HW8

AK

2024-10-24

R Markdown

1) Slide 26:- rejprobs <- sapply(mulist, function(mu) { pvals <- replicate(3000, t.test(rnorm(n, mean=mu, sd=sig), mu = mu_0)$p.value) mean(pvals < alp)

Slide 34:- rejprobs <- sapply(mulist, function(mu) { pvals <- pvalDist2(n, "exp", mu_0, alp=alp, rate=1/mu) mean(pvals < alp)

```
#2)
library(MASS)

set.seed(5400)

n <- 30
cov_matrix <- matrix(0.1, nrow=n, ncol=n)
diag(cov_matrix) <- 1

X <- mvrnorm(n=1, mu=rep(0, n), Sigma=cov_matrix)
X
```

```
##  [1] -2.0225365263 -0.0003287116 -0.1384650403 -0.6886167204  1.3308740520
##  [6] -0.0535414615  0.5065535243 -2.1194480484 -0.8165114324 -1.8558281800
## [11] -1.4869225564 -1.3302936980 -0.6712737414 -1.0337593544 -1.6083000621
## [16] -0.6198791704 -0.5485504307 -2.0811222957 -0.2791497316  0.6768892197
## [21] -1.1469035228  0.5324768778 -0.6079841454 -0.2039781701 -2.0174022123
## [26]  0.2493441674 -0.6357117827 -0.9361267696  0.3285488925 -0.6299566093
```

```
test_result <- t.test(X, mu = 0)

test_result$statistic
```

```
##         t
## -4.031398
```

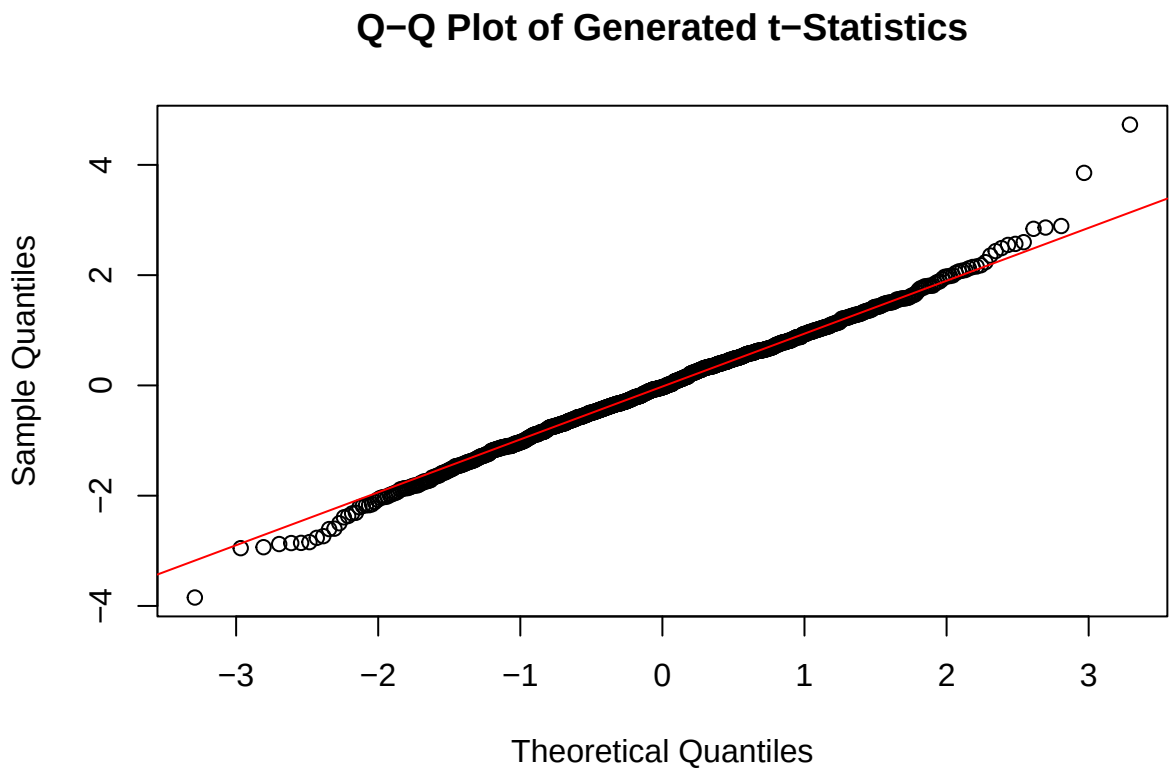
```
test_result$p.value
```

```
## [1] 0.0003673444
```

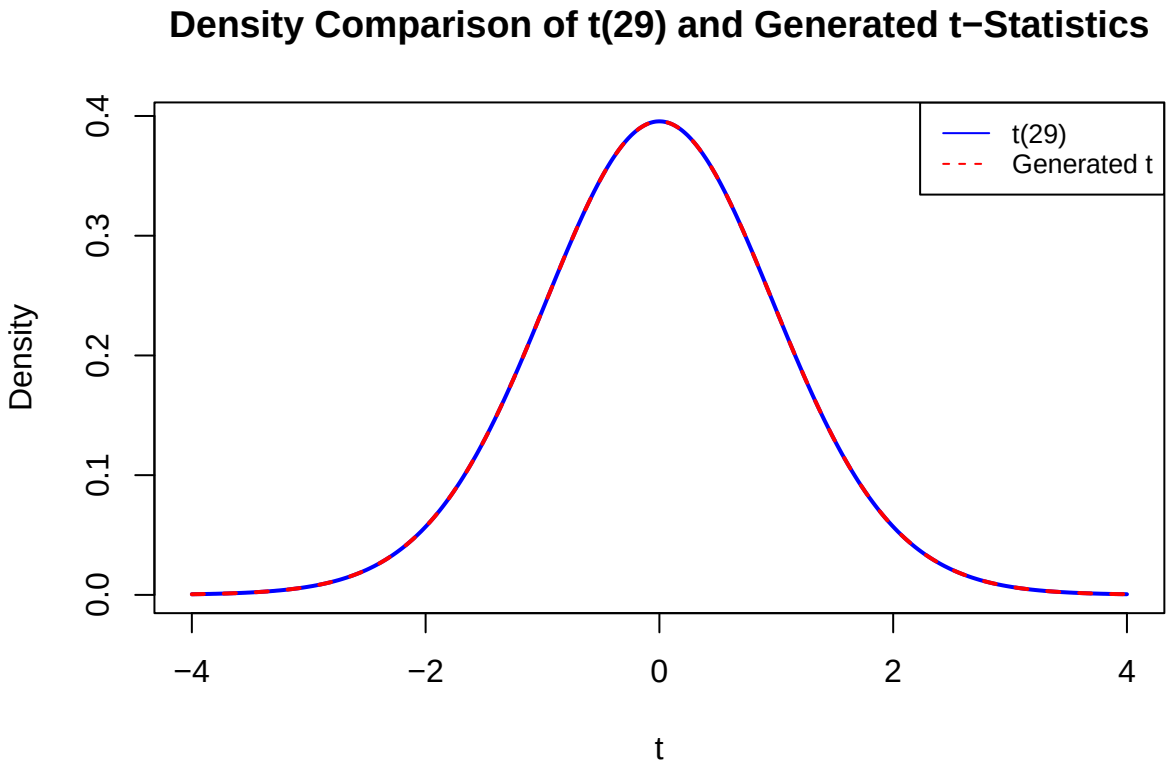
