All Roads Lead to the Depo

A comparison of VRP solving Algorithms

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Outline

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P and NP

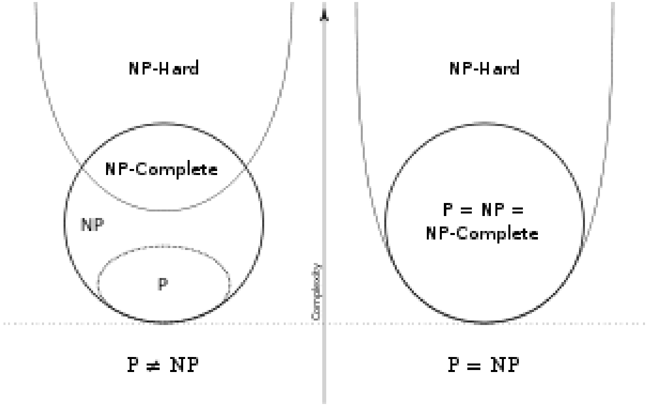

Figure: The possible relations of P and NP

VRP is NP-complete

Outline of the Proof(s)

- SAT is NP-complete
  Proof idea: We can reduce any NP problem to SAT in polynomial time
- 3-SAT is NP-complete
  Proof idea: We can turn an SAT into a 3-SAT
- HC is NP-complete
  Proof idea: Construct gadgets and represent as 3-SAT
- TSP is NP-complete
  Proof idea: Construct gadgets and represent as 3-SAT
- VRP is NP-complete
  Proof idea: TSP is a special case of the VRP VRP: Design a least cost set of routes satisfying length and other constraints and visiting every vertex.

Outline

Theoretical background
- Complexity Theory
- Algorithms

Experimentation
- Experiment design
- Results

Applied Algorithms

Tabu search

- Local Search method using GENI
  The important point is that it is easy to calculate the cost after the insertion
- Developed by Gendreau, Laporte, Hertz in 1994
- Also the typical move of any local perturbartion in MCMC
- Finds the locally best considering the *p-neighborhood* of a vertex
- Running time is p^4

Insertion Procedures

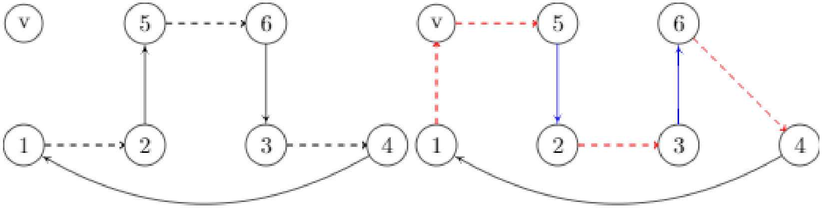

Figure: Type I. Insertion

Insertion Procedures

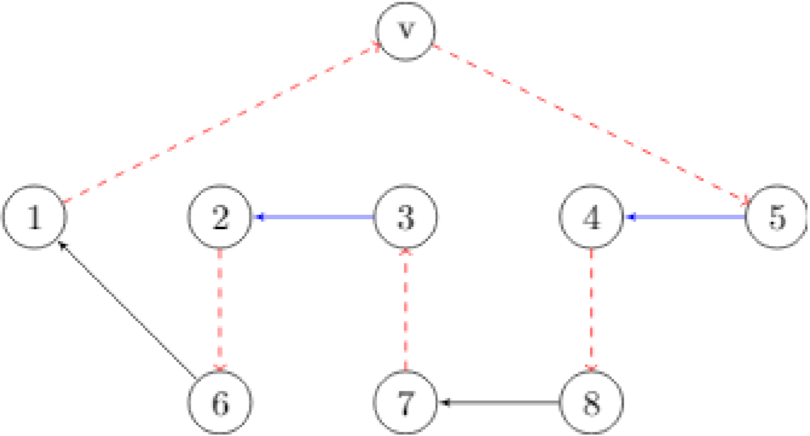

Figure: Type II. Insertion

Unstringing & Stringing

US

US is a useful post-optimization procedure. It can be applied to any TSP solution. It sequentially removes and reinserts some vertices to the same path

