

# Nonequilibrium Dynamics and Superadiabatic Fluxon Motion for Reversible Computing

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# Outline

- Resource theory of quantum thermodynamics.
- Modelling generalized reversible computing in quantum open systems.
- State space geometric properties of open systems and impact on generalized reversible computing.
- Application to the dissipation delay product.
- Other work: reversible computing via superconducting circuits and superadiabatic fluxon motion.

# Resource Theory

- Resource theory<sup>[1]</sup>: all free states  $\hat{\rho} \in \mathcal{D}(\mathcal{H})$  and free operations  $\Phi \in \text{Aut}(\mathcal{D}(\mathcal{H}))$  which can be implemented with no energetic cost or dissipation, with conditions.
  - Ex.: Resource theory of bipartite entanglement. Free states: separable states. Free operations: local unitaries & classical communication (LOCC). Necessary & sufficient condition:  $|\psi\rangle\langle\psi|_{AB} \mapsto |\phi\rangle\langle\phi|_{AB}$  iff  $S(\psi_A) \geq S(\phi_A)$ .
- Resource theory of quantum thermodynamics (RTQT):
  - Free states for nontrivial  $\beta\hat{H}$ : equilibrium thermal states  $\{\hat{\rho}_G\}$ .
  - Free operations for nontrivial  $\beta\hat{H}$ : all operations that preserve  $\{\hat{\rho}_G\}$ . *Thermal operations*.
  - Condition for  $\hat{\rho} \mapsto \hat{\sigma}$  under thermal operations: *thermomajorization*.

[1] – N. H. Y. Ng and M. P. Woods, in *Thermodynamics in the Quantum Regime*, edited by F. Binder *et al.* (Springer Nature, Cham, 2018); see also arXiv:1805.09564.

# Thermal Operations & Thermomajorization

- Start: system  $S$  coupled to environment  $E$ .  $E$  starts in thermal state:  $\hat{\rho}_{E,G}$ .
- Thermal operations: all quantum channels  $\mathcal{E}(\hat{\rho}_S): \hat{\rho}_S \mapsto \text{Tr}_E \left\{ \hat{U}_{SE} (\hat{\rho}_S \otimes \hat{\rho}_{E,G}) \hat{U}_{SE}^\dagger \right\}$ .
  - Energy-conserving unitary dynamics: global unitary evolution  $\hat{U}_{SE}$  across  $SE$ .
  - Require  $[\hat{U}_{SE}, \hat{H}_S + \hat{H}_E] = 0$  at all times.
  - Maps thermal states to thermal states (but not necessarily the same one).
- Thermomajorization: necessary & sufficient for  $\mathcal{E}(\hat{\rho})$  to map  $\hat{\rho}_S \mapsto \hat{\sigma}_S$ . For eigenvalues  $\lambda_i(\hat{\rho})$  of  $\hat{\rho}$  with corresponding energies  $E_{i,\hat{\rho}}$ , require:

$$\left( \sum_i e^{-\beta E_{i,\hat{\rho}}} , \sum_i \lambda_i(\hat{\rho})^\downarrow \right) \geq \left( \sum_i e^{-\beta E_{i,\hat{\sigma}}} , \sum_i \lambda_i(\hat{\sigma})^\downarrow \right)$$

- $\lambda_i(\hat{\rho})^\downarrow$ : order eigenvalues of  $\hat{\rho}$  by decreasing value.

# Nonequilibrium Landauer Limit

- Using RTQT, can get an explicit nonequilibrium Landauer bound<sup>[13]</sup>.

$$\beta \langle Q \rangle \geq -\ln \left[ \text{Tr}_S \text{Tr}_E \left\{ (\hat{\rho}_S(0) \otimes \mathbb{1}_E) \hat{U}_{SE}^\dagger (\mathbb{1}_S \otimes \hat{\rho}_{E,\beta}) \hat{U}_{SE} \right\} \right]$$

- Ejection of information in correlated bits<sup>[3]</sup>: loss of prior correlations to environment.
  - Ejection of *uncorrelated* bits to the environment does *not* contribute to change in entropy.
- Dissipation bound depends on subsystem non-unitarity of  $\hat{U}_{SE}$ .
  - Measure of non-unitarity<sup>[4]</sup> on  $E$ :  $\mathcal{N}_E := \left\| \sum_{jk} \lambda_k \langle s_k | \hat{U}_{SE} | s_j \rangle \langle s_j | \hat{U}_{SE} | s_k \rangle - \mathbb{1}_E \right\|$ .
  - $\mathcal{N}_E = 0$ : no lower bound on dissipation.
  - $\langle s_a | \hat{U}_{SE} | s_b \rangle$ : environment Kraus operators. Maps  $\mathcal{D}(\mathcal{H}_E)$  to itself via operator algebra.

[2] – J. Goold, M. Paternostro, and K. Modi, Phys. Rev. Lett. **114**, 060602 (2015).

[3] – M. Frank, arXiv:1806.10183.

[4] – G. Guarnieri *et al.*, New J. Phys. **19**, 103038 (2017).