The p-value is less than the significance level, you reject the null hypothesis.

```r
#install.packages("ggplot2")
library(ggplot2)
```

```r
set.seed(5400)

num_samples <- 1000
t_statistics <- rt(num_samples, df = 29)

qqnorm(t_statistics, main = "Q-Q Plot of Generated t-Statistics")
qqline(t_statistics, col = "red")
```




```r
x <- seq(-4, 4, length.out = 1000)
true_t_density <- dt(x, df = 29)
generated_t_density <- dt(x, df = 29)

plot(x, true_t_density, type = "l", col = "blue", lwd = 2,
     xlab = "t", ylab = "Density",
     main = "Density Comparison of t(29) and Generated t-Statistics")
lines(x, generated_t_density, col = "red", lty = 2, lwd = 2)
legend("topright", legend = c("t(29)", "Generated t"),
       col = c("blue", "red"), lty = 1:2, cex = 0.8)
```



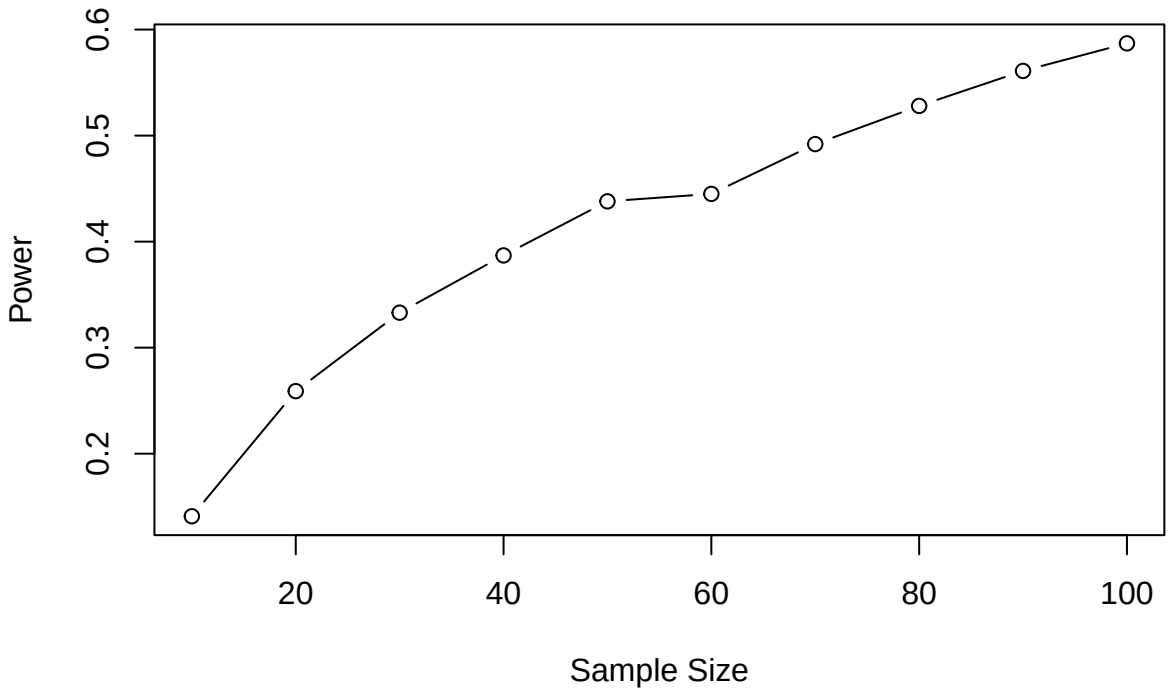
In the Q-Q plot, if the t-statistics follow a t-distribution with 29 degrees of freedom, the points should approximately lie on the red line. In the density plot, the red area represents the density of the generated t-statistics, and the blue line represents the density of a t-distribution with 29 degrees of freedom. The t-statistics follow a t-distribution with 29 degrees of freedom, the blue area and the red line should overlap significantly.

