Nonequilibrium Dynamics and Superadiabatic Fluxon Motion for Reversible Computing

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Outline

- Resource theory of quantum thermodynamics.
- Modelling generalized reversible computing in quantum open systems.
- State space geometric properties of open systems and impact on generalized reversible computing.
- Application to the dissipation delay product.
- Other work: reversible computing via superconducting circuits and superadiabatic fluxon motion.

Resource Theory

- *Resource theory*[1]: all *free states* $\hat{\rho} \in \mathcal{D}(\mathcal{H})$ and *free operations* $\Phi \in \mathrm{Aut}\big(\mathcal{D}(\mathcal{H})\big)$ which can be implemented with no energetic cost or dissipation, with conditions.
    - Ex.: Resource theory of bipartite entanglement. Free states: separable states. Free operations: local unitaries & classical communication (LOCC). Necessary & sufficient condition: $|\psi\rangle\langle\psi|_{AB} \mapsto |\phi\rangle\langle\phi|_{AB}$ iff $S(\psi_A) \geq S(\phi_A)$.
- Resource theory of quantum thermodynamics (RTQT):
    - Free states for nontrivial $\beta\widehat{H}$: equilibrium thermal states $\{\hat{\rho}_G\}$.
    - Free operations for nontrivial $\beta\widehat{H}$: all operations that preserve $\{\hat{\rho}_G\}$. *Thermal operations.*
    - Condition for $\hat{\rho} \mapsto \hat{\sigma}$ under thermal operations: *thermomajorization.*

[1] – N. H. Y. Ng and M. P. Woods, in *Thermodynamics in the Quantum Regime*, edited by F. Binder *et al.* (Springer Nature, Cham, 2018); see also arXiv:1805.09564.

Thermal Operations & Thermomajorization

- Start: system S coupled to environment E. E starts in thermal state: $\hat{\rho}_{E,G}$.
- Thermal operations: all quantum channels $\mathcal{E}(\hat{\rho}_S)$: $\hat{\rho}_S \mapsto \mathrm{Tr}_E\left\{\hat{U}_{SE}\left(\hat{\rho}_S \otimes \hat{\rho}_{E,G}\right)\hat{U}_{SE}{}^{\dagger}\right\}$.
  - Energy-conserving unitary dynamics: global unitary evolution $\hat{U}_{SE}$ across SE.
  - Require $\left[\hat{U}_{SE}, \hat{H}_S + \hat{H}_E\right] = 0$ at all times.
  - Maps thermal states to thermal states (but not necessarily the same one).
- Thermomajorization: necessary & sufficient for $\mathcal{E}\ \hat{\rho}$ to map $\hat{\rho}_S \mapsto \hat{\sigma}_S$. For eigenvalues $\lambda_i(\hat{\rho})$ of $\hat{\rho}$ with corresponding energies $E_{i,\hat{\rho}}$, require:

$$\left(\sum_i e^{-\beta E_{i,\hat{\rho}}}, \sum_i \lambda_i(\hat{\rho})^{\downarrow}\right) \geq \left(\sum_i e^{-\beta E_{i,\hat{\sigma}}}, \sum_i \lambda_i(\hat{\sigma})^{\downarrow}\right)$$

  - $\lambda_i(\hat{\rho})^{\downarrow}$: order eigenvalues of $\hat{\rho}$ by decreasing value.

Nonequilibrium Landauer Limit

- Using RTQT, can get an explicit nonequilibrium Landauer bound[13].

$$\beta\langle Q\rangle \geq -\ln\left[\mathrm{Tr}_S\,\mathrm{Tr}_E\left\{(\hat{\rho}_S(0)\otimes \mathbb{1}_E)\,\hat{U}_{SE}{}^{\dagger}\left(\mathbb{1}_S\otimes\hat{\rho}_{E,\beta}\right)\hat{U}_{SE}\right\}\right]$$

  - Ejection of information in correlated bits[3]: loss of prior correlations to environment.
    - Ejection of *uncorrelated* bits to the environment does *not* contribute to change in entropy.

- Dissipation bound depends on subsystem non-unitarity of $\hat{U}_{SE}$.
  - Measure of non-unitarity[4] on E: $\mathcal{N}_E \coloneqq \left\|\sum_{jk}\lambda_k\left\langle s_k\middle|\hat{U}_{SE}\middle|s_j\right\rangle\left\langle s_j\middle|\hat{U}_{SE}\middle|s_k\right\rangle - \mathbb{1}_E\right\|$.
  - $\mathcal{N}_E = 0$: no lower bound on dissipation.
  - $\left\langle s_a\middle|\hat{U}_{SE}\middle|s_b\right\rangle$: environment Kraus operators. Maps $\mathcal{D}(\mathcal{H}_E)$ to itself via operator algebra.

[2] – J. Goold, M. Paternostro, and K. Modi, Phys. Rev. Lett. **114**, 060602 (2015).
[3] – M. Frank, arXiv:1806.10183.
[4] – G. Guarnieri *et al.*, New J. Phys. **19**, 103038 (2017).

Modelling Reversible Computational Processes

- Standard[5] method: N density matrices encoding N-ary alphabet.
  - Bijection between physical and computational states.
  - Generalized classical reversible computing: more physical states than computational states.
- Need to generalize to GRC, thermal operations, and nonequilibrium dynamics.
  - Want physical state $\rightarrow$ computational state mapping to always commute with $\hat{U}_{SE(MW)}$. The same physical state always maps to the same computational state.
  - Can permit long-lived quantum coherences between physical states within an equiv. class.
  - Want to represent these as thermal operations, possibly within a larger open system.