# Modelling Reversible Computational Processes

- Standard<sup>[5]</sup> method:  $N$  density matrices encoding  $N$ -ary alphabet.
  - Bijection between physical and computational states.
  - Generalized classical reversible computing: more physical states than computational states.
- Need to generalize to GRC, thermal operations, and nonequilibrium dynamics.
  - Want physical state  $\rightarrow$  computational state mapping to always commute with  $\hat{U}_{SE(MW)}$ . The same physical state always maps to the same computational state.
  - Can permit long-lived quantum coherences between physical states within an equiv. class.
  - Want to represent these as thermal operations, possibly within a larger open system.

# (System) Kraus Operators

- $\infty$ -simal evolution for density matrices in open (noneq.) quantum systems:

$$\mathcal{E}_{dt}[\hat{\rho}_S(t)] = \hat{\rho}_S(t + dt) = \sum_{jk} \langle e_k | \hat{U}_{SE}(dt) | e_j \rangle \hat{\rho}_S(t) \langle e_j | \hat{U}_{SE}(dt) | e_k \rangle$$

- $\hat{E}_{jk} := \sqrt{\lambda_j} \langle e_k | \hat{U}_{SE} | e_j \rangle$ : (system) Kraus operators over environment eigenstates  $\{|e_i\rangle\}$ .
- Any map of the form  $\hat{\rho}_S(t) \mapsto \sum_{jk} \hat{E}_{jk}(t) \hat{\rho}_S(0) \hat{E}_{jk}^\dagger(t)$  is a *CPTP map*.
- $\hat{E}_{jk} \in \text{Op}(\mathcal{H}_S)$ : takes  $\mathcal{D}(\mathcal{H}_S)$  to itself, but expression depends on  $\{|e_i\rangle\}$ .
  - Done via algebra operation  $\text{Op}(\mathcal{H}_S) \times \text{Op}(\mathcal{H}_S) \rightarrow \text{Op}(\mathcal{H}_S)$ .  $\hat{E}_{jk} \notin L_1(\text{Op}(\mathcal{H}_S))$ .

# The GKSL Superoperator

- GKSL superop.: time evolution generator at lowest order.  $\mathcal{E}_{dt} = \mathcal{I} + dt \mathcal{L} + \dots$ , and  $\mathcal{L} := \lim_{dt \rightarrow 0} (\mathcal{E}_{dt} - \mathcal{I})/dt$ .

- Kraus operators give evolution of  $\hat{\rho}$  under quantum jump:

$$\hat{E}_0 := \mathbb{1}_S - i\hat{H}_S dt - \frac{1}{2} \sum_{j,k>0} \hat{F}_{jk}^\dagger \hat{F}_{jk}; \quad \hat{E}_{j,k>0} = \sqrt{\kappa_{jk} dt} \hat{F}_{jk}$$

- GKSL superoperator in terms of jump operators  $\{\hat{F}_{jk}\}$ :

$$\mathcal{L}[\hat{\rho}_S] := \frac{d\hat{\rho}_S}{dt} = -i[\hat{H}_S, \hat{\rho}_S] + \frac{1}{2} \sum_{j,k>0} \kappa_{jk} \left( 2\hat{F}_{jk} \hat{\rho}_S \hat{F}_{jk}^\dagger - \left\{ \hat{F}_{jk}^\dagger \hat{F}_{jk}, \hat{\rho}_S \right\} \right)$$



# Spectrum of the GKSL Superoperator

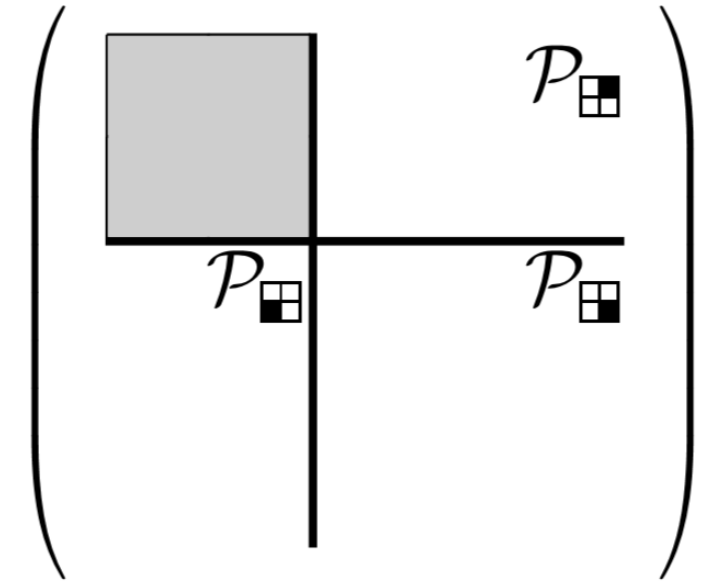
- Formal solution to GKSL equation:  $\hat{\rho}_S(t) = e^{t\mathcal{L}} \hat{\rho}_S(0)$ .
  - Examine spectral decomposition of  $\mathcal{L}$  via Choi-Jamiołkowski isomorphism: mapping of operators  $\mathbb{F}^{n \times m}$  to vector space  $\mathbb{F}^{nm}$ , employing Hilbert-Schmidt inner product.
    - $|a\rangle\langle b| \mapsto |a\rangle \otimes |b\rangle$ ;  $\hat{A} \mapsto |\hat{A}\rangle\rangle$ . Inner product  $\langle\langle \hat{A} | \hat{B} \rangle\rangle = \text{Tr}\{\hat{A}^\dagger \hat{B}\}$ .
- Steady state solutions: defined as  $|\hat{\rho}_{SS}\rangle\rangle := \lim_{t \rightarrow \infty} e^{t\mathcal{L}} |\hat{\rho}_S(0)\rangle\rangle$ .
- System with *multiple* steady states: assuming  $\mathcal{L}$  is unitarily diagonalizable, have:

$$|\hat{\rho}_S(t)\rangle\rangle = e^{t\mathcal{L}} |\hat{\rho}_S(0)\rangle\rangle = \sum_a e^{t\lambda_a} |\hat{x}_a\rangle\rangle \langle\langle \hat{y}_a | \hat{\rho}_S(0) \rangle\rangle$$

