.017, 2, 1.89, 0.002, 2, 0.034, 0.017, 2,
2, 0.002, 2, 0.002, 0.002, 1.988, 2, 0.002, 2, 0.002, 0.002, 2,
1.016, 0.002, 1.003, 0.002, 0.002, 0.9997, 1.113, 0.002, 1.102,
0.002, 0.002, 1.103, 0.923, 0.934, 0.929, 0.914, 0.905, 0.8945,
1.1746, 0.002, 1.121, 0.002, 0.002, 1.116)

XU = (2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2,
2, 2, 2, 2, -, -, -, -, -, -, -, -, -, -, -, -, -, -, -, -,
-, -, -, -, -, -, -, -)

TP 393:

```
      SUBROUTINE TP393(MODE)
      COMMON/L1/N,NILI,NINL,NELI,NENL
      COMMON/L2/X(48)
      COMMON/L3/G(3)
      COMMON/L6/FX
      COMMON/L9/INDEX1
      COMMON/L11/LXL
      COMMON/L12/LXU
      COMMON/L13/XL(48)
      COMMON/L14/XU(48)
      COMMON/L20/LEX,NEX,FEX,XEX(48)
      DIMENSION PSI(2),PHI(1)
      LOGICAL LEX,LXL(48),LXU(48),INDEX1(3)
      REAL*8 X,FX,XEX,FEX,G,XL,XU,PSI,F,PHI,C,E,DSQRT
      GOTO (1,2,3,4,5),MODE
    1 N=48
      NILI=0
      NINL=1
      NELI=2
      NENL=0
      DO 6 I=1,24
    6 X(I)=1.D+0
      DO 7 I=25,30
    7 X(I)=1.3D+0
      DO 8 I=31,48
    8 X(I)=1.D+0
      DO 9 I=1,48
      LXL(I)=.TRUE.
    9 XL(I)=.002D+0
      DO 10 I=1,24
      LXU(I)=.TRUE.
   10 XU(I)=2.D+0
      DO 11 I=25,48
   11 LXU(I)=.FALSE.
      LEX=.FALSE.
      NEX=1
      FEX=0.86337998D+0
      XEX(1)=2.D+0
      XEX(2)=.002D+0
      XEX(3)=2.D+0
      XEX(4)=.0339797D+0
      XEX(5)=.01657455D+0
      XEX(6)=2.D+0
      XEX(7)=1.8945347D+0
      XEX(8)=.002D+0
      XEX(9)=2.D+0
      XEX(10)=.03424074D+0
      XEX(11)=.016670308D+0
      XEX(12)=2.D+0
      XEX(13)=2.D+0
      XEX(14)=.002D+0
      XEX(15)=2.D+0
      XEX(16)=.002D+0
      XEX(17)=.002D+0
      XEX(18)=1.988000D+0
      XEX(19)=2.D+0
      XEX(20)=.002D+0
      XEX(21)=2.D+0
```

```
      XEX(22)=.002D+0
      XEX(23)=.002D+0
      XEX(24)=2.D+0
      XEX(25)=1.0159886D+0
      XEX(26)=.002D+0
      XEX(27)=1.003163D+0
      XEX(28)=.002D+0
      XEX(29)=.002D+0
      XEX(30)=.999691944D+0
      XEX(31)=1.11272844D+0
      XEX(32)=.002D+0
      XEX(33)=1.1024463D+0
      XEX(34)=.002D+0
      XEX(35)=.002D+0
      XEX(36)=1.1030764D+0
      XEX(37)=.92326572D+0
      XEX(38)=.9343325D+0
      XEX(39)=.92947437D+0
      XEX(40)=.91383802D+0
      XEX(41)=.90517162D+0
      XEX(42)=.89452569D+0
      XEX(43)=1.174573D+0
      XEX(44)=.002D+0
      XEX(45)=1.12080408D+0
      XEX(46)=.002D+0
      XEX(47)=.002D+0
      XEX(48)=1.1163321536D+0
      RETURN
    2 E=0.D+0
      DO 100 I=1,12
      C=1.D+0-X(I)
  100 E=E+10.D+0*C*C
      DO 120 I=25,36
      C=X(I)-1.D+0
  120 E=E+1000.D+0*(.1D+0+2.D+0*C*(C+DSQRT(.1D+0+C*C)))/4.D+0
      DO 140 I=37,42
      C=X(I)-1.D+0
  140 E=E+2000.D+0*(.1D+0+2.D+0*C*(C+DSQRT(.1D+0+C*C)))/4.D+0
      DO 160 I=43,48
  160 E=E+100.D+0*X(I)
      FX=E/1000.D+0
    3 RETURN
    4 IF (.NOT.INDEX1(1)) GOTO 12
      CALL TP393B(X,PHI)
      G(1)=PHI(1)
   12 IF (.NOT.INDEX1(2)) GOTO 14
      G(2)=12.D+0
      DO 13 I=1,12
   13 G(2)=G(2)-X(I)
   14 IF (.NOT.INDEX1(3)) GOTO 5
      G(3)=12.D+0
      DO 15 I=1,12
   15 G(3)=G(3)-X(I+12)
    5 RETURN
      END
```

```
      SUBROUTINE TP393B(X,PHI)
      DIMENSION A(18),U(18),X(48),PHI(1)
      DATA (A(I),I=1,18)/.9D+0,.8D+0,1.1D+0,1.D+0,.7D+0,1.1D+0,
     F     1.D+0,1.D+0,1.1D+0,.9D+0,.8D+0,1.2D+0,.9D+0,1.2D+0,
     F     1.2D+0,1.D+0,1.D+0,.9D+0/
      REAL*8 X,PHI,A,ALP,U,SUM,R
C     1ST TIER OF GASFIERS
      DO 20 I=1,6
      K1=I+24
      K2=I+42
      K3=I+12
      ALP=X(K1)*X(K1)*A(I)*2.D+0*X(K2)/(1.D+0+X(K2))*X(K3)
   20 U(I)=X(I)*X(I)/(X(I)+ALP)
C     2ND TIER OF GASFIERS
      DO 40 I=7,12
      K1=I+24
      K2=I+36
      K3=I+12
      ALP=X(K1)*X(K1)*A(I)*2.D+0*X(K2)/(1.D+0+X(K2))*X(K3)
      SUM=X(I)+U(I-6)
   40 U(I)=SUM*SUM/(SUM+ALP)
C     1ST TIER OF METHANATORS
      DO 60 I=13,15
      K1=2*(I-10)+1
      K2=I+24
      ALP=X(K2)*X(K2)*A(I)
      SUM=U(K1)+U(K1+1)
   60 U(I)=SUM*SUM/(SUM+ALP)
C     2ND TIER OF METHANATORS
      DO 80 I=16,18
      K1=I+24
      ALP=X(K1)*X(K1)*A(I)
      SUM=U(I-3)
   80 U(I)=SUM*SUM/(SUM+ALP)
      R=U(16)+U(17)+U(18)
      PHI(1)=1.5D+0-R
      RETURN
      END
```