[5] – N. Anderson, Eur. Phys. J. B **91**, 156 (2018).

(System) Kraus Operators

- ∞-simal evolution for density matrices in open (noneq.) quantum systems:

$$\mathcal{E}_{\mathrm{d}t}[\hat{\rho}_S(t)] = \hat{\rho}_S(t+\mathrm{d}t) = \sum_{jk} \langle e_k | \hat{U}_{SE}(\mathrm{d}t) | e_j \rangle \, \hat{\rho}_S(t) \, \langle e_j | \hat{U}_{SE}(\mathrm{d}t) | e_k \rangle$$

  - $\hat{E}_{jk} \coloneqq \sqrt{\lambda_j} \langle e_k | \hat{U}_{SE} | e_j \rangle$: (system) Kraus operators over environment eigenstates $\{|e_i\rangle\}$.
  - Any map of the form $\hat{\rho}_S(t) \mapsto \sum_{jk} \hat{E}_{jk}(t)\, \hat{\rho}_S(0)\, \hat{E}_{jk}{}^{\dagger}(t)$ is a *CPTP map.*
  - $\hat{E}_{jk} \in \mathsf{Op}(\mathcal{H}_S)$: takes $\mathcal{D}(\mathcal{H}_S)$ to itself, but expression depends on $\{|e_i\rangle\}$.
    - Done via algebra operation $\mathsf{Op}(\mathcal{H}_S) \times \mathsf{Op}(\mathcal{H}_S) \to \mathsf{Op}(\mathcal{H}_S)$. $\hat{E}_{jk} \notin \mathsf{L}_1(\mathsf{Op}(\mathcal{H}_S))$.

The GKSL Superoperator

- GKSL superop.: time evolution generator at lowest order. $\mathcal{E}_{\mathrm{d}t} = \mathcal{I} + \mathrm{d}t\,\mathcal{L} + \cdots$, and $\mathcal{L} \coloneqq \lim_{\mathrm{d}t \to 0} (\mathcal{E}_{\mathrm{d}t} - \mathcal{I})/\mathrm{d}t$.
- Kraus operators give evolution of $\hat{\rho}$ under quantum jump:

$$\hat{E}_0 \coloneqq \mathbb{1}_S - i\hat{H}_S\,\mathrm{d}t - \frac{1}{2}\sum_{j,k>0} \hat{F}_{jk}{}^{\dagger}\hat{F}_{jk}\,; \quad \hat{E}_{j,k>0} = \sqrt{\kappa_{jk}\,\mathrm{d}t}\,\hat{F}_{jk}$$

- GKSL superoperator in terms of jump operators $\{\hat{F}_{jk}\}$:

$$\mathcal{L}[\hat{\rho}_S] \coloneqq \frac{\mathrm{d}\hat{\rho}_S}{\mathrm{d}t} = -i\left[\hat{H}_S, \hat{\rho}_S\right] + \frac{1}{2}\sum_{j,k>0} \kappa_{jk}\left(2\hat{F}_{jk}\,\hat{\rho}_S\,\hat{F}_{jk}{}^{\dagger} - \left\{\hat{F}_{jk}{}^{\dagger}\,\hat{F}_{jk}, \hat{\rho}_S\right\}\right)$$

Spectrum of the GKSL Superoperator

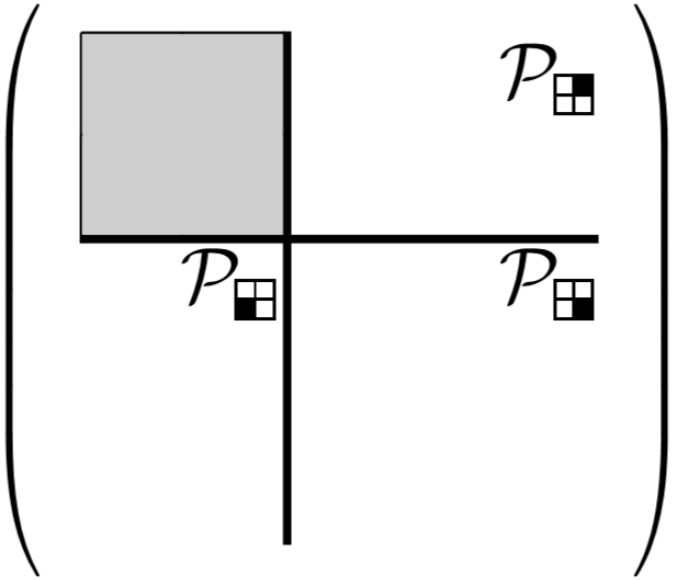
- Formal solution to GKSL equation: $\hat{\rho}_S(t) = e^{t\mathcal{L}}\,\hat{\rho}_S(0)$.
  - Examine spectral decomposition of $\mathcal{L}$ via Choi-Jamiołkowski isomorphism: mapping of operators $\mathbb{F}^{n\times m}$ to vector space $\mathbb{F}^{nm}$, employing Hilbert-Schmidt inner product.
    - $|a\rangle\langle b| \mapsto |a\rangle \otimes |b\rangle$; $\hat{A} \mapsto |\hat{A}\rangle\rangle$. Inner product $\langle\langle\hat{A}|\hat{B}\rangle\rangle = \mathrm{Tr}\{\hat{A}^\dagger\hat{B}\}$.
- Steady state solutions: defined as $|\hat{\rho}_{SS}\rangle\rangle \coloneqq \lim_{t\to\infty} e^{t\mathcal{L}}\,|\hat{\rho}_S(0)\rangle\rangle$.
- System with *multiple* steady states: assuming $\mathcal{L}$ is unitarily diagonalizable, have:

$$|\hat{\rho}_S(t)\rangle\rangle = e^{t\mathcal{L}}\,|\hat{\rho}_S(0)\rangle\rangle = \sum_a e^{t\lambda_a}\,|\hat{x}_a\rangle\rangle\langle\langle\hat{y}_a|\hat{\rho}_S(0)\rangle\rangle$$