- $|\hat{x}_a\rangle\rangle$ ,  $\langle\langle \hat{y}_a |$ ,  $\lambda_a$  are right eigenstates, left eigenstates, and eigenvalues of  $\mathcal{L}$ .
- $\Re\{\lambda_a\} < 0$ : damped states.  $\Re\{\lambda_a\} = 0$ : steady states. ( $\Re\{\lambda_a\} > 0$  blows up.)

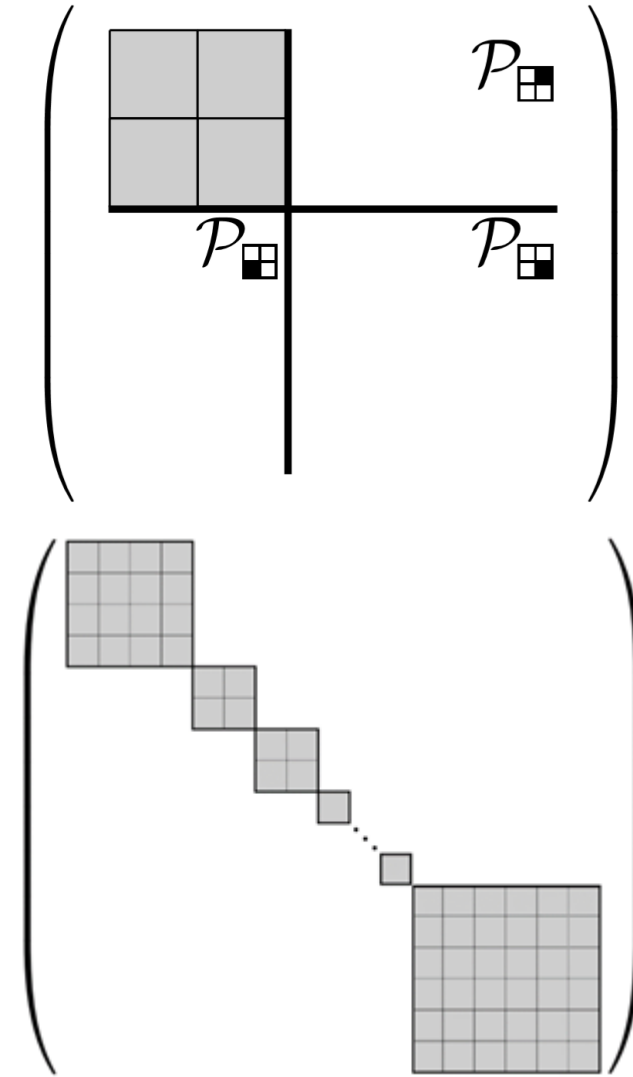
# Four-Corners Decomposition

- Multiple steady states form<sup>[6]</sup> *asymptotic subspace*  $As(\mathcal{H})$  of nonzero steady states.
  - Right eigenvectors of  $\mathcal{L}$  with pure imaginary eigenvalues.
    - *Corresponding* left eigenvectors are system conserved currents.
    - All initial states have components along these states.
  - $As(\mathcal{H})$  can have further nontrivial dynamics.
- *Four-corners decomposition*: decomposition of  $\mathcal{L}$  in terms of  $As(\mathcal{H})$  and conserved currents.



