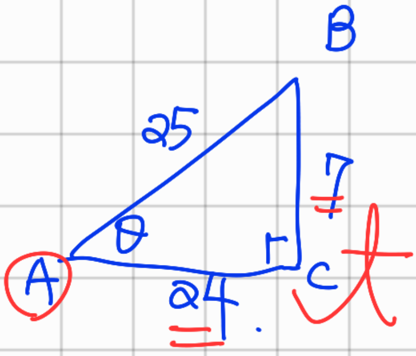
P.179

Remark: 已知一个三角函数值,即可求得另五个三角函数值.

Ex1. $\angle C = 90^\circ$. $\overline{AB} = 25$ $\overline{BC} = 7$.

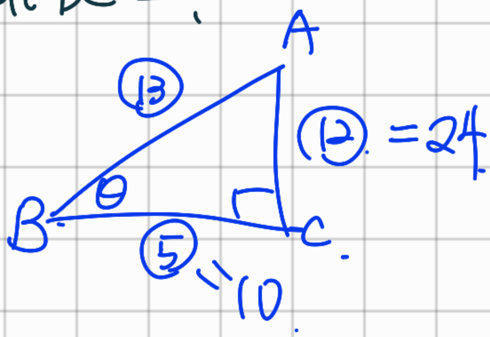


sol: $\Rightarrow \overline{AC} = 24$.

$\cos B = \frac{7}{25}$ $\tan A = \frac{7}{24}$

$\therefore \frac{7}{25} + \frac{7}{24} = \frac{343}{600}$ //

Ex2. $\angle C = 90^\circ$. $\cos B = \frac{5}{13}$. 且 $\overline{AC} = 24$. 求 $\overline{BC} = ?$

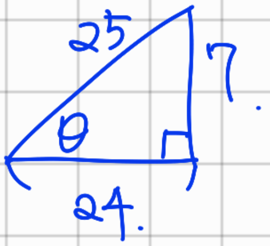


sol:

$\therefore \overline{BC} = 10$ //

Ex3. $0^\circ < \theta < 90^\circ$. $\sin\theta = \frac{7}{25}$. $\tan\theta + \frac{1}{\cos\theta} = ?$

sol:



$\therefore \tan\theta = \frac{7}{24}$

$\cos\theta = \frac{24}{25}$

$\therefore \frac{7}{24} + \frac{1}{\frac{24}{25}} = \frac{7+25}{24} = \frac{32}{24} = \frac{4}{3}$ //

Cor：特殊角的三角函数值（銳角） +1

	$15°$	$30°$	$45°$	$60°$	$75°$	
$\sin\theta$	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{6}+\sqrt{2}}{4}$	increasing (↗)
$\cos\theta$	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	$\frac{\sqrt{6}-\sqrt{2}}{4}$	decreasing (↘)
$\tan\theta$	$2-\sqrt{3}$	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$2+\sqrt{3}$	increasing (↗)
$\cot\theta$	$2+\sqrt{3}$	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	$2-\sqrt{3}$	decreasing (↘)
$\sec\theta$	$\sqrt{6}-\sqrt{2}$	$\frac{2\sqrt{3}}{3}$	$\sqrt{2}$	2	$\sqrt{6}+\sqrt{2}$	increasing (↗)
$\csc\theta$	$\sqrt{6}+\sqrt{2}$	2	$\sqrt{2}$	$\frac{2\sqrt{3}}{3}$	$\sqrt{6}-\sqrt{2}$	decreasing (↘)

臨界点

Cor：If $0° < \theta < 90°$. Then

1) increasing (↑)：$\sin\theta$. $\tan\theta$. $\sec\theta$

2) decreasing (↓)：$\cos\theta$. $\cot\theta$. $\csc\theta$.

3) $45°$是臨界点.

(i) $0° < \theta < 45°$ $\cos\theta > \sin\theta$. $\cot\theta > \tan\theta$. $\csc\theta > \sec\theta$

(ii) $45° < \theta < 90°$ $\cos\theta < \sin\theta$. $\cot\theta < \tan\theta$. $\csc\theta < \sec\theta$.

S3. $0^\circ < \theta < 90^\circ$. $\underline{\tan\theta = 1}$. $\theta = 45^\circ$.

<u>sol</u>: $\tan\theta = \dfrac{\sin\theta}{\cos\theta} = 1 \Longleftrightarrow \sin\theta = \cos\theta \Longleftrightarrow \theta = 45^\circ$.

$\therefore \sin\theta + \cos\theta = \dfrac{\sqrt{2}}{2} + \dfrac{\sqrt{2}}{2} = \sqrt{2}$ //

<u>p.81</u>

Ex4. $\pi = 180^\circ$.

$\sin^2\frac{\pi}{6} + \cos^2\frac{\pi}{4} + \tan^2\frac{\pi}{3} + \tan^2\frac{\pi}{4} + \cos^2\frac{\pi}{3}$

<u>sol</u>: $\sin^2 30^\circ + \cos^2 45^\circ + \tan^2 60^\circ + \tan^2 45^\circ + \cos^2 60^\circ$

$= \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2 + (\sqrt{3})^2 + 1^2 + \left(\frac{1}{2}\right)^2$

$= \frac{1}{4} + \frac{2}{4} + 3 + 1 + \frac{1}{4} = 5$ //

S4

$\left(1 + \frac{\sqrt{2}}{2} - \frac{1}{2}\right)\left(1 - \frac{1}{2} - \frac{\sqrt{2}}{2}\right)$

$= \left(\frac{1}{2} + \frac{\sqrt{2}}{2}\right)\left(\frac{1}{2} - \frac{\sqrt{2}}{2}\right)$

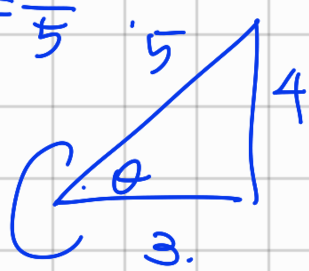
$= \left(\frac{1}{2}\right)^2 - \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{1}{4} - \frac{2}{4} = -\frac{1}{4}$ //

Ex5. (B) 反例. $\sin 30^\circ = \frac{1}{2} = 0.5$.

$\tan 30^\circ = \frac{\sqrt{3}}{3} = \frac{1.732}{3} \approx 0.57$.

Ex6. $\sin\theta = \frac{4}{5}$ $\sin\left(\frac{\pi}{2}-\theta\right)$

sol: $\sin\left(\frac{\pi}{2}-\theta\right) = \sin(90^\circ - \theta) = \cos\theta = \frac{3}{5}$



Ex7. $\frac{1-\sin^2 40^\circ}{\cos^2 50^\circ} \cdot \tan^2 40^\circ$

$$= \frac{\cancel{\cos^2 40^\circ}}{\cos^2 50^\circ} \cdot \frac{\sin^2 40^\circ}{\cancel{\cos^2 40^\circ}} = \frac{\sin^2 40^\circ}{\cos^2 50^\circ} = \frac{\sin^2 40^\circ}{\sin^2 40^\circ} = 1$$