```r
set.seed(5400)
num_simulations <- 1000
num_samples <- 30
mu_0 <- 0
alpha <- 0.05

type_i_error_count <- 0

for (i in 1:num_simulations) {
  dependent_data <- rt(num_samples, df = 29)

  t_test_result <- t.test(dependent_data, mu = mu_0)

  p_value <- t_test_result$p.value

  if (p_value < alpha) {
    type_i_error_count <- type_i_error_count + 1
  }
}
```

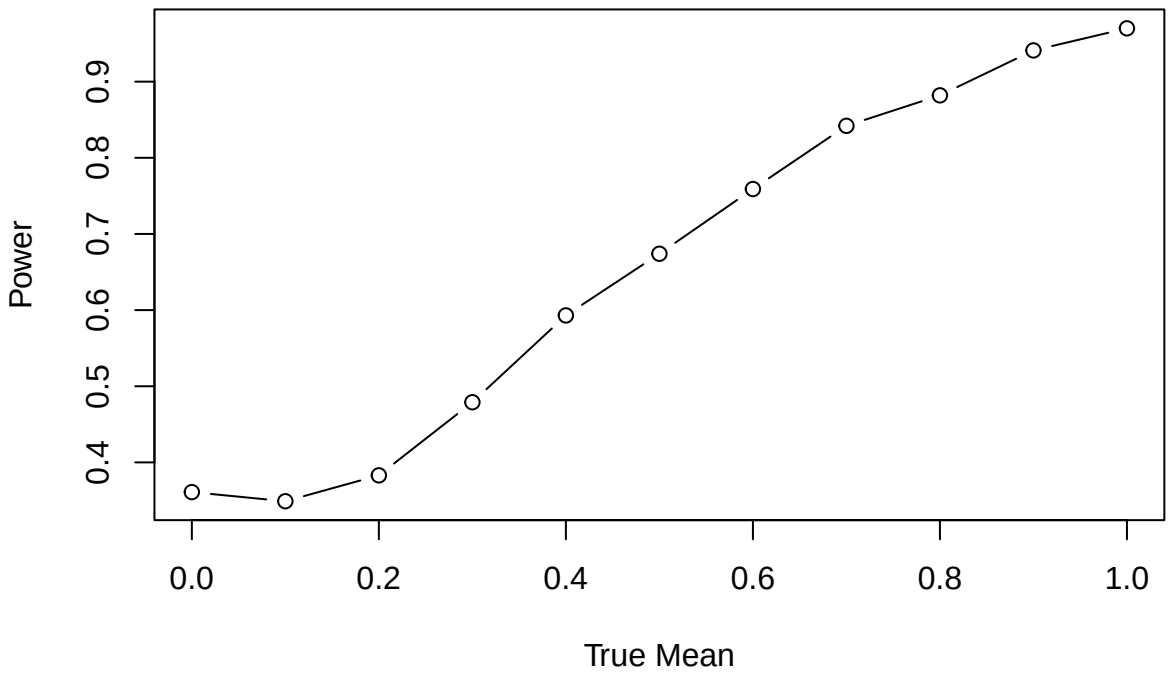
```r
estimated_type_i_error_rate <- type_i_error_count / num_simulations

cat("Estimated Type I Error Rate:", estimated_type_i_error_rate, "\n")
```

```
## Estimated Type I Error Rate: 0.051
```

```r
library(MASS)
library(ggplot2)

set.seed(5400)

n <- 30
cov_matrix <- matrix(0.1, nrow=n, ncol=n)
diag(cov_matrix) <- 1

calc_power <- function(n, mu, alpha=0.05, B=1000) {
  cov_matrix <- matrix(0.1, nrow=n, ncol=n)
  diag(cov_matrix) <- 1

  p_values <- replicate(B, {
    X <- mvrnorm(n=1, mu=rep(mu, n), Sigma=cov_matrix)
    t.test(X, mu = 0)$p.value
  })
  mean(p_values < alpha)
}

sample_sizes <- seq(10, 100, 10)
powers_sample_size <- sapply(sample_sizes, calc_power, mu=0)

plot(sample_sizes, powers_sample_size, type="b", xlab="Sample Size", ylab="Power")
```




```r
true_means <- seq(0, 1, 0.1)
powers_true_mean <- sapply(true_means, calc_power, n=30)

plot(true_means, powers_true_mean, type="b", xlab="True Mean", ylab="Power")
```




```r
#3)
#a)
set.seed(5400)
pval2SampleT <- function(mu1, mu2, var1, var2, n1, n2, alp = 0.05, B) {
  df <- n1 + n2 - 2
  t_alpha <- qt(1 - alp/2, df)
  p_values <- numeric(B)

  for (i in 1:B) {
    x <- rnorm(n1, mean = mu1, sd = sqrt(var1))
    y <- rnorm(n2, mean = mu2, sd = sqrt(var2))
    t_stat <- (mean(x) - mean(y)) / sqrt(var(x)/n1 + var(y)/n2)
    p_values[i] <- 2 * pt(abs(t_stat), df, lower.tail = FALSE)
  }

  return(p_values)
}
p_values <- pval2SampleT(mu1 = 5, mu2 = 7, var1 = 2, var2 = 3, n1 = 30, n2 = 40, B = 1000)
p_values
```

```
##    [1] 5.475076e-05 1.673822e-08 6.660335e-08 7.782652e-07 8.374035e-06
##    [6] 6.437103e-08 1.008553e-01 5.086850e-10 9.144337e-08 2.458471e-06
##   [11] 1.918582e-08 4.787150e-07 2.878145e-05 6.035534e-07 3.445514e-08
##   [16] 1.309807e-08 1.763287e-03 4.655091e-09 6.364852e-08 3.705925e-05
##   [21] 1.987349e-05 2.367420e-07 3.507522e-06 2.425838e-08 3.967285e-10
```

```
##   [26] 2.191375e-08 1.132205e-05 8.567491e-07 2.478495e-03 3.770126e-06
##   [31] 3.794815e-06 8.698442e-06 2.901288e-07 1.506377e-06 1.080903e-05
##   [36] 8.797494e-05 7.749991e-06 1.470016e-05 5.874496e-05 6.750765e-06
##   [41] 1.440932e-06 1.013562e-05 2.377668e-09 7.193365e-07 8.565462e-06
##   [46] 5.487843e-07 5.444177e-06 5.545862e-04 1.394585e-09 1.802681e-05
##   [51] 1.089576e-06 4.577651e-06 1.823545e-07 2.997677e-05 6.648849e-06
##   [56] 1.168924e-05 4.908659e-07 4.253285e-07 2.396366e-09 7.327105e-07
##   [61] 6.689991e-04 1.803673e-10 7.584467e-06 2.319800e-07 1.936380e-07
##   [66] 8.199418e-10 2.992789e-13 4.294620e-06 8.947382e-10 9.487668e-05
##   [71] 1.850802e-05 1.203696e-06 6.695326e-06 1.433150e-08 1.138260e-05
##   [76] 3.142630e-06 5.521298e-13 2.904509e-05 1.900800e-08 1.368804e-07
##   [81] 1.461958e-08 4.978767e-09 1.528031e-09 2.365605e-05 