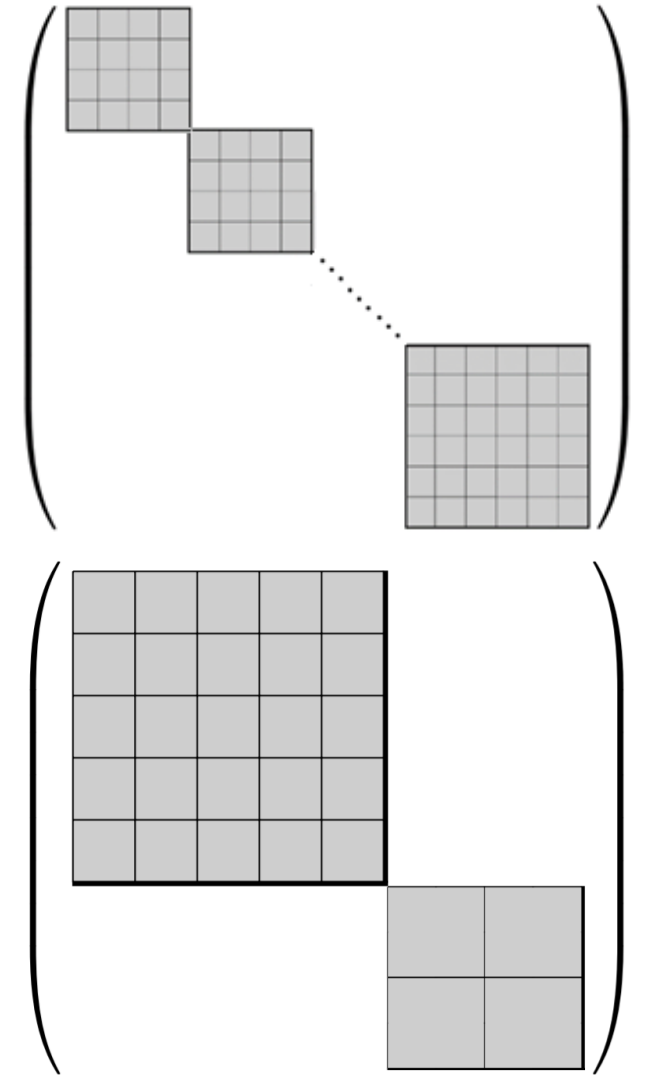
# Example Types of Asymptotic Subspaces

- Decoherent subspace: decoherence between steady states in dynamics purely within  $As(\mathcal{H})$ .
  - Decoherence-free subspace (DFS): no further decoherence.
- Noiseless subsystem (NS):  $As(\mathcal{H})$  is tensor product of DFS subspace and subspace spanned by different steady state.
- von Neumann algebra:  $As(\mathcal{H})$  is a direct sum of DFS or NS blocks.
  - von Neumann algebra:  $C^*$ -algebra which also includes all  $\hat{A}$  that satisfy  $\langle \phi | \hat{A}_n | \psi \rangle \rightarrow \langle \phi | \hat{A} | \psi \rangle$  for  $|\phi\rangle, |\psi\rangle \in \mathcal{H}$  and  $\{\hat{A}_n\} \in L_1(\mathcal{H})$ .



# Generalized Reversible Computing & Four Corners

- Classical reversible computing: surjective map from physical to computational states, equivalence classes.
  - All states *within* a class must have same noncomputational entropy: related by a unitary transformation.
  - We permit intra-class coherences: each class can be a DFS.
  - Each class in a given computational scheme must have same computational entropy: each DFS block has same dimension.
  - $\therefore$  quantum channels performing classical RC can be represented by the von Neumann algebra.
- Quantum GRC: permit inter-class coherences.



# Berry-Wilczek-Zee Connection

- Time evolution in adiabatic approximation, expressed in parameter space:  $t \mapsto \lambda^\mu \in \mathbb{R}^n$ .
- Nondegenerate eigenspaces: Berry phase & connection given by:

$$\phi = \oint_C d\lambda^\mu A_\mu; \quad A_\mu = i \langle n(\lambda) | \partial_\mu | n(\lambda) \rangle$$

- $A_\mu$  as U(1) gauge theory: under  $|n(t)\rangle \mapsto e^{i\xi} |n(t)\rangle$ , have  $A_\mu \mapsto A_\mu - \partial_\mu \xi$ .
- N-fold degenerate eigenspace: Berry-Wilczek-Zee connection is U(N) matrix. Eigenvalues of *Wilson loop* are generalizations of Berry phase:

$$W = \text{Tr } \mathcal{P} \exp \left\{ \oint_C d\lambda^\mu A_{ab;\mu} \right\}; \quad A_{\mu;ab} = i \langle n_a(\lambda) | \partial_\mu | n_b(\lambda) \rangle$$

- $A_{\mu;ab}$  as U(N) gauge theory: under  $|n_b(t)\rangle \mapsto \sum_a U_{ab} |n_a(t)\rangle$ , have  $A_\mu \mapsto iU^{-1} \partial_\mu U - U^{-1} A_\mu U$ .

# Berry-Wilczek-Zee to Operator Space

- Systems with multiple steady states: expect geometric effects to arise, especially in quantities not dependent on adiabatic path in parameter space.
- Basis for  $\text{Op}(\mathcal{H}_S)$ :  $\{|N_{ab}\rangle\rangle\} := |n_a\rangle\langle n_b|$ . BWZ connection and Wilson loops in decoherence-free subspace:

$$\mathcal{W} = \text{Tr } \mathcal{P} \exp \left\{ \oint_C d\lambda^\mu \mathcal{A}_{\mu; abcd} \right\}; \quad \mathcal{A}_{\mu; abcd} = \langle\langle N_{cd} | \partial_\mu | N_{ab} \rangle\rangle$$