  - $|\hat{x}_a\rangle\rangle$, $\langle\langle\hat{y}_a|$, λ_a are right eigenstates, left eigenstates, and eigenvalues of $\mathcal{L}$.
  - $\Re\{\lambda_a\} < 0$: damped states. $\Re\{\lambda_a\} = 0$: steady states. ($\Re\{\lambda_a\} > 0$ blows up.)

Four-Corners Decomposition

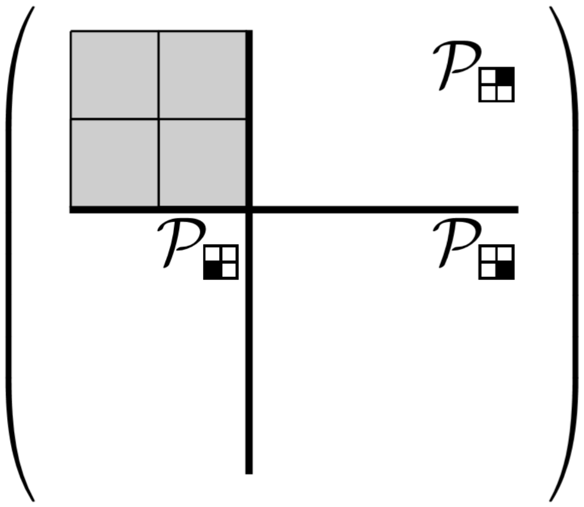
- Multiple steady states form[6] *asymptotic subspace* $\mathsf{As}(\mathcal{H})$ of nonzero steady states.
  - Right eigenvectors of $\mathcal{L}$ with pure imaginary eigenvalues.
    - *Corresponding* left eigenvectors are system conserved currents.
    - All initial states have components along these states.
  - $\mathsf{As}(\mathcal{H})$ can have further nontrivial dynamics.
- *Four-corners decomposition*: decomposition of $\mathcal{L}$ in terms of $\mathsf{As}(\mathcal{H})$ and conserved currents.

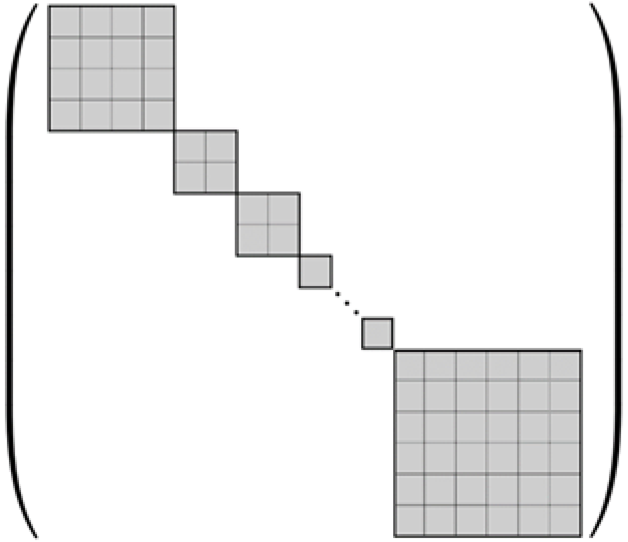


[6] – V. Albert, arXiv:1802.00010 (Ph.D. thesis, Yale). Image is from this.

Example Types of Asymptotic Subspaces

- Decoherent subspace: decoherence between steady states in dynamics purely within $\mathbf{As}(\mathcal{H})$.
  - Decoherence-free subspace (DFS): no further decoherence.
- Noiseless subsystem (NS): $\mathbf{As}(\mathcal{H})$ is tensor product of DFS subspace and subspace spanned by different steady state.
- von Neumann algebra: $\mathbf{As}(\mathcal{H})$ is a direct sum of DFS or NS blocks.
  - von Neumann algebra: C*-algebra which also includes all $\hat{A}$ that satisfy $\langle\phi|\hat{A}_n|\psi\rangle \to \langle\phi|\hat{A}|\psi\rangle$ for $|\phi\rangle, |\psi\rangle \in \mathcal{H}$ and $\{\hat{A}_n\} \in \mathsf{L}_1(\mathcal{H})$.