1.667163e-07
##   [86] 1.045799e-03 4.635163e-04 3.090471e-07 1.733482e-04 7.032766e-11
##   [91] 4.986404e-06 6.841815e-06 1.023887e-04 3.373973e-06 9.471553e-08
##   [96] 1.473240e-09 6.625890e-08 1.720777e-06 3.814217e-06 1.323736e-09
##  [101] 7.391876e-09 2.115801e-06 4.978989e-07 3.522635e-06 1.830483e-09
##  [106] 4.945971e-08 1.193535e-08 9.303689e-08 4.621776e-05 6.998589e-08
##  [111] 2.567779e-09 3.761307e-07 1.423675e-08 6.857420e-05 4.062621e-07
##  [116] 1.718538e-04 1.123081e-09 8.306150e-05 9.650969e-08 2.474230e-06
##  [121] 3.862731e-06 3.286485e-07 5.612138e-05 2.108651e-06 3.577923e-11
##  [126] 9.071618e-08 1.060434e-08 5.891507e-08 9.711773e-05 1.891782e-05
##  [131] 4.351859e-08 2.292667e-05 1.303903e-11 1.189921e-03 7.031631e-07
##  [136] 2.469181e-06 1.212380e-09 2.546804e-06 3.349761e-05 1.163243e-09
##  [141] 1.312899e-08 4.577784e-06 1.395060e-03 6.658967e-06 1.941695e-05
##  [146] 2.695755e-07 5.510866e-04 5.918014e-11 1.339628e-04 3.996421e-06
##  [151] 2.710069e-08 1.171885e-05 5.486848e-09 8.488235e-09 3.186582e-07
##  [156] 3.285138e-07 6.900278e-09 3.764470e-04 7.123998e-08 7.785994e-08
##  [161] 1.983577e-10 4.069834e-03 2.414158e-10 1.947642e-02 1.042485e-05
##  [166] 1.417816e-09 1.440448e-06 1.199969e-06 1.026911e-05 6.180775e-06
##  [171] 4.693489e-06 8.014958e-07 2.228932e-05 1.525269e-03 1.824973e-05
##  [176] 8.595129e-05 5.221541e-07 1.030202e-07 2.813063e-05 4.981464e-10
##  [181] 1.937554e-10 3.711804e-10 1.641275e-09 2.375782e-05 3.197286e-07
##  [186] 8.553061e-06 2.187992e-06 6.173022e-06 8.105986e-05 4.756587e-06
##  [191] 8.209611e-10 1.605852e-05 3.449415e-06 1.272537e-08 1.084835e-05
##  [196] 2.360011e-05 7.219657e-06 1.550025e-07 2.503552e-07 5.674845e-06
##  [201] 7.864740e-08 9.283169e-05 1.351832e-05 5.686829e-08 2.135718e-08
##  [206] 4.457198e-06 1.432282e-07 5.567492e-08 5.543611e-05 1.011121e-05
##  [211] 6.093949e-07 7.116499e-09 1.410557e-04 1.154271e-06 6.133656e-05
##  [216] 8.951279e-09 1.918369e-08 9.517480e-07 1.929887e-06 3.267453e-06
##  [221] 1.336370e-04 1.339613e-05 5.070847e-07 4.733185e-04 9.188361e-05
##  [226] 3.496602e-05 1.269788e-06 4.036453e-05 1.772817e-07 2.084793e-05
##  [231] 7.490551e-04 3.251225e-07 1.989841e-07 4.300947e-06 2.282325e-05
##  [236] 5.452065e-05 1.015001e-03 9.368289e-07 3.427276e-07 5.588880e-06
##  [241] 1.057247e-03 3.714194e-09 3.644253e-04 9.825454e-06 2.608432e-08
##  [246] 2.039895e-06 2.028719e-07 1.694076e-06 1.227179e-05 3.345385e-08
##  [251] 1.234339e-07 6.018123e-05 3.595254e-05 1.720797e-07 3.523771e-09
##  [256] 3.665900e-05 2.682172e-07 8.029517e-08 7.622334e-08 8.800632e-06
##  [261] 3.459878e-08 1.022264e-08 2.186998e-04 1.360041e-07 8.041762e-10
##  [266] 5.088427e-04 9.051931e-04 3.473240e-07 6.324208e-03 2.546644e-06
##  [271] 1.287062e-06 4.450676e-07 4.090594e-07 2.763967e-06 3.174669e-06
##  [276] 6.820756e-03 7.974134e-08 4.810507e-08 7.077333e-06 2.346694e-06
##  [281] 8.476660e-07 3.379799e-07 6.669372e-06 8.893348e-06 1.706664e-06
##  [286] 1.959971e-07 5.641577e-09 6.109021e-08 1.791388e-07 5.012685e-09
##  [291] 2.959951e-08 9.628617e-06 2.361753e-07 4.385652e-07 7.765777e-07
```

```