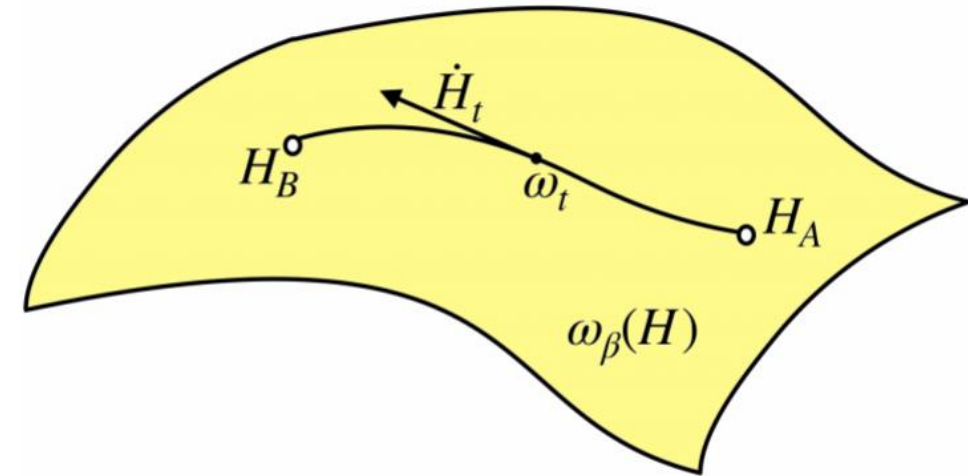
- Noiseless subsystem:  $\text{As}(\mathcal{H})$  is tensor product of DFS subspace and subspace spanned by different steady state.  $\mathcal{W}$  and  $\mathcal{A}_{\mu; abcd}$  can mix these!
- von Neumann algebra:  $\mathcal{A}_{\mu; abcd}$  is distinct on each block. No mixing between directly summed blocks.

# Dissipation Delay Product

- Major quantity of interest for characterizing efficiency of reversible operations.
  - Goal: characterizing *general* (protocol-based, device-independent) efficiency properties of reversible operations.
- Dissipation-delay product: product of energy dissipation incurred by transition process and delay of process.
  - For perfectly unitary time evolution, dissipation is zero.
  - Want to quantify energy dissipation for reversible operation. Dissipation as a function of delay.
  - Geometric approach: both dissipation (via this approach to calculating bound on average dissipation) and delay (preliminarily, can see this from quantum speed limit<sup>[7]</sup>) will depend on geometry of states.

# Dissipation Length for Single Steady States

- *Thermodynamic* dissipation length: minimal dissipation amount for processes.
  - Construct manifold for set of all possible Gibbs states. (*Not* the same manifold!)
  - Geodesics: minimal dissipation process.
- Derived<sup>[9]</sup> for GKSL dynamics with a *single* steady state.
  - Current status: extending perturbation theory<sup>[10]</sup> of steady states for  $\mathcal{L}$  with slowly varying parameters to multiple steady states.



[8] – M. Scandi and M. Peranau-Llobet, Quantum **3**, 197 (2019). Image from here.

[9] – V. Cavina, A. Mari, and V. Giovannetti, Phys. Rev. Lett. **119**, 050601 (2017).



# Superadiabaticity / Shortcuts to Adiabaticity (STA)

- Goal: more quickly reach the final results of adiabatic time evolution of  $\hat{H}$ .
  - Ideally, want to reproduce full spectrum (states and their respective densities) that we'd have gotten from adiabatic evolution.
    - Alternately, want to find all of the invariants of the adiabatic motion.
    - *No* excitation (perturbations) allowed in final  $\hat{H}$  (relative to the time-evolved form of the unperturbed initial  $\hat{H}$ , but such perturbations permitted along the way.
    - Mapping  $\{|\psi_i\rangle\} \rightarrow \{|\psi_f\rangle\}$  should be state independent: holds for *any* possible  $|\psi_i\rangle$ .

# STA Techniques

- Counterdiabatic driving (CD): add perturbations, not necessarily adiabatic, to  $\hat{H}_0(t)$ . Dynamics should result in same spectral decomposition at end.

$$\hat{H}_{CD} = i\hbar \sum_n \{|\partial_t n\rangle\langle n| - \langle n|\partial_t n\rangle \cdot |n\rangle\langle n|\}$$

- Invariant-based engineering (IE): construct dynamical invariants of the full Hamiltonian from 1<sup>st</sup> integrals of motion. examine spectral decomposition of this invariant.
  - Lewis-Riesenfeld invariant has  $d_t \hat{I} = 0$ , so its eigenvalues are time independent and time evolved expectation values of its eigenvectors are constant. Calculate  $\hat{H}$  from time evolution.

$$d\hat{I}/dt = \partial\hat{I}/\partial t + i[\hat{H}, \hat{I}]/\hbar = 0$$

# Quantum-Classical Correspondence for STA

- In parameter (Berry) space, define<sup>[11]</sup>  $\hat{\xi} := i\hbar \sum_n \{ |\partial_\mu n\rangle \langle n| - \langle n | \partial_\mu n \rangle \cdot |n\rangle \langle n| \}$ . Full  $\hat{H}$  is then:

$$\hat{H} = \hat{H}_0 + \hat{H}_{CD} = \hat{H}_0 + i\hbar \sum_n \{ |\partial_\mu n\rangle \langle n| - \langle n | \partial_\mu n \rangle \cdot |n\rangle \langle n| \} = \hat{H}_0 + \frac{dR_\mu}{dt} \cdot \hat{\xi}$$

- $\hat{\xi}$  satisfies (and is specified by):

$$[\hat{\xi}, \hat{H}_0] = i\hbar \left( \partial_\mu \hat{H}_0 - \sum_n |n\rangle \langle n | \partial_\mu \hat{H}_0 | n \rangle \langle n| \right); \quad \langle n | \hat{\xi} | n \rangle = 0$$

