Linear Algebra Exercise 5

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■ **1.** A invertible matrix can be similar to a singular matrix.

■ **2.** $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$ are similar.

■ **3.** The matrix $A = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ are similar.

■ **4.** If matrices A and B are row equivalent, then they are also similar.

■ **5.** If B is similar to A, then $AB = BA$.

■ **6.** A symmetric mattrix can be similar to a nonsymmetric matrix.

■ **7.** A can't be similar to $A + I$.

■ **8.** If A is similar to $-A$, then $A = O$.

■ **9.** It is possible that T does not have T-invariant subspaces.

■ **10.** The sum of two eigenvectors of a matrix A is also an eigenvector of A.

■ **11.** A nonzero vector cannot correspond to two different eigenvalues of A.

■ **12.** If A contains a row or column of zeros, then 0 is an eigenvalue of A.

■ **13.** If $\lambda = 0$ is an eigenvalue of A, then A^2 is singular.

■ **14.** Let A be a 5×5 matrix with the characteristic polynomial $p_A(x) = -x^5 + 1$, then A is invertible.

■ **15.** Matrix A has eigenvectors and the corresponding eigenvalues x_1, x_2 and λ_1, λ_2, respectively, where $\lambda_1 \neq \lambda_2$. Is $x_1 + x_2$ also an eigenvector of A ?

■ **16.** The sum of two eigenvalues of an operator T is always an eigenvalue of T.

■ **17.** All eigenvalues of a real symmetry matrix are real and distinct.

■ **18.** If λ is an eigenvalue of A and μ is an eigenvalue of B, then $\lambda\mu$ is an eigenvalue of AB.

■ **19.** Let A and B be $n \times n$ matrices. Let λ be an eigenvalue for both A and B. Then λ is an eigenvalue for $A + B$.

■ **20.** Suppose $A, B \in \mathbb{R}^{n \times n}$. If x is the common eigenvector of A and B, then x must be an eigenvector of $C = \alpha A + \beta B$, where α and β are two constants.

■ **21.** Similar matrices always have the same eigenvectors.

■ **22.** If v is an eigenvector of A^2, then v is an eigenvector of A.

■ **23.** Every $n \times n$ matrix has n eigenvalues and n eigenvectors.

■ **24.** If the matrix A is 3×3 and has eigenvalues 7,5,2, then the matrix A is symmetric.

■ **25.** Let A be an $n \times n$ matrix. The eigenvalues of A are nonzero solutions of $\det(A - \lambda I) = 0$.

■ **26.** If A is an $n \times n$ matrix, then the rank of A is equal to nonzero eigenvalues.

■ **27.** Any two eigenvectors are linearly independent.

■ **28.** Distinct eigenvectors are linearly independent.

■ **29.** Two eigenvectors corresponding to the same eigenvalue are always linearly dependent.

■ **30.** If each vector e_j in the standard basis for $\mathbb{R}^n$ is an eigenvector of A, then A is a diagonal matrix.

■ **31.** Let A be an $n \times n$ real matrix. If A^2 is diagonalizable, then A is diagonalizable.

■ **32.** If the only eigenvectors of A are multiples of $(1,4)$, then A has no diagonalization of SAS^{-1}.

■ **33.** If A is a 3×3 matrix with 3 distinct eigenvalues 0,1,2, then the matrix $(A+I)$ must be invertible.

■ **34.** If v is an eigenvector of an invertible matrix A, then cv is an eigenvector of A^{-1} for all nonzero scalars c.

■ **35.** If A is similar to a diagonalizable matrix B, then A is also diagonalizable.

■ **36.** If a triangular matrix is similar to a diagonal matrix, it is already diagonal.

■ **37.** If A is an $n \times n$ matrix and has fewer than n distinct eigenvalues, then A is not diagonalizable.

■ **38.** A diagonalizable $n \times n$ matrix must always have n distinct eigenvalues.

■ **39.** Let A be an $n \times n$ matrix. If $a_{ij} = i$ for $i = j$, $a_{ij} = 0$ for $i < j$, and $a_{ij} = i - 1$ for $i > j$, then A is diagonalizable.

■ **40.** An $n \times n$ matrix A is diagonalizable if and only if BAB^T is diagonalizable.

■ **41.** All skew-symmetric matrices are diagonalizable.

