

0526 週考解析

一、填充題 A

1. 設 $\vec{v} = (a, b)$ ，則 $|\vec{v}| =$ _____。Ans: $\sqrt{a^2 + b^2}$
2. 設 $\vec{a} = (x_1, y_1)$ 、 $\vec{b} = (x_2, y_2)$ ，若 $\vec{a} // \vec{b}$ ，則 $\vec{a} =$ _____。(平行分量呈比例)Ans: $r\vec{b}$
3. 設 $\vec{a} = (a_1, a_2)$ ，則與 \vec{a} 同向的單位向量為 $\vec{u}_a = \frac{\vec{a}}{|\vec{a}|} =$ _____。Ans: $\frac{(a_1, a_2)}{\sqrt{a_1^2 + a_2^2}}$
4. 設 \vec{a} 、 \vec{b} 為平面上的兩個非零向量，且兩向量的夾角為 θ ，則 \vec{a} 與 \vec{b} 的內積符號定為 $\vec{a} \cdot \vec{b} =$ _____。Ans: $|\vec{a}||\vec{b}|\cos\theta$
若 $\vec{a} = (x_1, y_1)$ 、 $\vec{b} = (x_2, y_2)$ ，則 \vec{a} 與 \vec{b} 的內積符號定為 $\vec{a} \cdot \vec{b} =$ _____。Ans: $x_1x_2 + y_1y_2$
5. 設 \vec{a} 、 \vec{b} 為平面上的兩個非零向量，且兩向量的夾角為 θ ，則 $\cos\theta =$ _____。Ans: $\frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$
6. $\vec{a} \cdot \vec{a} =$ _____。Ans: $|\vec{a}|^2$
7. 設 $\vec{a} = (x_1, y_1)$ 、 $\vec{b} = (x_2, y_2)$ ，若 $\vec{a} \perp \vec{b}$ ，則 _____。(即內積為 0)Ans: $\vec{a} \cdot \vec{b} = 0$ / $x_1x_2 + y_1y_2 = 0$

二、填充題 B

$$1. \vec{AB} = (0-12, 5-0) \quad \therefore |\vec{AB}| = \sqrt{(-12)^2 + 5^2} = 13$$

$$= (-12, 5)$$

$$2. \vec{AB} = (4, 0)$$

A為起點, B為終點, \vec{AB} 為A到B的向量

$$\therefore B = A + \vec{AB} = (-3, 2) + (4, 0) = (1, 2)$$

$$3. ABCD \Rightarrow A+C=B+D$$

$$\text{令 } D(x, y) \Rightarrow (-4, -2) + (2, 3) = (1, 0) + (x, y)$$

$$(-2, 1) = (x+1, y)$$

$$\therefore \begin{cases} x = -3 \\ y = 1 \end{cases} \quad \therefore D(-3, 1)$$

$$4. (1) \vec{AB} + \vec{BC} = \vec{AC}$$

$$\therefore \vec{BC} = \vec{AC} - \vec{AB}$$

$$= (4, 0) - (0, 3)$$

$$= (4, -3)$$

$$(2) \text{ 周長} = |\vec{AB}| + |\vec{BC}| + |\vec{AC}|$$

$$= 3 + 4 + 5$$

$$= 12$$

$$5. \vec{a} \parallel \vec{b} \Rightarrow \vec{a} = r\vec{b} \Rightarrow \vec{a}, \vec{b} \text{ 呈比例關係}$$

$$\therefore \frac{4}{-6} = \frac{2}{k} \Rightarrow 4k = -12$$

$$\therefore k = -3$$

$$6. \vec{AB} = (-1-3, -2-(-5)) \quad \therefore \text{同向的單位向量} = \frac{(-4, 3)}{\sqrt{4^2+3^2}} = \frac{(-4, 3)}{5}$$

$$= (-4, 3)$$

$$= \left(-\frac{4}{5}, \frac{3}{5}\right)$$

$$\begin{aligned}
 7. \vec{a} \cdot \vec{b} &= a_1 b_1 + a_2 b_2 \\
 &= 3 \cdot (-2) + 5 \cdot 1 \\
 &= -6 + 5 = -1
 \end{aligned}$$

$$\begin{aligned}
 8. \text{ 設 } \vec{AB}, \vec{AC} \text{ 的夾角為 } \theta, \quad \vec{AB} = (1, 1), \vec{AC} = (-3, 3) \\
 \text{則 } \cos \theta &= \frac{\vec{AB} \cdot \vec{AC}}{|\vec{AB}| |\vec{AC}|} \\
 \therefore \cos \theta &= \frac{1 \times (-3) + 1 \times 3}{\sqrt{1^2 + 1^2} \sqrt{(-3)^2 + 3^2}} = \frac{0}{\sqrt{2} \cdot 3\sqrt{2}} = 0 \\
 \therefore \theta &= 90^\circ
 \end{aligned}$$

$$\begin{aligned}
 9. \vec{a} \perp \vec{b} &\Rightarrow \vec{a} \cdot \vec{b} = 0 \\
 \therefore (k-3) \cdot k + 1 \cdot (-4) &= 0 \\
 k^2 - 3k - 4 &= 0
 \end{aligned}$$

$$\begin{aligned}
 (k+1)(k-4) &= 0 \\
 \therefore k &= -1, 4
 \end{aligned}$$

$$\begin{aligned}
 10. \therefore |\vec{a} - \vec{b}|^2 &= |\vec{a}|^2 - 2\vec{a} \cdot \vec{b} + |\vec{b}|^2 \\
 &= 4|\vec{a}|^2 - 2\vec{a} \cdot \vec{b} + 9|\vec{b}|^2 \\
 &= 4 \cdot 1^2 - 2 \cdot (|\vec{a}| \cdot |\vec{b}| \cdot \cos 0^\circ) + 9 \cdot 3^2 \\
 &= 4 - 12 \cdot 1 \cdot 3 \cdot 1 + 81 \\
 &= 85 - 36 = 49 \\
 \therefore |\vec{a} - \vec{b}| &= \sqrt{49} = 7
 \end{aligned}$$