##  [296] 2.644833e-06 1.462558e-06 7.823422e-06 1.364856e-09 5.187602e-06
##  [301] 4.194759e-08 7.998728e-11 3.082341e-07 1.057698e-08 7.878808e-04
##  [306] 3.492683e-07 1.536747e-10 3.743926e-08 2.380383e-06 7.974373e-06
##  [311] 1.498463e-07 1.945282e-07 1.645214e-05 6.609132e-08 3.020427e-07
##  [316] 2.476947e-06 3.932321e-07 3.234753e-07 2.184629e-06 1.538941e-04
##  [321] 4.674681e-07 2.696493e-07 4.068861e-06 2.808547e-06 1.501834e-08
##  [326] 6.614834e-07 1.085378e-07 1.119278e-06 6.675143e-08 4.284441e-10
##  [331] 5.696589e-08 1.937727e-10 1.337508e-07 2.488145e-05 1.379081e-06
##  [336] 1.172128e-06 1.262657e-05 6.224513e-07 5.238953e-05 1.204203e-11
##  [341] 1.014494e-06 3.504123e-04 2.197328e-07 2.029483e-08 1.332915e-04
##  [346] 1.226833e-08 7.098656e-07 7.500725e-07 4.542142e-09 1.450383e-07
##  [351] 3.206193e-06 8.937067e-07 9.436403e-05 3.254841e-04 7.403567e-06
##  [356] 1.481620e-05 8.942912e-09 4.517966e-04 1.118344e-05 6.826128e-06
##  [361] 8.791102e-07 2.340813e-07 1.155772e-05 8.895339e-08 1.567118e-08
##  [366] 2.220245e-05 5.759933e-07 2.557171e-05 1.150834e-06 1.601946e-04
##  [371] 2.147468e-08 1.091025e-05 6.802439e-05 3.278673e-05 1.169106e-07
##  [376] 4.397897e-06 8.141933e-06 9.649154e-07 1.547404e-03 4.983849e-07
##  [381] 1.790337e-05 8.431629e-08 7.091315e-09 1.316086e-06 3.751827e-05
##  [386] 5.210863e-04 3.360325e-07 1.340440e-07 5.502388e-07 8.369914e-10
##  [391] 4.038789e-07 7.171785e-08 2.280535e-06 3.690674e-11 2.027276e-08
##  [396] 1.524397e-05 3.488277e-09 6.546027e-05 5.045523e-09 6.461249e-06
##  [401] 2.131071e-09 4.392260e-05 6.642910e-08 1.986279e-05 2.495855e-07
##  [406] 7.330967e-04 2.466530e-05 3.264137e-08 3.009720e-05 1.750417e-06
##  [411] 1.960872e-06 2.552961e-07 5.566463e-08 8.759521e-07 1.906994e-07
##  [416] 2.408806e-06 4.544390e-05 6.620009e-06 7.047874e-06 2.091166e-08
##  [421] 4.378016e-05 8.616245e-04 2.504740e-04 5.329733e-05 1.411309e-08
##  [426] 1.252820e-09 1.881536e-08 1.394795e-09 1.869653e-06 1.571768e-05
##  [431] 7.584591e-07 5.030402e-03 4.243263e-05 1.233213e-04 1.102496e-04
##  [436] 7.780071e-06 5.779936e-06 2.126308e-08 2.106588e-05 1.931751e-03
##  [441] 4.263430e-05 6.522767e-06 1.738423e-05 2.629413e-07 1.583719e-10
##  [446] 7.688438e-05 6.713718e-06 3.215735e-07 1.597651e-04 1.333275e-05
##  [451] 1.674913e-05 2.243191e-03 2.557157e-06 2.544846e-06 1.417578e-07
##  [456] 9.776217e-06 8.382982e-06 8.352325e-05 9.445774e-05 4.729105e-07
##  [461] 5.591266e-04 2.144989e-06 1.993365e-06 4.123220e-04 3.637560e-07
##  [466] 2.289457e-07 1.612542e-09 3.784053e-06 3.676462e-07 8.922656e-07
##  [471] 2.095265e-04 4.344179e-06 2.353539e-07 2.380621e-04 4.220535e-07
##  [476] 9.539585e-07 1.006552e-04 5.875041e-08 4.036791e-09 1.988408e-06
##  [481] 1.304261e-07 6.615933e-08 1.225870e-07 2.208854e-08 1.932905e-03
##  [486] 1.166948e-07 8.775878e-05 1.271089e-06 1.223512e-07 5.709832e-06
##  [491] 1.880741e-08 1.281103e-08 9.059004e-04 1.648453e-06 6.131586e-08
##  [496] 4.707078e-11 5.594956e-09 1.343147e-03 6.046979e-05 1.980111e-07
##  [501] 2.544074e-07 2.626961e-04 8.496654e-07 2.416485e-06 6.180829e-07
##  [506] 3.132511e-08 1.050330e-06 7.173746e-07 1.080655e-09 