Tabu Search typical behaviour

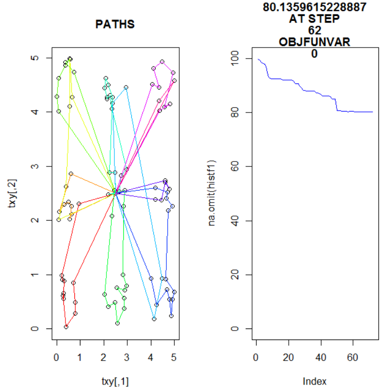

Figure: Typical behaviour of Tabu Search

Simulated Annealing

SA is an MCMC method based on the Metropolis-Hastings algorithm. It is developed by Kirkpatrick et al. in 1983.

Metropolis-Hastings Algorithm (1953)

In optimalization, I use the MH algorithm as follows: Given a solution, use the GENI moves to propose a new solution. Given the original and the new objective function, accept the new solution with a given probability.

Reversibility condition:

$$\pi(x_i)T(x_i, x_j)A(x_i, x_j) = \pi(x_j)T(x_j, x_i)$$

Acceptance probability based on temperature:

$$\exp(-(e' - e)/T)$$

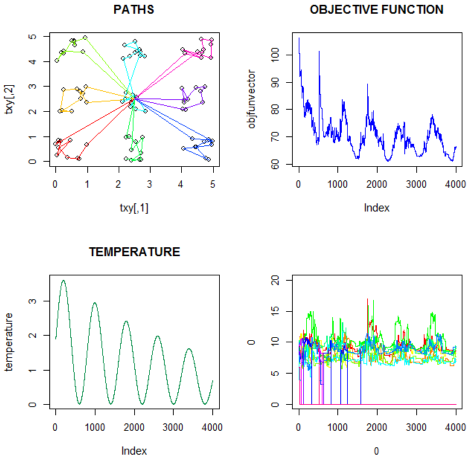

Figure: Typical behaviour of SA. Notice the cooling schedule

Parallel Tempering

Basically the same as SA, but with a twist. Run a number of MCMC chains parallel but with different temperatures, and allow the chains to switch states.
The idea is that the high temperature chains explore a wide area of the search space, while good solutions tend to get "frozen" in small temperatures.
The acceptance probability is the following:

$$p = \min\left(1, \frac{\exp\left(-\frac{E_j}{kT_i} - \frac{E_i}{kT_j}\right)}{\exp\left(-\frac{E_i}{kT_i} - \frac{E_j}{kT_j}\right)}\right) = \min\left(1, e^{(E_i - E_j)\left(\frac{1}{kT_i} - \frac{1}{kT_j}\right)}\right)$$

Algorithms

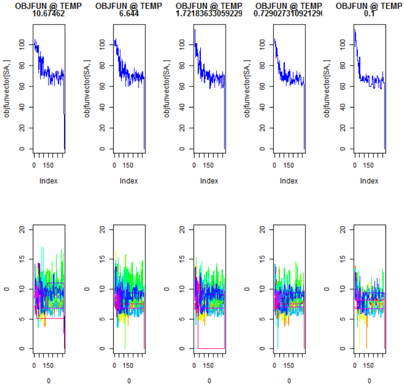

Figure: Typical behaviour of PT. Notice the difference of the variation of the chains

The problem with MCMC

The most important problem I encountered with using SA and PT is that when proposing new solutions, the solution is subject to length and capacity constraints. To obtain a good solution, I need to have a mixture of feasible and unfeasible solutions. However, the chains (especially on high temperatures) tend to get lost in the search space without finding any new feasible solutions.

Solution

Use US postoptimization proc. to reduce individual pathlengths. This does not solve the capacity issue, therefore I could only loosen the length constraint this way.