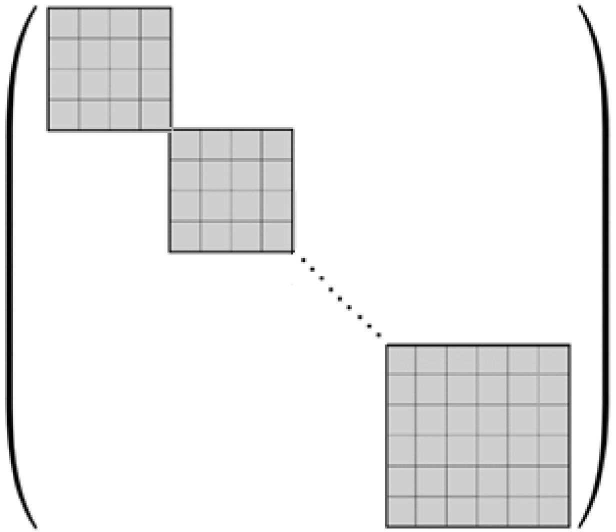


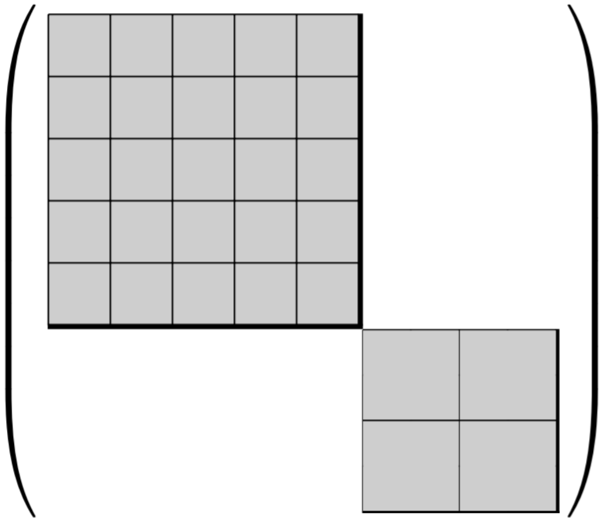


Images from (or modified from) V. Albert, arXiv:1802.00010 (Ph.D. thesis, Yale).

Generalized Reversible Computing & Four Corners

- Classical reversible computing: surjective map from physical to computational states, equivalence classes.
  - All states *within* a class must have same noncomputational entropy: related by a unitary transformation.
  - We permit intra-class coherences: each class can be a DFS.
  - Each class in a given computational scheme must have same computational entropy: each DFS block has same dimension.
  - ∴ quantum channels performing classical RC can be represented by the von Neumann algebra.
- Quantum GRC: permit inter-class coherences.





Images modified from V. Albert, arXiv:1802.00010 (Ph.D. thesis, Yale).

Berry-Wilczek-Zee Connection

- Time evolution in adiabatic approximation, expressed in parameter space: $t \mapsto \lambda^\mu \in \mathbb{R}^n$.
- Nondegenerate eigenspaces: Berry phase & connection given by:

$$\phi = \oint_C \mathrm{d}\lambda^\mu \, A_\mu ; \quad A_\mu = i\langle n(\lambda) | \partial_\mu | n(\lambda) \rangle$$

  - A_μ as U(1) gauge theory: under $|n(t)\rangle \mapsto e^{i\xi}|n(t)\rangle$, have $A_\mu \mapsto A_\mu - \partial_\mu \xi$.

- N-fold degenerate eigenspace: Berry-Wilczek-Zee connection is U(N) matrix. Eigenvalues of *Wilson loop* are generalizations of Berry phase:

$$W = \mathrm{Tr}\, \mathcal{P} \exp\left\{ \oint_C \mathrm{d}\lambda^\mu \, A_{ab;\,\mu} \right\}; \quad A_{\mu;\,ab} = i\langle n_a(\lambda) | \partial_\mu | n_b(\lambda) \rangle$$

  - $A_{\mu;\,ab}$ as U(N) gauge theory: under $|n_b(t)\rangle \mapsto \sum_a U_{ab}\, |n_a(t)\rangle$, have $A_\mu \mapsto i U^{-1} \partial_\mu U - U^{-1} A_\mu U$.

Berry-Wilczek-Zee to Operator Space

- Systems with multiple steady states: expect geometric effects to arise, especially in quantities not dependent on adiabatic path in parameter space.
- Basis for $\mathsf{Op}(\mathcal{H}_S)$: $\{|\mathrm{N}_{ab}\rangle\!\rangle\} \coloneqq |n_a\rangle\langle n_b|$. BWZ connection and Wilson loops in decoherence-free subspace:

$$\mathcal{W} = \operatorname{Tr} \mathcal{P} \exp\left\{\oint_C \mathrm{d}\lambda^\mu \, \mathcal{A}_{\mu;\,abcd}\right\}; \quad \mathcal{A}_{\mu;\,abcd} = \langle\!\langle \mathrm{N}_{cd} | \partial_\mu | \mathrm{N}_{ab} \rangle\!\rangle$$

  - Noiseless subsystem: $\mathsf{As}(\mathcal{H})$ is tensor product of DFS subspace and subspace spanned by different steady state. $\mathcal{W}$ and $\mathcal{A}_{\mu;\,abcd}$ can mix these!
  - von Neumann algebra: $\mathcal{A}_{\mu;\,abcd}$ is distinct on each block. No mixing between directly summed blocks.