- Correspondence: commutator  $\rightarrow$  Poisson bracket. For a classical  $H_0$ , microcanonical average  $\langle \alpha \rangle_\mu$  of classical observable  $\alpha$  on constant energy shells given by:

$$\langle \alpha \rangle_\mu = \frac{\int d^{3n}p \, d^{3n}q \, \alpha \, \delta(E - H_0)}{\partial_E \int d^{3n}p \, d^{3n}q \, \theta(E - H_0)}; \quad \{\xi, H_0\} = \nabla H_0 - \langle \nabla H_0 \rangle_\mu; \quad \langle \xi \rangle_\mu = 0$$

# Classical STA and Completely Integrable Systems

- Procedure<sup>[12]</sup>: for Hamiltonian  $H_0$  that depends on parameters  $R$ , determine  $\xi$  that satisfies

$$\frac{\partial H_0}{\partial \vec{R}} = \{\xi, H_0\} + \frac{dE_0}{d\vec{R}}$$

- Goal for classical STA (and invariant-based engineering): finding invariants.
  - Classical STA:  $\xi$  is an invariant used to solve a classical PDE.
  - Invariant-based engineering: Lewis-Riesenfeld invariants used to find a convenient eigenbasis for  $\hat{H}$ .
- Lewis-Riesenfeld equation  $\partial \hat{I} / \partial t + i[\hat{H}, \hat{I}] / \hbar = 0$  can be directly related to soliton theory: all soliton equations can be reformulated as  $dL/dt + [P, L] = 0$ , with  $L$  determined from the PDE's symmetry group.

# Quantum and Classical STA of KdV Equation

- Korteweg-de Vries equation:  $6 \cdot \partial u / \partial R^\mu - 6u \cdot \partial u / \partial x + \partial u^3 / \partial x^3 = 0$
- Quantum STA developed by Lewis-Riesenfeld invariants:  $i\hbar \partial \hat{H}_0 / \partial R^\mu = [\hat{\xi}, \hat{H}_0]$ 
  - Solution<sup>[12]</sup>: for  $\hat{H}_0 = \hat{p}^2 + u$ , have  $\hat{\xi} = -2\hat{p}/3 - (\hat{p}u + u\hat{p})/2$
- Classical STA:  $\partial H_0 / \partial R^\mu = \{\xi, \hat{H}_0\}$ 
  - Solution<sup>[12]</sup>: for  $\hat{H}_0 = p^2 + u$ , have  $\xi = -2p/3 - pu$
  - Relies on solving a classical PDE for  $\xi$ .
- Notice that these provide somewhat different results. Can't just naively go from commutator to Poisson!
- Next step: apply this technique to sine Gordon equation.

# Conclusions and Next Steps

- To develop reversible computing and minimal dissipation properties, can embed framework of generalized reversible computing within GKSL dynamics.
- Current work: extending RC to a proper open system / nonequilibrium quantum thermodynamic footing. (Sandia Tracking #1068410)
  - Gives us fundamental quantum thermo results “for free”; e.g. thermodynamic length for multiple SSs.
  - Can give us a device-independent, protocol-based metric for efficiency of reversible operations!
- Current work: shortcut to adiabaticity for fluxons.
  - Relies intrinsically on infinite degrees of freedom (complete integrability) of sine-Gordon equation.



# Landauer's Limit

- Landauer's limit<sup>[13]</sup>: one bit of information lost in computational process dissipates  $\Delta E \geq k_B T \ln 2$  of energy as heat.
  - Dissipation due to increase in entropy:  $\Delta S \geq k_B \ln 2$ . Links information and physics!
  - Ejection of information in correlated bits<sup>[14]</sup>: loss of prior correlations to environment.
    - Ejection of *uncorrelated* bits to the environment does *not* contribute to change in entropy.
- No-hiding theorem<sup>[14]</sup>: information can't be destroyed.
  - Moves from system  $S$  to environment  $E$ . Global unitary evolution over  $\mathcal{H}_S \otimes \mathcal{H}_E$ .
  - Information lost from original system can't remain in  $SE$  correlations.
- Conventional computing: *entirely* irreversible. (Ex.: clearing memory.)

[13] – R. Landauer, IBM J. Res. Dev. **5**, 163 (1961).

[3] – M. Frank, arXiv:1806.10183.

[14] – S. Braunstein and A. K. Pati, Phys. Rev. Lett. **98**, 080502 (2007).



# Practical Motivation for Reversible Computing

- Dennard scaling: power density of transistors remains constant as transistor size decreases.
- Moore's law: number of transistors in given integrated circuit area doubles (approximately) every 18 months.
  - Dennard: ended by 2008<sup>[15]</sup>. Moore: ending now<sup>[16]</sup>.
- *Main* challenge: energy efficiency! Reversible computing: avoids major source of energy dissipation.
  - Almost all quantum computing is reversible, but classical reversible computing is the *lower bound* case on dissipation.

[15] – B. Deng *et al.*, ACM Trans. Arch. Code Opt. **15**, 8:1 (2018).

[16] – H. Khan, D. Hounshell, and E. Fuchs, Nature Electronics **1**, 14 (2018).