Outline

Experiment design

I ran 3 experiments with 3 differently characterized problem instances. The dataset was 90 points in 9 clusters, and the difference was the distance and size of the clusters.
The idea behind the experiment is that different methods will produce different results, and this can lead to a better understanding of the underlying principles.

- Experiment 1. The basic experiment with distance and size of the clusters were medium.

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- Experiment 2. Small, easily identifiable clusters

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- Experiment 1. The basic experiment with distance and size of the clusters were medium.
- Experiment 2. Small, easily identifiable clusters
- Experiment 3. Overlapping, hardly identifiable clusters

Experiment design

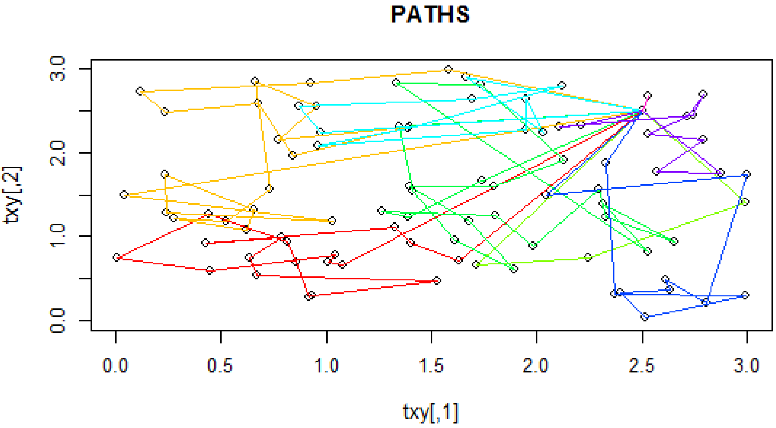

Figure: Experiment 1

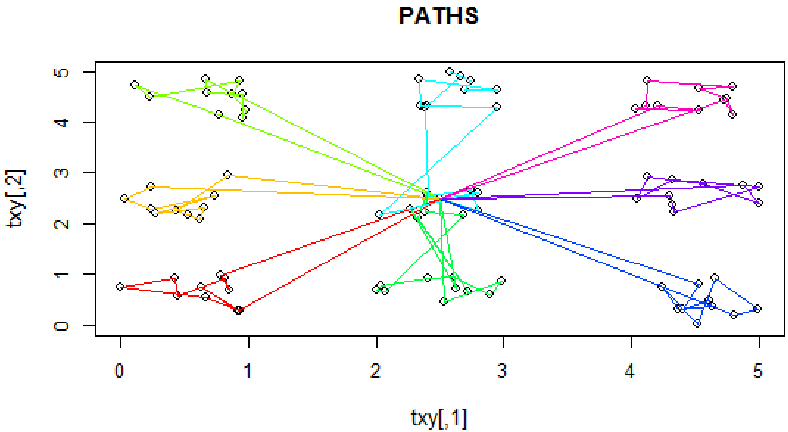

Figure: Experiment 2

Experiment design

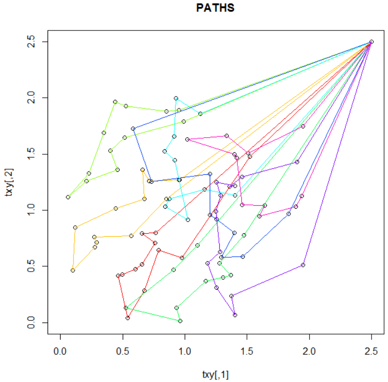

Figure: Experiment 3

Outline

Experiment 1

Problem Instance	BEST SOLUTION	BEST ALG	SWEEP	Reduction from SWEEP	Length of best solution			Number of routes			Running Time (seconds)		
					SA US	PT US	TABU SEARCH	SA US	PT US	TABU SEARCH	SA	PT	TABU SEARCH
1	46	TABU SEARCH	74.22	62%	51.1	51	46	8	8	8	201	632	1302
2	50	TABU SEARCH	73.24	68%	55.7	51	50	8	8	8	186	686	988
3	50	PT US	80.91	62%	61.3	50	54	9	8	9	206	698	900
4	46	SA US	75.46	61%	45.8	49	64	7	8	9	186	586	25
5	48	PT US	75.88	63%	57.0	48	50	9	8	9	236	879	1076
6	46	TABU SEARCH	69.49	66%	55.7	46	46	8	8	8	170	697	715
7	55	SA US	85.24	64%	54.6	55	56	9	8	9	230	759	1214
8	47	TABU SEARCH	72.14	65%	52.4	49	47	8	8	8	213	676	726
9	47	TABU SEARCH	72.07	65%	50.7	48	47	8	8	8	197	738	1353
10	48	PT US	75.29	63%	49.2	48	53	9	7	9	165	712	389

Figure: Experiment 1

Experiment 2

Problem Instance	BEST SOLUTION	BEST ALG	SWEEP	Reduction from SWEEP	Length of best solution			Number of routes			Running Time (seconds)		