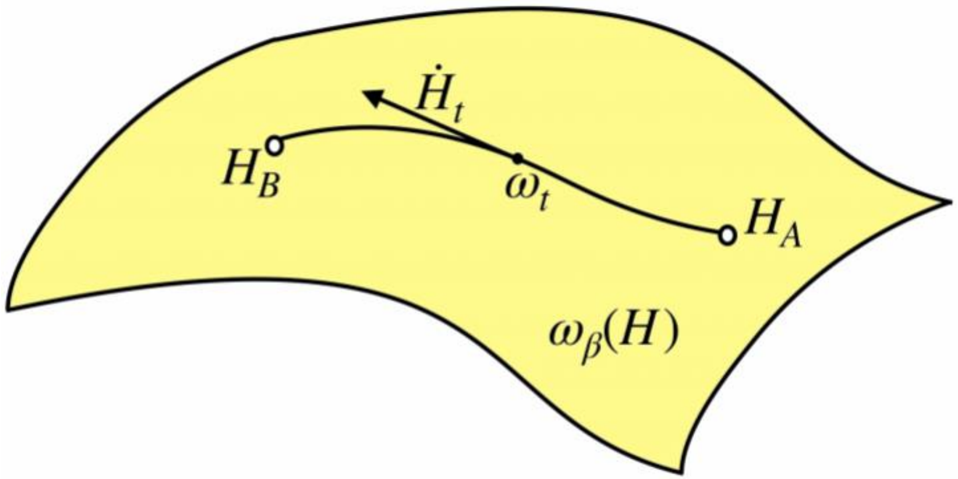
Dissipation Delay Product

- Major quantity of interest for characterizing efficiency of reversible operations.
  - Goal: characterizing *general* (protocol-based, device-independent) efficiency properties of reversible operations.
- Dissipation-delay product: product of energy dissipation incurred by transition process and delay of process.
  - For perfectly unitary time evolution, dissipation is zero.
  - Want to quantify energy dissipation for reversible operation. Dissipation as a function of delay.
  - Geometric approach: both dissipation (via this approach to calculating bound on average dissipation) and delay (preliminarily, can see this from quantum speed limit[7]) will depend on geometry of states.

[7] – S. Deffner and S. Campbell, J. Phys. A: Math. Theor. **50**, 453001 (2017).

Dissipation Length for Single Steady States

- *Thermodynamic* dissipation length: minimal dissipation amount for processes.
  - Construct manifold for set of all possible Gibbs states. (*Not* the same manifold!)
  - Geodesics: minimal dissipation process.
- Derived[9] for GKSL dynamics with a *single* steady state.
  - Current status: extending perturbation theory[10] of steady states for $\mathcal{L}$ with slowly varying parameters to multiple steady states.



[8] – M. Scandi and M. Peranau-Lloblet, Quantum **3**, 197 (2019). Image from here.
[9] – V. Cavina, A. Mari, and V. Giovannetti, Phys. Rev. Lett. **119**, 050601 (2017).

Superadiabaticity / Shortcuts to Adiabaticity (STA)

- Goal: more quickly reach the final results of adiabatic time evolution of $\widehat{H}$.
  - Ideally, want to reproduce full spectrum (states and their respective densities) that we'd have gotten from adiabatic evolution.
    - Alternately, want to find all of the invariants of the adiabatic motion.
    - *No* excitation (perturbations) allowed in final $\widehat{H}$ (relative to the time-evolved form of the unperturbed initial $\widehat{H}$, but such perturbations permitted along the way.
    - Mapping $\{|\psi_i\rangle\} \rightarrow \{|\psi_f\rangle\}$ should be state independent: holds for *any* possible $|\psi_i\rangle$.

STA Techniques

- Counterdiabatic driving (CD): add perturbations, not necessarily adiabatic, to $\hat{H}_0(t)$. Dynamics should result in same spectral decomposition at end.

$$\hat{H}_{CD} = i\hbar \sum_n \{|\partial_t n\rangle\langle n| - \langle n|\partial_t n\rangle \cdot |n\rangle\langle n|\}$$

- Invariant-based engineering (IE): construct dynamical invariants of the full Hamiltonian from 1st integrals of motion. examine spectral decomposition of this invariant.
  - Lewis-Riesenfeld invariant has $\mathbf{d}_t \hat{I} = \mathbf{0}$, so its eigenvalues are time independent and time evolved expectation values of its eigenvectors are constant. Calculate $\hat{H}$ from time evolution.

$$\mathrm{d}\hat{I}/\mathrm{d}t = \partial\hat{I}/\partial t + i[\hat{H}, \hat{I}]/\hbar = 0$$

Quantum-Classical Correspondence for STA

- In parameter (Berry) space, define[11] $\hat{\xi} \coloneqq i\hbar \sum_n \{ |\partial_\mu n\rangle\langle n| - \langle n|\partial_\mu n\rangle \cdot |n\rangle\langle n| \}$. Full $\widehat{H}$ is then:

$$\widehat{H} = \widehat{H}_0 + \widehat{H}_{CD} = \widehat{H}_0 + i\hbar \sum_n \{ |\partial_\mu n\rangle\langle n| - \langle n|\partial_\mu n\rangle \cdot |n\rangle\langle n| \} = \widehat{H}_0 + {}^{\mathrm{d}R_\mu}/_{\mathrm{d}t} \cdot \hat{\xi}$$

- $\hat{\xi}$ satisfies (and is specified by):

$$[\hat{\xi}, \widehat{H}_0] = i\hbar \left(\partial_\mu \widehat{H}_0 - \sum_n |n\rangle\langle n|\partial_\mu \widehat{H}_0|n\rangle\langle n| \right); \quad \langle n|\hat{\xi}|n\rangle = 0$$

- Correspondence: commutator → Poisson bracket. For a classical H_0, µcanonical average $\langle \alpha \rangle_\mu$ of classical observable α on constant energy shells given by:

$$\langle \alpha \rangle_\mu = \frac{\int \mathrm{d}^{3n}p\ \mathrm{d}^{3n}q\ \alpha\ \delta(E - H_0)}{\partial_E \int \mathrm{d}^{3n}p\ \mathrm{d}^{3n}q\ \theta(E - H_0)}; \quad \{\xi, H_0\} = \nabla H_0 - \langle \nabla H_0 \rangle_\mu; \quad \langle \xi \rangle_\mu = 0$$

[11] – C. Jarzynski, Phys. Rev. A **88**, 040101(R) (2013).