■ **42.** A square matrix with linearly independent column vectors is diagonlizable.

■ **43.** If A is row equivalent to the identity matrix I, then A is diagonalizable.

■ **44.** If A is diagonalizable, then the columns of A are linearly independent.

■ **45.** An $n \times n$ matrix with n linearly independent eigenvectors is invertible.

■ **46.** If A is diagonalizable, then A is invertible.

■ **47.** Every invertible square matrix can be diagonalized.

■ **48.** If A is an $n \times n$ diagonalizable matrix, then each vector in $\mathbb{R}^n$ can be written as a linear combination of eigenvectors of A.

■ **49.** If the eigenvalues of A are 2,2,5, then the matrix is certainly not diagonalizable.

■ **50.** $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 1 & 2 \end{bmatrix}$ is diagonalizable.

■ **51.** $A = \begin{bmatrix} 2 & 0 & 2 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ is not defective.

■ **52.** If matrix A is diagonalizable, then there is a unique matrix P such that $P^{-1}AP$ is a diagonal matrix.

■ **53.** Let A be a real 2×2 matrix, whose characteristic polynomial does not have real roots. Then A is diagonalizable.

■ **54.** If A and B are diagonalizable, so is AB.

■ **55.** If A is diagonalizable, then the rank of A equals the number of nonzero eigenvalues of A.

■ **56.** Two diagonalizable matrices A and B with the same eigenvalues and eigenvectors must be the same.

■ **57.** If A^2 is diagonalizable, then A is diagonalizable.

■ **58.** Let A be diagonalizable and $AB = BA$. Is B diagonalizable ?

■ **59.** If A and B are both diagonalizable, then A and B commute.

■ **60.** e^A is invertible for any diagonalizable matrix A.

■ **61.** For any $x \in V$, then the T-cyclic subspace generated by x is the same as that T-cyclic subspace generated by $T(x)$.

■ **62.** Let T be a linear operator on a finite-dimensional vector space V, and let x and y be elements of V. If W is T-cyclic subspace generated by x, W' is the T-cyclic subspace generated by y, and $W = W'$, then $x = y$.

■ **63.** Every matrix is similar to its Jordan canonical form.

■ **64.** If an operator has a Jordan canonical form, then there is a unique Jordan canonical basis for that operator.

■ **65.** All 3×3 real matrix has a corresponding Jordan canonical form.

■ **66.** A can't be similar to $-A$ unless $A = O$.

■ **67.** The Jordan canonical form of a diagonal matrix is the matrix itself.

■ **68.** Let A be a matrix representation of the linear mapping $T: \mathbb{R}^n \to \mathbb{R}^n$. As we knnow, the characteristic polynomial of $T: \mathbb{R}^n \to \mathbb{R}^n$ is defined as $f(x) = \det(A - xI)$. Therefore, it is easy to have that $f(A) = \det(A - AI) = O$.

■ **69.** Let T be the linear operator on $\mathbb{R}^3$, $f(x) = (x+1)(x+2)^2$ and $g(x) = (x+1)^2(x+2)$. If $f(T) = g(T) = O$ is zero transformation, then T is diagonalizable.

■ **70.** If T satisfies the condition $T^2 = I$, then T is diagonalizable.

■ **71.** Let A be a real n by n matrix. If $A^5 - 3A^3 + 2A = O$, then A is diagonalizable over real field.

■ **72.** Let T be a linear operator on an n-dimensional vector space V. Suppose that $p_T(x)$ splits. Then $p_T(x)$ devides $[m_T(x)]^n$.

■ **73.** Let T be a linear operator on a vector space V such that T has n distinct eigenvalues, where $\dim(V) = n$. Then the degree of the minimal polynomial for T equals n.

■ **74.** Suppose $A, B \in \mathbb{R}^{n \times n}$. If A and B have same minimal polynomial, then they have the same Jordan canonical form.

■ **75.** If the characteristic polynomial of a 3×3 matrix is $-(x-1)^2(x-3)$, then $(A - I)(A - 3I) = O$.

■ **76.** Let T be linear operator on a finite-dimensional vector space V, and let W_1 and W_2 be T-invariant subspaces of V such that $V = W_1 \oplus W_2$. Then $m_{T_{W_1}}(x) m_{T_{W_2}}(x) = m_T(x)$.