1.969003e-04
##  [511] 1.981753e-05 4.118282e-06 8.985552e-08 5.686609e-13 1.423687e-05
##  [516] 3.126739e-08 1.126979e-05 2.343494e-11 1.751152e-09 2.279540e-08
##  [521] 5.643329e-05 6.986714e-05 3.930406e-07 2.687805e-05 8.094758e-05
##  [526] 2.144321e-07 7.151154e-08 4.497660e-07 5.784911e-06 1.136111e-06
##  [531] 5.492516e-08 7.672512e-07 9.764923e-05 8.311932e-08 9.149754e-07
##  [536] 6.520256e-03 5.773638e-05 1.457908e-02 2.180346e-08 1.296336e-07
##  [541] 1.843284e-05 2.902069e-06 2.058399e-07 9.002751e-05 2.859804e-09
##  [546] 1.117207e-03 1.246708e-05 8.929045e-05 9.696664e-07 1.002418e-08
##  [551] 1.635004e-05 5.353474e-07 2.455185e-07 6.749480e-07 1.053554e-04
##  [556] 1.607721e-07 6.790792e-07 6.846390e-07 1.543256e-07 1.437626e-09
##  [561] 4.791782e-08 2.614070e-05 8.802458e-10 2.760785e-06 1.360087e-06
```

```
##  [566] 5.660251e-08 1.495314e-04 8.573293e-06 9.708680e-08 2.629285e-05
##  [571] 7.881417e-07 9.063401e-08 2.044468e-04 2.126603e-07 1.842297e-07
##  [576] 4.644319e-06 2.830670e-08 2.480466e-07 2.178330e-04 1.289655e-06
##  [581] 5.890279e-05 2.008007e-06 6.371005e-07 1.321038e-09 1.250233e-06
##  [586] 2.383792e-07 1.671861e-08 3.785550e-05 9.260873e-07 9.010316e-06
##  [591] 1.432467e-07 7.949730e-13 5.444651e-06 1.136600e-06 2.948079e-07
##  [596] 1.933810e-11 5.985816e-08 2.163761e-05 3.159953e-10 2.871984e-05
##  [601] 2.019414e-05 1.168756e-04 1.985940e-08 4.121211e-04 2.160275e-03
##  [606] 2.668761e-07 3.734042e-11 7.368627e-07 1.675294e-05 4.519092e-11
##  [611] 2.934671e-10 1.298602e-06 2.173129e-13 3.528150e-05 8.598100e-07
##  [616] 1.013326e-06 2.641381e-04 3.404393e-06 5.096862e-06 3.680477e-06
##  [621] 3.269986e-11 3.253266e-04 7.211645e-04 1.276148e-09 1.966924e-08
##  [626] 1.245112e-07 3.793714e-06 1.135289e-05 8.096452e-06 5.984337e-03
##  [631] 1.208155e-06 2.410761e-04 1.773269e-07 2.459206e-05 8.648225e-11
##  [636] 1.176465e-04 5.743039e-06 4.455432e-06 6.484321e-07 2.692027e-06
##  [641] 4.158359e-09 3.176617e-07 1.186255e-06 1.137590e-03 3.032423e-06
##  [646] 7.044884e-09 1.070538e-07 4.021679e-06 2.066316e-08 1.010068e-06
##  [651] 2.153242e-04 5.913199e-05 1.274793e-06 1.390822e-04 7.693046e-05
##  [656] 2.346537e-08 9.642925e-10 7.888269e-05 1.335201e-05 3.355940e-09
##  [661] 6.666955e-08 9.421761e-05 1.387110e-04 1.027611e-09 1.808218e-08
##  [666] 1.108917e-05 5.777702e-08 1.766315e-07 3.436427e-11 3.138386e-05
##  [671] 2.128031e-06 1.938112e-06 1.873485e-06 8.809827e-08 4.528585e-06
##  [676] 5.534254e-05 8.799015e-06 2.169470e-07 4.313200e-07 9.239222e-06
##  [681] 8.329880e-04 3.964590e-05 5.213542e-03 2.009159e-07 1.448294e-07
##  [686] 9.681644e-07 1.968454e-06 1.052838e-07 2.854563e-10 6.021893e-07
##  [691] 1.773220e-04 2.616786e-03 3.320029e-06 1.275096e-06 4.986908e-07
##  [696] 1.345694e-08 1.229167e-07 1.166114e-04 6.857998e-05 3.800173e-07
##  [701] 3.447842e-08 1.894361e-07 1.298670e-07 2.622435e-09 8.565998e-09
##  [706] 1.118113e-10 3.526770e-07 2.865002e-07 3.072640e-07 6.192541e-09
##  [711] 2.806269e-07 2.045913e-04 4.744096e-05 1.328579e-09 2.195910e-08
##  [716] 4.014024e-05 1.037791e-05 8.723092e-07 4.320082e-06 2.572142e-07
##  [721] 2.456201e-04 