Classical STA and Completely Integrable Systems

- Procedure[12]: for Hamiltonian H_0 that depends on parameters R, determine ξ that satisfies

$$\frac{\partial H_0}{\partial \vec{R}} = \{\xi, H_0\} + \frac{\mathrm{d}E_0}{\mathrm{d}\vec{R}}$$

- Goal for classical STA (and invariant-based engineering): finding invariants.
  - Classical STA: ξ is an invariant used to solve a classical PDE.
  - Invariant-based engineering: Lewis-Riesenfeld invariants used to find a convenient eigenbasis for $\hat{H}$.
- Lewis-Riesenfeld equation $\partial\hat{I}/\partial t + i[\hat{H}, \hat{I}]/\hbar = 0$ can be directly related to soliton theory: all soliton equations can be reformulated as $\mathrm{d}L/\mathrm{d}t + [P, L] = 0$, with L determined from the PDE's symmetry group.

[12] – M. Okuyama and K. Takahashi, J. Phys. Society Japan **86**, 043002 (2017).

Quantum and Classical STA of KdV Equation

- Korteweg-de Vries equation: $6 \cdot \partial u/\partial R^\mu - 6u \cdot \partial u/\partial x + \partial u^3/\partial x^3 = 0$
- Quantum STA developed by Lewis-Riesenfeld invariants: $i\hbar\, \partial \hat{H}_0/\partial R^\mu = [\hat{\xi}, \hat{H}_0]$
  - Solution[12]: for $\hat{H}_0 = \hat{p}^2 + u$, have $\hat{\xi} = -2\hat{p}/3 - (\hat{p}u + u\hat{p})/2$
- Classical STA: $\partial H_0/\partial R^\mu = \{\xi, \hat{H}_0\}$
  - Solution[12]: for $\hat{H}_0 = p^2 + u$, have $\xi = -2p/3 - pu$
  - Relies on solving a classical PDE for ξ.
- Notice that these provide somewhat different results. Can't just naively go from commutator to Poisson!
- Next step: apply this technique to sine Gordon equation.

[12] – M. Okuyama and K. Takahashi, J. Phys. Society Japan **86**, 043002 (2017).

Conclusions and Next Steps

- To develop reversible computing and minimal dissipation properties, can embed framework of generalized reversible computing within GKSL dynamics.
- Current work: extending RC to a proper open system / nonequilibrium quantum thermodynamic footing. (Sandia Tracking #1068410)
  - Gives us fundamental quantum thermo results “for free”; e.g. thermodynamic length for multiple SSs.
  - Can give us a device-independent, protocol-based metric for efficiency of reversible operations!
- Current work: shortcut to adiabaticity for fluxons.
  - Relies intrinsically on infinite degrees of freedom (complete integrability) of sine-Gordon equation.

Landauer's Limit

- *Landauer's limit*[13]: one bit of information lost in computational process dissipates $\Delta E \geq k_B T \ln 2$ of energy as heat.
  - Dissipation due to increase in entropy: $\Delta S \geq k_B \ln 2$. Links information and physics!
  - Ejection of information in correlated bits[14]: loss of prior correlations to environment.
    - Ejection of *uncorrelated* bits to the environment does *not* contribute to change in entropy.
- No-hiding theorem[14]: information can't be destroyed.
  - Moves from system S to environment E. Global unitary evolution over $\mathcal{H}_S \otimes \mathcal{H}_E$.
  - Information lost from original system can't remain in SE correlations.
- Conventional computing: *entirely* irreversible. (Ex.: clearing memory.)

[13] – R. Landauer, IBM J. Res. Dev. **5**, 163 (1961).
[3] – M. Frank, arXiv:1806.10183.
[14] – S. Braunstein and A. K. Pati, Phys. Rev. Lett. **98**, 080502 (2007).

Practical Motivation for Reversible Computing

- Dennard scaling: power density of transistors remains constant as transistor size decreases.
- Moore's law: number of transistors in given integrated circuit area doubles (approximately) every 18 months.
  - Dennard: ended by 2008[15]. Moore: ending now[16].
- *Main* challenge: energy efficiency! Reversible computing: avoids major source of energy dissipation.
  - Almost all quantum computing is reversible, but classical reversible computing is the *lower bound* case on dissipation.

[15] – B. Deng *et al.*, ACM Trans. Arch. Code Opt. **15**, 8:1 (2018).
[16] – H. Khan, D. Hounshell, and E. Fuchs, Nature Electronics **1**, 14 (2018).