# Classical Computing as a Lower Dissipative Bound

- Information processing expressed as a thermal operation<sup>[17]</sup>. Dissipation:

$$\Delta E_Q \geq k_B T (S(\hat{\rho}_S) - S(\hat{\varrho}_S)) + S \left( \hat{U}_{SME} (\hat{\rho}_S \otimes \hat{\rho}_M \otimes \hat{\rho}_E) \hat{U}_{SME}^\dagger \parallel \hat{\varrho}_S \otimes \hat{\rho}_M \otimes \hat{\rho}_E \right)$$

- System  $S$  coupled to environment  $E$  and catalyst  $M$ ; same as splitting  $E$  into  $M$  and  $E$ .
- Channel:  $\mathcal{E}(\hat{\rho}_S): \hat{\rho}_S \mapsto \hat{\varrho}_S := \text{Tr}_M \text{Tr}_E \left\{ \hat{U}_{SME} (\hat{\rho}_S \otimes \hat{\rho}_M \otimes \hat{\rho}_E) \hat{U}_{SME}^\dagger \right\}$ .
- First term: information cost of classical IP. Second term: quantum IP.
- Classical IP is a *lower* dissipative bound! Quantum IP can be equal at best.
  - Classical IP: signal states correspond to orthogonal quantum states.
  - Pure unitaries and single input & output operations match classical IP dissipation bound.

# Resource Theory & Reversible Computing

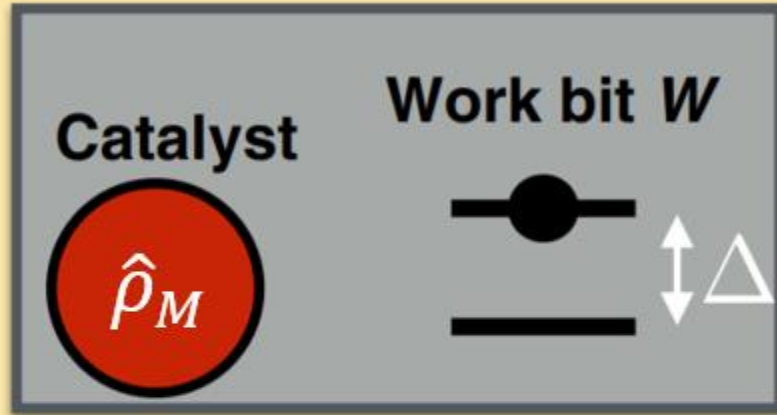
- Key issue: IP operations usually involve a catalyst machine, which sometimes needs to be reset. Reset destroys correlations (mutual info.): dissipation!
  - This is also the source of dissipation in irreversible (classical) computing.
- Thermal operations *can* provide a framework for quantum RC with dissipation arbitrarily close to classical RC.
  - Same idea as classical RC: preserve correlations. Rigorously proved<sup>[18]</sup>: examine the transition  $\hat{\rho}_S \otimes \hat{\rho}_M \otimes |w\rangle\langle w| \mapsto \hat{\varrho}_{SM} \otimes |w - \Delta\rangle\langle w - \Delta|$  for  $SME$  coupled to work bit  $|w\rangle$ . Resetting condition:  $\text{Tr}_S \hat{\varrho}_{SM} = \hat{\rho}_M$ .
  - $S(\hat{\rho}_S \| \text{Tr}_S \hat{\varrho}_{SM})$  can be arbitrarily close to zero: arbitrarily close to classical IP bound.

Thermal bath

$$\hat{\rho}_E, \hat{H}_E$$

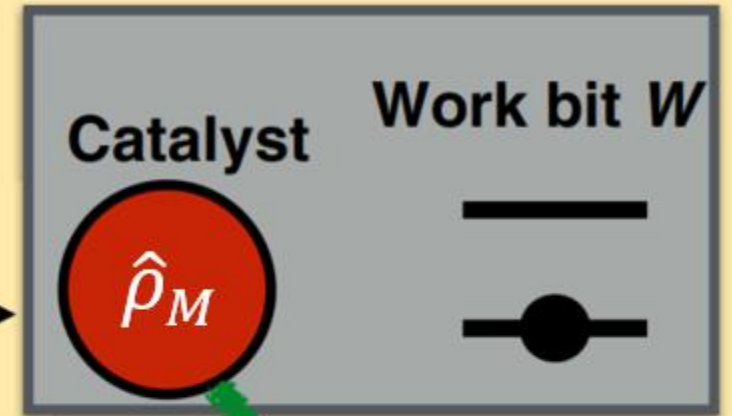
Machine

Machine



controls

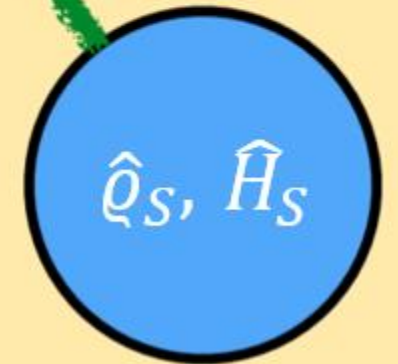
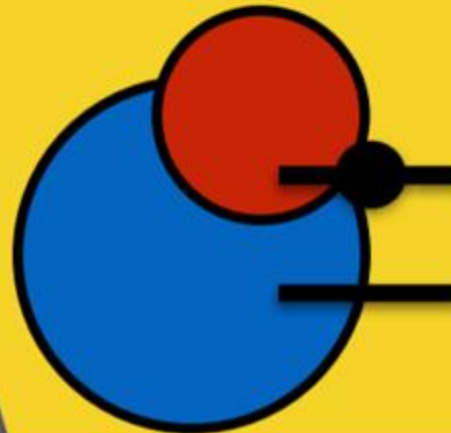
Energy-  
preserving  
transformation



