Ex6. $S_n = n^2+3n.$ $a_n = ?$

$$S_n = a_1 + \cdots + a_n$$
$$-)\ S_{n-1} = a_1 + \cdots + a_{n-1}$$
$$\boxed{S_n - S_{n-1} = a_n}$$

sol:

$$a_n = S_n - S_{n-1}$$
$$= n^2+3n - ((n-1)^2+3(n-1))$$
$$= n^2+3n - (n^2-2n+1+3n-3)$$

$S_6.$

$$= n^2+3n-n^2-n+2 = 2n+2 \ //$$

$$S_{10} = a_1 + \cdots + a_9 + a_{10}$$
$$-)\ S_9 = a_1 + \cdots + a_9$$
$$S_{10} - S_9 = a_{10}.$$

Ex7*. $\langle a_n \rangle : 2 | a_n \Rightarrow 2, 4, \cdots, 100$ $n = \frac{100-2}{2}+1 = 50$項

$$S_{a_n} = \frac{(2+100)50}{2} = 51 \times 50 = 2550$$

$\langle b_n \rangle : 5 | b_n \Rightarrow 5, 10, \cdots, 100$ $n = \frac{100-5}{5}+1 = 20$項

$$S_{b_n} = \frac{(5+100)\times 20}{2} = 105 \times 10 = 1050$$

$\langle c_n \rangle : 10 | c_n \Rightarrow 10, 20, \cdots, 100$ $n = \frac{100-10}{10}+1 = 10$項

$$S_{c_n} = \frac{(10+100)\times 10}{2} = 55 \times 10 = 550$$

Total: $2550 + 1050 - 550.$

$$= 3050 \ //$$

7.2 等比數列與等比級數

Def: 等比 sequence

$$a_1 \underbrace{\quad\quad}_{\times r} a_2 \underbrace{\quad\quad}_{\times r} a_3 \quad \cdots \quad \underbrace{\quad\quad}_{\times r} a_n$$

$(n-1)$次 r.

1) r: 公比 $= \dfrac{a_2}{a_1} = \dfrac{a_n}{a_{n-1}}$

2) $a_n = a_1 \cdot r^{n-1}$.

Thm: 等比 sequence 的級數和

$$S_n = \frac{a_1 \cdot (1-r^n)}{1-r} \quad (r<1) \iff \frac{a_1(r^n-1)}{r-1} \quad (r>1)$$

Ex: 1. 3. 9. 27. 81

$r=3$. $S_n = \dfrac{1\cdot(3^5-1)}{3-1} = \dfrac{242}{2} = 121$ //

Lem: If a, b, c 成等比, then $b = \pm\sqrt{ac}$.

Ex1. $a_n > 0.\ \forall n.$ $\quad a_1 = 6.\quad a_7 = \frac{3}{32}\quad a_9 = ?$

Sol:

$$a_7 = a_1 \cdot r^6. \qquad \frac{3}{32} = 6 \cdot r^6.$$

$$\therefore \frac{1}{64} = r^6 \qquad \therefore r = \pm\frac{1}{2} \qquad \because a_n > 0. \qquad \therefore r = \frac{1}{2}$$

$$\therefore a_n = 6 \cdot \left(\frac{1}{2}\right)^{n-1}$$

$$\therefore a_9 = 6 \cdot \left(\frac{1}{2}\right)^8 = 6 \cdot \frac{1}{256} = \frac{3}{128} \ ///$$

Ex2. $\underbrace{\frac{2}{9} + \frac{2}{3}}_{3} \underbrace{+ 2}_{3} + \cdots + 4374.\quad S_n = ?$

Sol:

$$r = 3. \qquad a_n = \frac{2}{9} \cdot 3^{n-1} = 4374 \qquad \therefore 3^{n-1} = 19683.$$

$$3^{n-1} = 3^9 \qquad \therefore n = 10.$$

$$\therefore S_{10} = \frac{\frac{2}{9} \cdot (3^{10} - 1)}{3 - 1} = \frac{59048}{9} \ ///$$