Classical Computing as a Lower Dissipative Bound

- Information processing expressed as a thermal operation[17]. Dissipation:

$$\Delta E_Q \geq k_B T\left(S(\hat{\rho}_S) - S(\hat{\varrho}_S)\right) + S\left(\hat{U}_{SME}\left(\hat{\rho}_S \otimes \hat{\rho}_M \otimes \hat{\rho}_E\right)\hat{U}_{SME}{}^{\dagger} \middle\| \hat{\varrho}_S \otimes \hat{\rho}_M \otimes \hat{\rho}_E\right)$$

  - System S coupled to environment E and catalyst M; same as splitting E into M and E.
  - Channel: $\mathcal{E}(\hat{\rho}_S): \hat{\rho}_S \mapsto \hat{\varrho}_S \coloneqq \mathrm{Tr}_M \, \mathrm{Tr}_E \left\{\hat{U}_{SME}\left(\hat{\rho}_S \otimes \hat{\rho}_M \otimes \hat{\rho}_E\right)\hat{U}_{SME}{}^{\dagger}\right\}$.
  - First term: information cost of classical IP. Second term: quantum IP.
- Classical IP is a *lower* dissipative bound! Quantum IP can be equal at best.
  - Classical IP: signal states correspond to orthogonal quantum states.
  - Pure unitaries and single input & output operations match classical IP dissipation bound.

[17] – D. Bedingham and O. Maroney, New J. Phys. **18**, 113050 (2016)

Resource Theory & Reversible Computing

- Key issue: IP operations usually involve a catalyst machine, which sometimes needs to be reset. Reset destroys correlations (mutual info.): dissipation!
  - This is also the source of dissipation in irreversible (classical) computing.
- Thermal operations *can* provide a framework for quantum RC with dissipation arbitrarily close to classical RC.
  - Same idea as classical RC: preserve correlations. Rigorously proved[18]: examine the transition $\hat{\rho}_S \otimes \hat{\rho}_M \otimes |w\rangle\langle w| \mapsto \hat{\varrho}_{SM} \otimes |w-\Delta\rangle\langle w-\Delta|$ for SME coupled to work bit $|w\rangle$. Resetting condition: $\mathrm{Tr}_S\, \hat{\varrho}_{SM} = \hat{\rho}_M$.
  - $S(\hat{\rho}_S \| \mathrm{Tr}_S\, \hat{\varrho}_{SM})$ can be arbitrarily close to zero: arbitrarily close to classical IP bound.

[18] – M. Müller, Phys. Rev. X **8**, 041051 (2018).

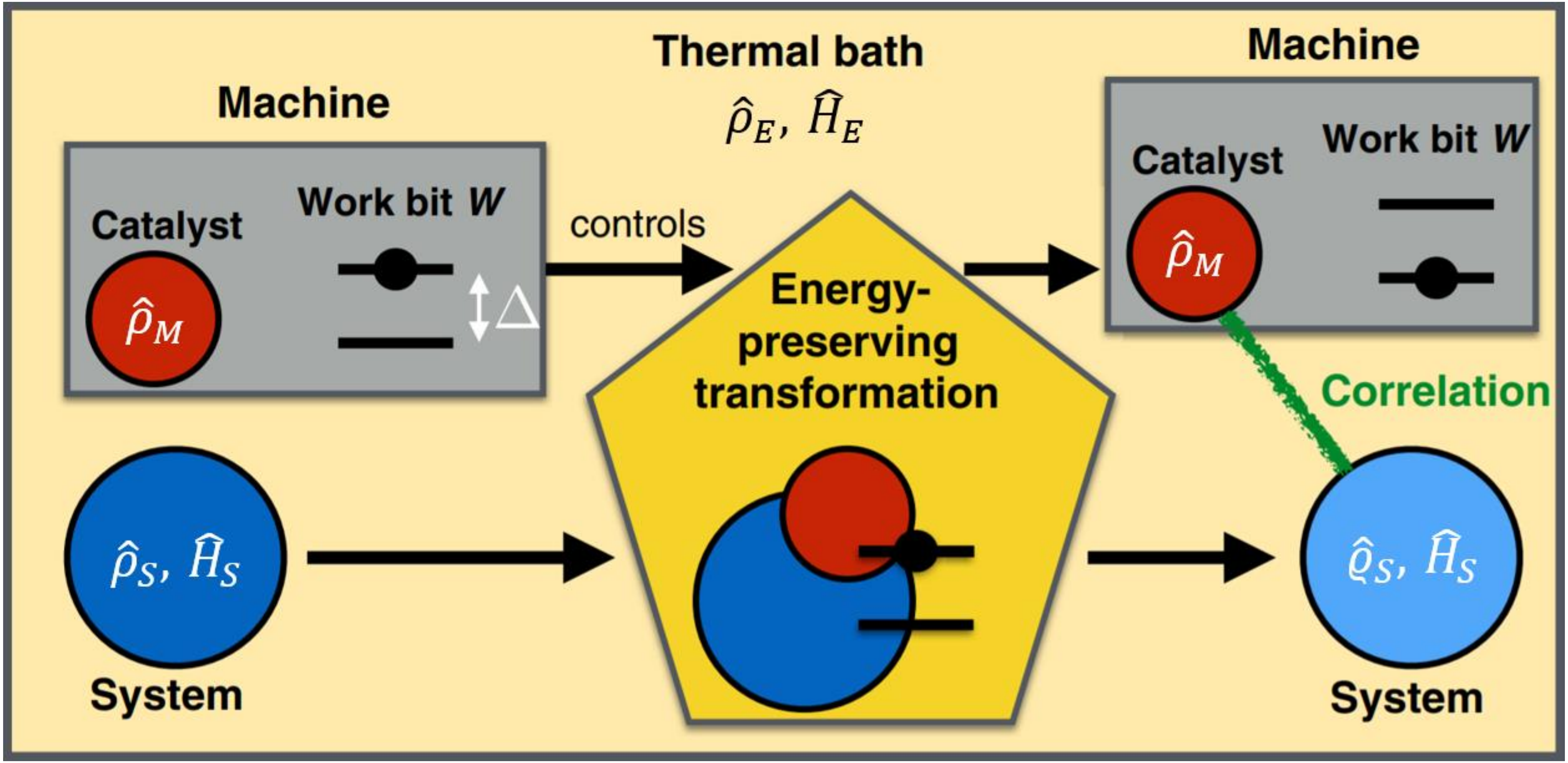


Image taken from M. Müller, Phys. Rev. X **8**, 041051 (2018).