5.836596e-06 9.654110e-13 6.440691e-07 2.943740e-10
##  [726] 1.603989e-04 1.414071e-07 3.391704e-07 1.445514e-07 4.592702e-09
##  [731] 3.727672e-06 1.080274e-06 3.504710e-08 1.724668e-06 2.868184e-06
##  [736] 1.940679e-07 3.081078e-09 6.387489e-08 1.520849e-07 2.544594e-07
##  [741] 5.889353e-08 4.699308e-07 3.461216e-04 1.185276e-06 9.817285e-06
##  [746] 3.475457e-07 3.085791e-07 8.464098e-04 6.302710e-11 1.973949e-05
##  [751] 2.872408e-06 1.795275e-05 4.627250e-07 7.231094e-10 1.766056e-05
##  [756] 6.459944e-07 1.454444e-04 1.075523e-06 2.725016e-03 1.575032e-12
##  [761] 1.504102e-09 3.155381e-05 3.406928e-07 5.260984e-08 7.721905e-09
##  [766] 4.285244e-06 1.227587e-06 1.665181e-09 4.069385e-11 4.128499e-05
##  [771] 5.590521e-08 1.862654e-06 5.022024e-06 4.472063e-06 6.626908e-07
##  [776] 1.617476e-08 2.275714e-05 6.802377e-05 2.782118e-05 4.668915e-06
##  [781] 7.588683e-05 8.318301e-08 1.407671e-03 2.589554e-05 1.054281e-06
##  [786] 9.346208e-07 4.842471e-06 9.813360e-04 2.614374e-07 2.047188e-06
##  [791] 1.404182e-03 2.877165e-03 4.147464e-05 2.715517e-05 2.772610e-07
##  [796] 2.972984e-06 6.210095e-10 3.698211e-06 2.175814e-06 5.620453e-06
##  [801] 8.508144e-07 1.432970e-10 4.533088e-04 3.946066e-06 4.523889e-06
##  [806] 1.674884e-04 5.271291e-06 1.574717e-04 6.819514e-05 1.310404e-07
##  [811] 4.259614e-05 7.725989e-07 8.286180e-08 3.208524e-06 3.368604e-05
##  [816] 2.578774e-06 4.861581e-07 5.484329e-07 3.819386e-06 1.179267e-04
##  [821] 1.761741e-05 4.452482e-09 2.182349e-06 1.093955e-05 1.443902e-06
##  [826] 1.265963e-06 5.241455e-07 2.724456e-06 2.894522e-06 1.175759e-06
##  [831] 2.485863e-07 2.158975e-07 2.617581e-06 1.558242e-07 2.184051e-10
```

```
##  [836] 8.583435e-09 7.729485e-08 2.068311e-09 8.507486e-05 9.234120e-08
##  [841] 9.620854e-04 1.637550e-06 4.240229e-07 9.547570e-09 3.210537e-08
##  [846] 6.331797e-09 3.185354e-06 1.538644e-07 6.118409e-09 5.416655e-05
##  [851] 1.264043e-09 5.922942e-06 5.235201e-07 2.998496e-09 4.793647e-06
##  [856] 1.697242e-07 5.551002e-06 1.855981e-11 5.250725e-07 8.382582e-06
##  [861] 3.773567e-07 4.299468e-07 1.280323e-05 2.576437e-05 1.163075e-10
##  [866] 2.562161e-06 2.539963e-08 2.613846e-07 6.386408e-06 5.281654e-08
##  [871] 3.190953e-07 9.735573e-10 4.906571e-06 7.827380e-08 7.134969e-06
##  [876] 1.480172e-04 8.607539e-05 1.037180e-04 5.728861e-07 5.872921e-08
##  [881] 2.705112e-08 1.856544e-06 2.037572e-08 3.725737e-07 1.683062e-08
##  [886] 1.449843e-06 2.319019e-04 1.899626e-05 7.785835e-05 2.121352e-06
##  [891] 7.110013e-08 1.465775e-05 2.630860e-06 1.081170e-07 2.627913e-07
##  [896] 7.190544e-07 3.197578e-07 1.937605e-04 5.498507e-05 2.326322e-07
##  [901] 3.248935e-05 2.399248e-06 4.672895e-09 8.515116e-07 1.924167e-07
##  [906] 4.393719e-09 9.919110e-07 2.463654e-06 6.285778e-07 8.337684e-07
##  [911] 2.282205e-05 2.642225e-11 3.947328e-06 3.234393e-07 1.939624e-07
##  [916] 2.785251e-05 1.034976e-05 2.624144e-05 1.211142e-06 2.740666e-09
##  [921] 3.015346e-04 8.227255e-05 1.262497e-06 6.284937e-06 3.317340e-08
##  [926] 2.479801e-07 8.344415e-06 1.135793e-05 2.845742e-07 1.189270e-07
##  [931] 5.991426e-10 4.072805e-07 1.148305e-03 