					SA US	PT US	TABU SEARCH	SA US	PT US	TABU SEARCH	SA	PT	TABU SEARCH
1	57	SA US	90.02	63%	56.8	57	88	8	8	10	141	495	11
2	60	PT US	96.95	62%	59.7	60	91	9	9	11	128	463	119
3	59	PT US	104.17	57%	61.1	59	79	9	9	11	125	565	431
4	57	PT US	96.94	58%	57.1	57	74	9	9	11	142	507	753
5	56	PT US	113.41	50%	59.2	56	111	8	8	13	155	592	11
6	57	PT US	100.59	57%	58.3	57	80	8	8	11	133	555	903
7	57	PT US	105.74	54%	59.1	57	103	8	8	12	140	548	17
8	59	PT US	100.83	59%	64.0	59	81	9	9	11	154	673	721
9	59	PT US	90.80	65%	59.3	59	69	9	9	10	147	600	707
10	58	PT US	96.84	60%	58.2	58	86	8	9	11	168	590	473

Figure: Experiment 2

Experiment 3

Problem Instance	BEST SOLUTION	BEST ALG	SWEEP	Reduction from SWEEP	Length of best solution			Number of routes			Running Time (seconds)		
					SA US	PT US	TABU SEARCH	SA US	PT US	TABU SEARCH	SA	PT	TABU SEARCH
1	48	TABU SEARCH	74.10	65%	49.9	51	48	8	8	8	296	616	853
2	50	SA US	81.65	62%	50.4	56	53	8	9	9	352	661	1076
3	48	SA US	92.64	52%	48.3	58	60	7	9	10	201	655	1221
4	51	TABU SEARCH	73.53	69%	52.6	52	51	8	8	8	166	769	801
5	52	TABU SEARCH	82.09	63%	52.3	52	52	8	8	9	176	735	1970
6	49	TABU SEARCH	76.30	64%	55.3	56	49	8	8	8	124	426	1322
7	50	SA US	83.53	60%	49.9	54	55	7	8	9	178	667	1234
8	45	PT US	75.32	60%	52.6	45	51	8	7	8	266	655	1173
9	52	PT US	79.08	66%	55.3	52	53	8	8	9	183	660	1089
10	50	TABU SEARCH	76.29	65%	57.9	59	50	8	8	8	114	387	1062

Figure: Experiment 3

Summary

- VRP with capacity and length constraints is an NP-complete problem
- I used a local search heuristic and two MCMC methods to solve it
- MCMC methods (especially PT) outperformed Tabu Search in Experiment 2.
- The main difference between MCMC and Tabu Search is the number of steps they consider before making a move
- My main conclusion is that combining these methods (note that the MCMC methods used US) yields the best result
- Possible future research: The structure of the search space in terms of feasibility and objective function

Thank you for your attention

Figure: GoodBye!