Generalized Reversible Computing

- *Reversible computing*: reversibly transform computational states, instead of destructively overwriting them.
  - Computational states c: states representing computational information.
    - $\{c\}$ partitions $\{\hat{\rho}\}$ in equiv. classes. All $\{\hat{\rho}\}$ in same class are linked by unitary transform: have same entropy.
  - GRC: Bijections on the probability-1 subset of $\{c\}$.
  - Lower dissipative bound on classical RC can be zero!
    - Quantum RC: bounded only by mutual entropy term.

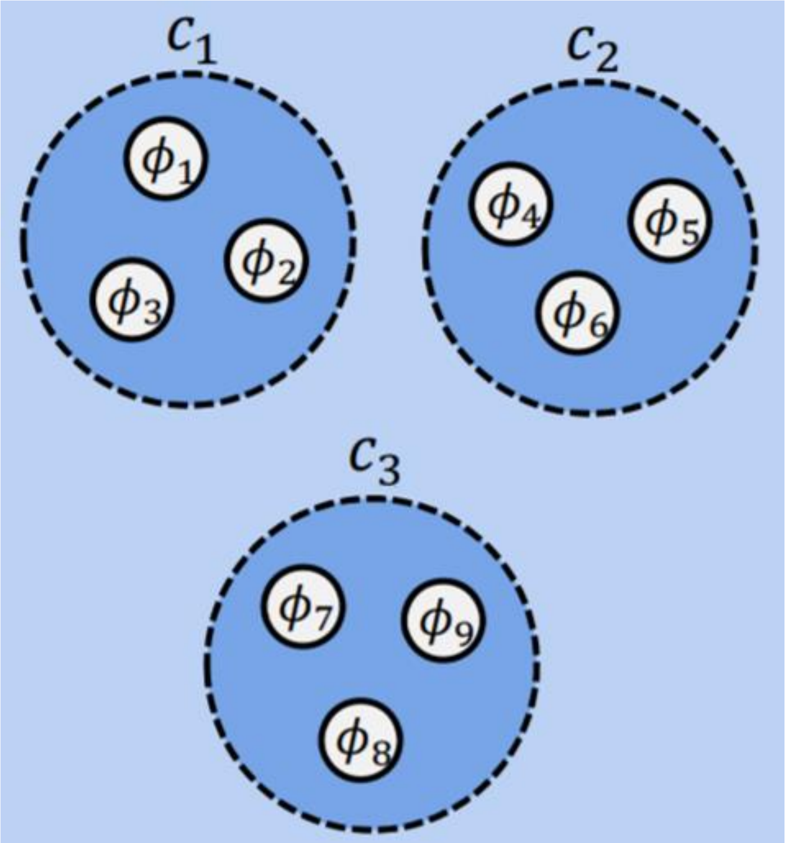


Image modified from M. Frank, arXiv:1806.10183.

Quantum Geometric Tensor from GKSL Dynamics

- Distance in parameter space:

$$\mathrm{d}s^2 = \|\psi(\lambda + \mathrm{d}\lambda) - \psi(\lambda)\|^2 = \langle \partial_\mu \psi | \partial_\nu \psi \rangle \, \mathrm{d}\lambda^\mu \, \mathrm{d}\lambda^\nu = \left(\gamma_{\mu\nu} + i\sigma_{\mu\nu}\right) \mathrm{d}\lambda^\mu \, \mathrm{d}\lambda^\nu$$

  - $\sigma_{\mu\nu}$: Berry curvature. $\sigma_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.
  - $g_{\mu\nu} = \gamma_{\mu\nu} - A_\mu \wedge A_\nu$: quantum geometric tensor. Metric on the manifold of states.

- Interest is in quantum geometric tensor induced by GKSL dynamics.
  - Wilczek-Zee-Berry connection: $\mathcal{A}_{\mu;\,abcd} = \langle\!\langle \mathrm{N}_{cd} | \partial_\mu | \mathrm{N}_{ab} \rangle\!\rangle = \mathrm{Tr}\{\widehat{\mathrm{N}}_{cd} \, \partial_\mu \, \widehat{\mathrm{N}}_{ab}\}$.
  - Induced QGT for vN algebra: $\mathcal{Q}_{\alpha\beta;\,abcd} = \partial_\alpha \mathcal{A}_{\beta;abcd} + \mathcal{A}_{\alpha;abcd} \wedge \mathcal{A}_{\beta;abcd} - \langle\!\langle \hat{\rho}_{cd} | \partial_\alpha \partial_\beta | \hat{\rho}_{ab} \rangle\!\rangle$.

More STA Techniques

- Variational counterdiabatic driving: same as CD, except use variational method in quantum to find perturbative term.
- Lie algebraic invariant-based engineering: same as IE, except construct dynamical invariants based on Lie algebra of underlying symmetries.
- Scaling laws: same as IE, except construct invariant from overarching scaling laws of $\hat{H}$.
- Variational method: construct approximate dynamics using Lagrangian and calculate via saddle-point approximation.
- Fast forward: same as scaling law approach, but also requires renormalization.