TP 394:

X* = (0.91287, 0.408268, -0.000017, -0.0000054, 0.000002, -0.0000089, 0.0000082, -0.000014, 0.000022, -0.000014, 0.0000135, -0.000004, -0.000011, -0.000013, 0.0000079, 0.00002, 0.00000456, -0.000009, -0.00001, -0.0000014)

TP 395:

X* = (0.91285, 0.40829, -0.0000065, -0.0000991, 0.000119, -0.0000465, 0.0000576, -0.000048, 0.0000257, 0.0000117, -0.000031, 0.0000087, 0.00002, -0.000012, -0.0000164, 0.00000734, 0.000017, 0.0000044, -0.0000059, -0.0000025, 0.0000046, 0.00000325, -0.00000666, -0.0000144, -0.000012, -0.0000039, 0.00000099, 0.00000015, -0.00000068, 0.0000024, 0.0000054, 0.0000027, -0.00000293, -0.0000038, 0.00000061, 0.0000044, 0.0000041, 0.000001455, -0.00000126, -0.000003, -0.00000386, -0.00000426, -0.00000451, -0.0000045, -0.0000038, -0.00000234, -0.00000075, -0.0000000546, -0.0000011, -0.0000021)

Appendix B

SOME NUMERICAL TEST RESULTS

To give a test designer the possibility to compare his own performance results with those of the sequential quadratic programming code NLPQL of Schittkowski [3], some numerical results are presented in this Appendix. Although all solution data of the "practical" test problems are computed by NLPQL, there are nevertheless some differences in the results. To be able to present a solution as accurate as possible, NLPQL was executed with very low stopping tolerances and sometimes also with more appropriate starting values. But the presented test results are evaluated by the standard version of NLPQL, in particular starting from the given initial values and a fixed stopping criterion (1.E-7). In some cases, local solutions were approximated different from those known in advance.

The numerical tests were performed on a VAX11/780 of the Institute for Computer Science of the University of Stuttgart. All calculations within the driving program, test problems and the optimization code were carried out in double precision FORTRAN. Table 6 summarizes the numerical results using the following abbreviations:

TP: Number of the test problem

NF: Number of objective function evaluations

NG: Number of restriction function evaluations (each restriction counted individually)

NDF: Number of gradient evaluations of the objective function

NDG: Number of gradient evaluations of the objective function (each restriction counted individually)

FV: Objective function value

VC: Sum of constraint violations.