Correlation



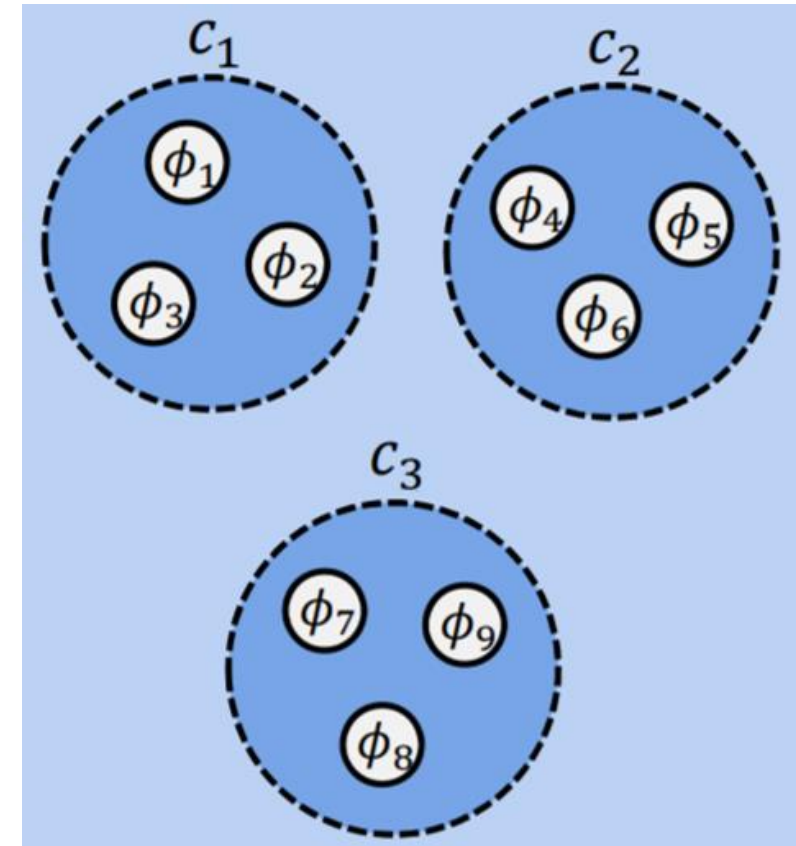
System



System

# Generalized Reversible Computing

- Reversible computing: reversibly transform computational states, instead of destructively overwriting them.
- Computational states  $c$ : states representing computational information.
  - $\{c\}$  partitions  $\{\hat{\rho}\}$  in equiv. classes. All  $\{\hat{\rho}\}$  in same class are linked by unitary transform: have same entropy.
- GRC: Bijections on the probability-1 subset of  $\{c\}$ .
- Lower dissipative bound on classical RC can be zero!
  - Quantum RC: bounded only by mutual entropy term.



# Quantum Geometric Tensor from GKSL Dynamics

- Distance in parameter space:

$$ds^2 = \|\psi(\lambda + d\lambda) - \psi(\lambda)\|^2 = \langle \partial_\mu \psi | \partial_\nu \psi \rangle d\lambda^\mu d\lambda^\nu = (\gamma_{\mu\nu} + i\sigma_{\mu\nu}) d\lambda^\mu d\lambda^\nu$$

- $\sigma_{\mu\nu}$ : Berry curvature.  $\sigma_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ .
- $g_{\mu\nu} = \gamma_{\mu\nu} - A_\mu \wedge A_\nu$ : quantum geometric tensor. Metric on the manifold of states.
- Interest is in quantum geometric tensor induced by GKSL dynamics.
  - Wilczek-Zee-Berry connection:  $\mathcal{A}_{\mu;abcd} = \langle\langle N_{cd} | \partial_\mu | N_{ab} \rangle\rangle = \text{Tr}\{\hat{N}_{cd} \partial_\mu \hat{N}_{ab}\}$ .
  - Induced QGT for vN algebra:  $\mathcal{Q}_{\alpha\beta;abcd} = \partial_\alpha \mathcal{A}_{\beta;abcd} + \mathcal{A}_{\alpha;abcd} \wedge \mathcal{A}_{\beta;abcd} - \langle\langle \hat{\rho}_{cd} | \partial_\alpha \partial_\beta | \hat{\rho}_{ab} \rangle\rangle$ .

# More STA Techniques

- Variational counterdiabatic driving: same as CD, except use variational method in quantum to find perturbative term.
- Lie algebraic invariant-based engineering: same as IE, except construct dynamical invariants based on Lie algebra of underlying symmetries.
- Scaling laws: same as IE, except construct invariant from overarching scaling laws of  $\hat{H}$ .
- Variational method: construct approximate dynamics using Lagrangian and calculate via saddle-point approximation.
- Fast forward: same as scaling law approach, but also requires renormalization.