家族計畫 Quiz(2) 解析 (2023.05.31)

1. 試求 $\sqrt{3}\tan 30^\circ + \sqrt{2}\sin 45^\circ - \cos 60^\circ$.

Sol: $\tan 30^\circ = \frac{1}{\sqrt{3}}$ $\sin 45^\circ = \frac{1}{\sqrt{2}}$. $\cos 60^\circ = \frac{1}{2}$

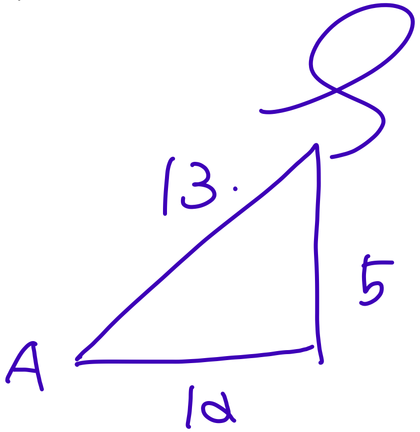
$\therefore$ 原式 $= \sqrt{3}\cdot\frac{1}{\sqrt{3}} + \sqrt{2}\cdot\frac{1}{\sqrt{2}} - \frac{1}{2} = (1+1) - \frac{1}{2} = \frac{3}{2}$ //

2. $\angle C = 90^\circ$. $12\sin A = 5\cos A$, 求 $\sin A$.

sol:

$\because 12\sin A = 5\cos A$.

$\therefore \dfrac{\sin A}{\cos A} = \tan A = \dfrac{5}{12}$. $\Rightarrow$

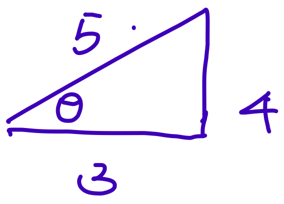


$\therefore \sin A = \dfrac{5}{13}$ //

3. $0^\circ<\theta<90^\circ$ 且 $\tan\theta=\frac{4}{3}$. 求 $\sin^2\theta-4\sin\theta\cos\theta+3\cos^2\theta$.

sol:

$\tan\theta=\frac{4}{3} \Longrightarrow$



$\Longrightarrow \begin{cases} \sin\theta=\frac{4}{5} \\ \cos\theta=\frac{3}{5} \end{cases}$

$\therefore$ 原式 $=\left(\frac{4}{5}\right)^2-4\cdot\frac{4}{5}\cdot\frac{3}{5}+3\cdot\left(\frac{3}{5}\right)^2$

$=\frac{16}{25}-\frac{48}{25}+\frac{27}{25}=-\frac{5}{25}=-\frac{1}{5}$ //

4. $\angle A$ 為銳角. $\sin A\cos A=\frac{7}{18}$. 求 $\sin^3 A+\cos^3 A$.

sol: Recall: $a^3+b^3=(a+b)^3-3ab(a+b)$.

Step.1 求 $\sin A+\cos A$.

$$(\sin A+\cos A)^2=\sin^2 A+2\sin A\cos A+\cos^2 A$$

$$=1+2\sin A\cos A$$

$$=1+2\cdot\frac{7}{18}=1+\frac{7}{9}=\frac{16}{9}.$$

$$\therefore \sin A+\cos A=\frac{4}{3}.$$

Step.2. 求原式 $=(\frac{4}{3})^3-3\cdot\frac{7}{18}\cdot\frac{4}{3}$

$$=\frac{64}{27}-\frac{14}{9}=\frac{64-42}{27}=\frac{22}{27}\ ///$$

5. $0^\circ < \theta < 90^\circ$. $\dfrac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta} = \dfrac{1}{7}$. 求 $\tan\theta$.

Sol: Hint: $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$$\because \frac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta} \cdot \frac{1/\cos\theta}{1/\cos\theta} = \frac{\dfrac{\sin\theta - \cos\theta}{\cos\theta}}{\dfrac{\sin\theta + \cos\theta}{\cos\theta}}$$

$$= \frac{\tan\theta - 1}{\tan\theta + 1} = \frac{1}{7}$$

$$\therefore 7(\tan\theta - 1) = \tan\theta + 1 \Rightarrow 7\tan\theta - 7 = \tan\theta + 1$$

$$\Rightarrow 6\tan\theta = 8 \quad \Rightarrow \tan\theta = \frac{8}{6} = \frac{4}{3} \text{ //}$$

6. 求 $\cos^2 10^\circ + \cos^2 20^\circ + \cos^2 30^\circ + \cdots + \cos^2 80^\circ = ?$

<u>sol</u>: Hint: $\sin^2\theta + \cos^2\theta = 1$.

$\because \cos^2 10^\circ + \cos^2 20^\circ + \cdots + \cos^2 80^\circ$

$= \cos^2 10^\circ + \cdots + \cos^2 40^\circ + \sin^2 40^\circ + \cdots + \sin^2 10^\circ$

$= 1 \times 4 = 4$ //

7. $\cos\theta = \tan\theta$. 求 $\dfrac{\sin\theta}{1+\cos\theta} + \dfrac{1+\cos\theta}{\sin\theta}$ 之值.

Sol: Step. 1

$$\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta + (1+\cos\theta)^2}{\sin\theta(1+\cos\theta)}$$

$$= \frac{\sin^2\theta + 1 + 2\cos\theta + \cos^2\theta}{\sin\theta(1+\cos\theta)}$$

$$= \frac{1+1+2\cos\theta}{\sin\theta(1+\cos\theta)} = \frac{2(1+\cos\theta)}{\sin\theta(1+\cos\theta)} = \frac{2}{\sin\theta} \cdots\cdots (*)$$

Step.2. $\because \cos\theta = \tan\theta$

$\therefore \cos\theta = \dfrac{\sin\theta}{\cos\theta} \Rightarrow \sin\theta = \cos^2\theta$.

$\because \sin^2\theta + \cos^2\theta = 1 \quad \therefore \cos^2\theta = 1 - \sin^2\theta$.

$\therefore \sin\theta = 1 - \sin^2\theta \Rightarrow \sin^2\theta + \sin\theta - 1 = 0$.

Step.3.

$\because \sin^2\theta + \sin\theta - 1 = 0$

$\therefore \sin\theta = \dfrac{-1 \pm \sqrt{1-(-4)}}{2} = \dfrac{-1 \pm \sqrt{5}}{2}$.

$\because \cos\theta = \tan\theta$. $\quad \therefore \sin\theta = \dfrac{\sqrt{5}-1}{2}$.

Step.4 $\sin\theta = \dfrac{\sqrt{5}-1}{2}$ 代$\boxed{2}$ (*).

$$\therefore \frac{2}{\sin\theta} = \frac{2}{\frac{\sqrt{5}-1}{2}} = \frac{4}{\sqrt{5}-1} \cdot \frac{\sqrt{5}+1}{\sqrt{5}+1} = \frac{4(\sqrt{5}+1)}{4} = \sqrt{5}+1 \,//$$