If the gradients are approximated numerically by a forward difference formula, the additional function evaluations are counted for NF and

NG, respectively. NLPQL reports a few failures if round-off errors in the neighbourhood of a solution prevent satisfaction of the stopping criteria, e.g. TP 332, TP 382, or if the problem is very ill-conditioned, e.g. the Hilbert-problems TP 277 to TP 279 (although TP 280 is solved successfully) and in particular a couple of nonlinear least squares problems. To show the capability of a special purpose code for some of the least squares problems, Table 7 reports a few comparative test data obtained by DFNLP and NLPQL. The data fitting algorithm realized in DFNLP, transforms a nonlinear least squares problem into a constrained nonlinear programming problem by introducing additional variables and constraints, cf. Schittkowski [4] for more details. The transformed problem is solved by NLPQL.

TP	F	NF	NG	NDF	NDG	FV	VC
201	0	5	0	3	0	0.	0.
202	0	17	0	13	0	0.489842E+2	0.
203	0	12	0	10	0	0.780804E-8	0.8E-8
204	0	7	0	6	0	0.183601	0.2E-6
205	0	14	0	12	0	0.156232E-7	0.
206	0	10	0	7	0	0.219173E-10	0.
207	0	14	0	13	0	0.270343E-7	0.
208	0	47	0	34	0	0.112238E-7	0.
209	4	179	0	119	0	0.613716E-5	0.
210	1	263	0	200	0	0.248516	0.
211	0	26	0	20	0	0.478140E-8	0.
212	0	22	0	15	0	0.165129E-9	0.
213	0	52	0	48	0	0.742919E-7	0.
214	0	169	0	79	0	0.734645	0.
215	0	7	7	7	7	-0.265923E-8	0.3E-8
216	0	12	12	10	10	0.489694E+2	0.3E-8
217	0	9	18	9	17	-0.800000	0.3E-8
218	0	14	14	14	12	0.	0.7E-4
219	0	18	36	17	34	-0.100000E+1	0.1E-8
220	0	63	63	63	63	0.100049E+1	0.1E-9
221	0	29	29	29	29	-0.999991	0.
222	0	7	7	6	6	-0.150000E+1	0.3E-12
223	0	9	18	9	10	-0.834032	0.3E-12
224	0	4	16	4	7	-0.304000E+3	0.
225	0	7	35	7	17	0.200000E+1	0.
226	0	7	14	7	8	-0.500000	0.9E-10
227	0	6	12	5	9	0.100000E+1	0.9E-9
228	0	8	16	8	8	-0.300000E+1	0.2E-13
229	0	32	0	24	0	0.946011E-9	0.
230	0	5	10	4	8	0.375000	0.
231	0	47	94	35	7	0.117573E-7	0.
232	0	5	15	5	7	-0.100000E+1	0.8E-16
233	0	18	18	14	4	0.471850E-9	0.
234	0	1	1	1	1	-0.800000	0.
235	0	21	21	18	18	0.400000E-1	0.4E-7
236	0	9	18	9	4	-0.589034E+2	0.
237	0	12	36	12	9	-0.589034E+2	0.
238	0	11	33	9	9	-0.589034E+2	0.
239	0	23	23	16	10	-0.589034E+2	0.
240	0	8	0	8	0	0.837389E-11	0.
241	0	34	0	25	0	0.958716E-8	0.
242	0	11	0	11	0	0.119600E-10	0.
243	0	11	0	7	0	0.796579	0.
244	0	18	0	18	0	0.416012E-8	0.
245	0	16	0	16	0	0.133417E-3	0.
246	4	18	0	3	0	0.248683E+1	0.
247	0	5	0	2	0	0.	0.
248	0	28	56	18	33	-0.800000	0.6E-9
249	0	14	14	14	14	0.100000E+1	0.
250	0	5	10	5	5	-0.330000E+4	0.