5.955011e-04 1.849665e-07
##  [936] 9.177894e-06 4.770710e-08 1.832167e-07 6.068630e-07 2.886582e-10
##  [941] 7.986974e-06 1.268236e-07 1.101039e-05 3.107897e-08 1.089345e-06
##  [946] 4.054301e-10 9.881930e-03 1.114633e-05 2.624836e-10 2.323101e-06
##  [951] 7.746655e-07 2.556835e-09 1.046728e-06 4.948539e-05 2.240939e-10
##  [956] 1.227382e-04 6.645823e-06 4.340337e-07 8.582409e-09 3.940248e-08
##  [961] 5.880159e-06 4.859724e-06 1.638566e-11 4.810578e-06 1.982867e-02
##  [966] 3.754061e-05 3.766583e-09 3.390868e-09 7.386094e-08 5.179805e-08
##  [971] 9.065760e-10 1.133903e-09 2.916185e-04 1.072415e-05 1.585370e-05
##  [976] 1.863169e-07 3.086859e-04 9.046703e-04 1.019630e-06 1.970949e-11
##  [981] 9.667114e-08 1.715032e-08 1.565660e-09 2.337260e-08 2.902548e-05
##  [986] 3.546573e-06 5.827441e-05 2.899819e-05 3.995295e-07 9.178925e-10
##  [991] 7.410716e-04 1.382881e-09 7.673590e-09 7.391527e-03 3.261623e-09
##  [996] 1.852808e-06 2.903280e-09 6.401388e-05 6.144479e-09 2.141220e-05
```

```r
#b)
set.seed(5400)
mu1 <- 10
mu2 <- 10
var1 <- 4
var2 <- 4
n1 <- 50
n2 <- 50
alp <- 0.05
B <- 1000

p_values <- pval2SampleT(mu1, mu2, var1, var2, n1, n2, alp, B)

type_I_error <- mean(p_values < alp)

cat("Estimated Type I Error Rate:", type_I_error)
```

```
## Estimated Type I Error Rate: 0.041
```

```r
#4)
set.seed(5400)
lambda <- 5
n <- 20
alpha <- 0.05
B <- 10000

set.seed(5400)
test_stats <- replicate(B, {
  X <- rpois(n, lambda)
  t_stat <- (mean(X) - lambda) / (sd(X) / sqrt(n))
  t_stat
})

critical_value <- qt(1 - alpha/2, df = n-1)

type_I_error_rate <- mean(abs(test_stats) > critical_value)

cat("Estimated Type I Error Rate:", type_I_error_rate, "\n")
```

```
## Estimated Type I Error Rate: 0.0511
```

```r
score_interval <- binom.test(round(type_I_error_rate * B), B, conf.level = 0.99)$conf.int

cat("99% Score Confidence Interval for Type I Error Rate:", score_interval, "\n")
```

```
## 99% Score Confidence Interval for Type I Error Rate: 0.04559289 0.05703924
```

```r
alpha_in_interval <- alpha > score_interval[1] & alpha < score_interval[2]

cat("Is alpha captured in the confidence interval?", alpha_in_interval, "\n")
```

```
## Is alpha captured in the confidence interval? TRUE
```

```r
set.seed(5400)
conf_intervals <- replicate(B, {
  X <- rpois(n, lambda)
  mean_X <- mean(X)
  sd_X <- sd(X)
  error <- qt(1 - alpha/2, df = n-1) * sd_X / sqrt(n)
  c(lower = mean_X - error, upper = mean_X + error)
}, simplify = "matrix")

coverage_prob <- mean(lambda > conf_intervals[1,] & lambda < conf_intervals[2,])

cat("Estimated Coverage Probability:", coverage_prob, "\n")
```

```
## Estimated Coverage Probability: 0.9489
```

```r
score_interval <- binom.test(round(coverage_prob * B), B, conf.level = 0.99)$conf.int

cat("99% Score Confidence Interval for Coverage Probability:", score_interval, "\n")
```

```
## 99% Score Confidence Interval for Coverage Probability: 0.9429608 0.9544071
```

```r
coverage_in_interval <- (1 - alpha) > score_interval[1] & (1 - alpha) < score_interval[2]
cat("Is 1 - alpha captured in the confidence interval?", coverage_in_interval, "\n")
```

```
## Is 1 - alpha captured in the confidence interval? TRUE
```