Table 2: Test results for NLPQL

TP	F	NF	NG	NDF	NDG	FV	VC
251	0	8	8	8	7	-0.345600E+4	0.
252	0	15	15	14	14	0.400000E-1	0.3E-4
253	0	6	6	6	2	0.692820E+2	0.
254	0	9	18	9	18	-0.173205E+1	0.2E-4
255	0	10	0	10	0	-0.188112E+11	0.
256	0	36	0	27	0	0.531126E-7	0.
257	0	29	0	19	0	0.237535E-7	0.
258	0	76	0	63	0	0.359922E-7	0.
259	0	27	0	15	0	0.358444E-7	0.
260	0	76	0	63	0	0.359923E-7	0.
261	0	41	0	36	0	0.184501E-6	0.
262	0	7	28	7	16	-0.100000E+2	0.2E-15
263	0	15	60	15	57	-0.100000E+1	0.1E-4
264	0	14	42	11	28	-0.440000E+2	0.2E-7
265	0	2	4	2	4	0.190363E+1	0.
266	0	31	0	21	0	0.996817	0.
267	0	35	0	32	0	0.159553E-4	0.
268	0	28	140	22	23	0.420890E-10	0.
269	0	8	24	7	21	0.409302E+1	0.
270	0	9	9	8	8	-0.100000E+1	0.2E-6
271	0	25	0	7	0	0.731853E-31	0.
272	0	36	0	33	0	0.565572E-2	0.
273	0	45	0	35	0	0.764049E-6	0.
274	0	6	0	5	0	0.175540E-35	0.
275	0	5	0	4	0	0.139756E-6	0.
276	0	9	0	8	0	0.131843E-10	0.
277	4	14	56	4	12	0.543516E+1	0.1
278	4	15	90	5	26	0.745414E+1	0.4
279	4	20	160	7	49	0.105124E+2	0.9E-1
280	0	11	110	11	49	0.133754E+2	0.2E-17
281	4	254	0	88	0	0.187167E-3	0.
282	0	182	0	131	0	0.458011E-8	0.
283	0	107	0	103	0	0.327441E-4	0.
284	0	73	730	32	155	-0.184000E+4	0.5E-5
285	0	55	550	20	173	-0.825200E+4	0.4E-7
286	0	183	0	112	0	0.152545E-4	0.
287	0	28	0	27	0	0.669341E-9	0.
288	0	33	0	29	0	0.183602E-6	0.
289	0	8	0	7	0	0.100871E-9	0.
290	0	19	0	17	0	0.107889E-6	0.
291	0	46	0	43	0	0.111618E-5	0.
292	0	87	0	83	0	0.880737E-5	0.
293	0	146	0	130	0	0.624188E-4	0.
294	0	61	0	48	0	0.148495E-7	0.
295	0	98	0	78	0	0.180818E-7	0.
296	0	121	0	92	0	0.175103E-7	0.
297	0	185	0	140	0	0.258286E-7	0.
298	1	267	0	200	0	0.219352E-5	0.
299	0	199	0	101	0	0.141069E-7	0.
300	0	41	0	21	0	-0.200000E+2	0.

Table 2: Test results for NLPQL

TP	F	NF	NG	NDF	NDG	FV	VC
301	0	101	0	51	0	-0.500000E+2	0.
302	0	201	0	101	0	-0.100000E+3	0.
303	0	23	0	19	0	0.420711E-7	0.
304	0	37	0	32	0	0.267789E-8	0.
305	0	35	0	30	0	0.647403E-4	0.
307	0	29	0	22	0	0.124362E+3	0.
308	4	22	0	8	0	0.793171	0.
309	0	14	0	10	0	-0.398717E+1	0.
311	0	16	0	10	0	0.434753E-8	0.
312	0	66	0	49	0	0.761467E-10	0.
314	0	8	0	6	0	0.169043	0.
315	0	10	30	9	17	-0.800000	0.9E-11
316	0	10	10	8	8	0.334315E+3	0.1E-10
317	0	14	14	10	10	0.372467E+3	0.2E-9
318	0	15	15	12	12	0.412750E+3	0.2E-12
319	0	16	16	12	12	0.452404E+3	0.4E-9
320	0	16	16	12	12	0.485531E+3	0.3E-13
321	0	15	15	12	12	0.496112E+3	0.2E-10
322	0	23	23	15	15	0.499960E+3	0.9E-11
323	0	9	18	7	8	0.379894E+1	0.3E-8
324	0	8	16	8	9	0.500000E+1	0.4E-7
325	0	7	21	6	12	0.379134E+1	0.3E-8
326	0	8	16	7	13	-0.798078E+2	0.4E-11
327	0	5	5	3	2	0.306463E-1	0.
328	0	11	0	11	0	0.174415E+1	0.
329	0	7	21	7	15	-0.696181E+4	0.7E-13
330	0	10	10	10	10	0.162058E+1	0.3E-9
331	0	10	10	7	4	0.425838E+1	0.
332	4	164	254	0	0	0.114950E+1	0.2E-7
333	0	8	0	3	0	0.560827E+1	0.
334	0	33	0	24	0	0.821492E-2	0.
335	0	25	50	21	42	-0.447214E-2	0.9E-8
336	0	14	28	11	22	-0.337896	0.3E-8
337	0	11	11	8	8	0.600000E+1	0.2E-9
338	0	17	34	12	24	-0.720570E+1	0.8E-11
339	0	11	11	10	9	0.336168E+1	0.
340	0	8	8	8	8	-0.540000E-1	0.
341	0	14	14	12	11	-0.226274E+2	0.3E-7
342	0	30	30	28	28	-0.226274E+2	0.2E-7
343	0	5	10	5	10	-0.568478E+1	0.3E-12
344	0	11	11	10	10	0.325682E-1	0.7E-8
345	0	29	29	20	20	0.325682E-1	0.8E-8
346	0	5	10	5	10	-0.568478E+1	0.3E-12
347	0	2	2	2	2	0.173746E+5	0.
348	0	76	76	0	0	0.369708E+2	0.2E-10
350	0	14	0	13	0	0.385043E-3	0.

Table 2: Test results for NLPQL

TP	F	NF	NG	NDF	NDG	FV	VC
351	0	43	0	39	0	0.318572E+3	0.
352	0	16	0	15	0	0.903234E+3	0.
353	0	2	6	2	5	-0.399337E+2	0.
354	0	20	20	14	13	0.113784	0.
355	0	345	345	170	170	0.696755E+2	0.2E-12
356	0	55	203	0	0	0.238116E+1	0.8E-15
357	0	55	0	0	0	0.358457	0.
358	0	23	0	15	0	0.547297E-4	0.
359	0	7	98	7	39	-0.528034E+7	0.
360	0	30	60	21	22	-0.528034E+7	0.
361	0	8	48	8	13	-0.776412E+6	0.
362	0	6	24	0	0	0.270800	0.
364	0	186	222	0	0	0.606002E-1	0.1E-4
365	0	120	502	0	0	0.233137E+2	0.3E-8
366	0	209	1162	0	0	0.122723E+4	0.8E-13
367	4	59	295	16	67	-0.373705E+2	0.2E-5
368	0	1	0	1	0	0.	0.
369	0	81	486	71	321	0.704925E+4	0.2E-11
370	0	224	0	0	0	0.228771E-2	0.
371	0	330	0	0	0	0.380890E-4	0.
372	4	14	168	3	17	0.706560E+6	0.5E+2
373	0	237	1422	112	672	0.133901E+5	0.6E-8
374	0	381	4415	0	0	0.233263	0.7E-7
375	0	46	414	25	225	-0.151610E+2	0.7E-8
376	0	275	2275	0	0	-0.443009E+4	0.6E-13
377	0	22	66	0	0	-0.795001E+3	0.3E-9
378	0	561	1683	0	0	-0.477610E+2	0.2E-6
379	0	62	0	50	0	0.401377E-1	0.
380	0	1160	2978	0	0	0.316832E+1	0.
381	0	7	28	7	22	0.101490E-1	0.7E-16
382	4	23	92	11	33	0.103831E+1	0.8E-5
383	0	95	95	48	48	0.728594E+6	0.1E-16
384	0	49	490	20	174	-0.831L26E+4	0.4E-8
385	0	63	630	26	216	-0.831529E+4	0.6E-9
386	0	75	825	34	293	-0.816437E+4	0.9E-8
387	0	77	847	33	314	-0.825014E+4	0.1E-7
388	0	52	780	25	274	-0.582108E+4	0.7E-7
389	0	53	795	25	261	-0.580972E+4	0.8E-9
391	0	1050	0	0	0	0.162831E-6	0.
392	0	3288	65730	0	0	-0.169607E+7	0.3E-13
393	0	4362	13086	0	0	0.863380	0.1E-6
394	0	50	50	49	49	0.191666E+1	0.7E-8
395	0	85	85	78	78	0.191667E+1	0.7E-8

Table 2: Test results for NLPQL

TP	DFNLP			NLPQL		
	NF	NDF	FV	NF	NDF	FV
210	41	23	.34E-11	263	200	.24931
246	4	3	.48E-14		failure	
261	16	13	.27E-7	41	36	.18E-6
282	34	30	.10E-6	182	131	.46E-8
286	4	3	.34E-10	183	112	.15E-4
294	34	21	.13E-7	61	48	.15E-7
308	12	10	.77325		failure	

Table 4: Comparative test results for some least squares problems

References

The list of references is divided into two parts. First those references are shown which are used to describe the documentation scheme and the evaluation of the numerical solutions. Subsequently the sources of the test examples are collected, where the problems were found and where a reader finds additional information on the problems, their practical background or performence results of other algorithms.

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