統測數學 Test 2

計算題：每題 10 分，共 100 分

切記：計算過程比答案更重要，沒有計算過程不給分。

1. 已知$\sin\theta+\cos\theta=\sqrt{2}$，則$\tan\theta+\frac{1}{\tan\theta}=?$ 2

2. $\frac{\cos(-\theta)}{\sin(\frac{3\pi}{2}-\theta)}+\frac{\sin(\pi-\theta)}{\cos(\frac{3\pi}{2}-\theta)}=?$ -2

3. 已知$0\le\theta\le 2\pi$，$\cos\theta=-\frac{1}{2}$且$\tan\theta>0$，求$\theta=?$ $\frac{4}{3}\pi$

4. 函數$f(x)=\cos^2 x+3\sin x$，則$f(x)$的最大值為何？ 3

5. 在ΔABC中，a、b、c分別表示三邊長。已知$(a+b):(b+c):(a+c)=3:4:5$，則$\sin A:\sin B:\sin C=?$ $2:1:3$

6. 某人從 A 處測得山峰的仰角為 45°，水平前進 100 公尺至 B 點，測得山峰的仰角為 60°，則此山的高度為多少公尺？ $150+50\sqrt{3}$

7. 已知$\vec{\imath}=(1,0)$、$\vec{\jmath}=(0,1)$為單位向量，設向量$\vec{a}=3\vec{\imath}+2\vec{\jmath}$、$\vec{b}=2\vec{\imath}-p\vec{\jmath}$、$\vec{c}=q\vec{\imath}-6\vec{\jmath}$，若$\vec{a}\perp\vec{b}$，且$\vec{a}//\vec{c}$，則$p+q=?$ -6

8. 已知$\vec{\alpha}$、$\vec{\beta}$兩向量之夾角為 60°，且$|\vec{\alpha}|=1$、$|\vec{\beta}|=2$，則$|2\vec{\alpha}+\vec{\beta}|$之值為何？ $2\sqrt{3}$

9. 設$x^2+y^2+2x-ky+2=0$表一圓，求k之範圍為何？ $k<-2$ 或 $k>2$

10. 過點$P(1,2)$且與圓$(x+1)^2+(y+2)^2=20$相切之直線方程式為何？ $x+2y-5=0$
【直線方程式必須化成$ax+by+c=0$的形式，$a:b:c$為最簡單整數比且$a>0$，否則扣 3 分】

Solution:

1. $\tan\theta+\frac{1}{\tan\theta}=\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}=\frac{1}{\sin\theta\cos\theta}$

$\because 1+2\sin\theta\cos\theta=2 \quad \therefore \sin\theta\cos\theta=\frac{1}{2}$

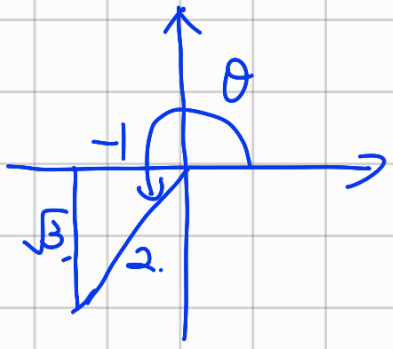
$\therefore \tan\theta+\frac{1}{\tan\theta}=\frac{1}{\frac{1}{2}}=2$ //

2. $\frac{\cos(-\theta)}{\sin(\frac{3\pi}{2}-\theta)}+\frac{\sin(\pi-\theta)}{\cos(\frac{3\pi}{2}-\theta)}=\frac{\cos\theta}{-\cos\theta}+\frac{\sin\theta}{-\sin\theta}$

$=-1-1=-2$ //

3. $0\le\theta\le 2\pi$. $\cos\theta=-\frac{1}{2}$. $\tan\theta>0$.

$\because \cos\theta<0 \wedge \tan\theta>0 \quad \therefore \theta\in$ III.



$\therefore \theta=\pi+\frac{\pi}{3}=\frac{4}{3}\pi$ //

4. $f(x)=\cos^2x+3\sin x=1-\sin^2x+3\sin x$

$=-(\sin^2x-3\sin x+\frac{9}{4})+1+\frac{9}{4}$

$=-(\sin x-\frac{3}{2})^2+\frac{13}{4}$

When $\sin x=\frac{3}{2}$. 有 $Max=\frac{13}{4}$.

$\because -1 \le \sin x \le 1$.

$\therefore$ 取 $\sin x = 1$ $(x = \frac{\pi}{2})$. $f(x) = -(-\frac{1}{2})^2 + \frac{13}{4} = 3$ 为 Max ///

5. $a+b : b+c : a+c = 3 : 4 : 5$.

$a+b = 3r$

$b+c = 4r$

$+)\ a+c = 5r$

$2a+2b+2c = 12r$.

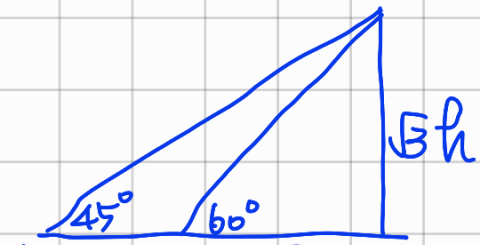
$\therefore a+b+c = 6r$.

$\therefore a : b : c = \sin A : \sin B : \sin C$

$= 2r : r : 3r$

$= 2 : 1 : 3$ ///

6,



$\tan 45° = \frac{\sqrt{3}h}{100+h}$

$\therefore 100+h = \sqrt{3}h$. $h(\sqrt{3}-1) = 100$

$\therefore h = \frac{100}{\sqrt{3}-1} \cdot \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{100\sqrt{3}+100}{2} = 50\sqrt{3}+50$.

$\therefore \overline{AB} = \sqrt{3}h = 150+50\sqrt{3}$ ///

1. $\vec{a} = 3\vec{i}+2\vec{j}$. $\vec{b} = 2\vec{i}-p\vec{j}$. $\vec{c} = 8\vec{i}-6\vec{j}$.

$\vec{a} = (3,2)$ $\vec{b} = (2,-p)$. $\vec{c} = (8,-6)$.

$\because \vec{a} \perp \vec{b}$ $\therefore \vec{a} \cdot \vec{b} = 0$. $\Rightarrow 6-2p = 6$ $\therefore p = 3$.

$\because \vec{a} \parallel \vec{c} \quad \therefore \frac{q}{3} = \frac{-6}{2} \Rightarrow 2q = -18 \quad \therefore q = -9$

$\therefore p+q = 3+(-9) = -6$ ///

8. $\theta = 60°$. $|\vec{\alpha}| = 1$. $|\vec{\beta}| = 2$.

$$|2\vec{\alpha}+\vec{\beta}|^2 = |2\vec{\alpha}|^2 + 2\cdot 2\vec{\alpha}\cdot\vec{\beta} + |\vec{\beta}|^2$$

$$= 4|\vec{\alpha}|^2 + 4|\vec{\alpha}||\vec{\beta}|\cos 60° + |\vec{\beta}|^2$$

$$= 4\cdot 1 + 4\cdot 1\cdot 2\cdot \frac{1}{2} + 2^2 = 12$$

$\therefore |2\vec{\alpha}+\vec{\beta}| = \sqrt{12} = 2\sqrt{3}$ ///

9. $x^2+y^2+2x-ky+2=0$ 表一圓.

$$(x+1)^2 + (y-\frac{k}{2})^2 = -2+1+\frac{k^2}{4} > 0.$$

$\therefore -4+k^2 > 0 \Rightarrow (k+2)(k-2) > 0$

$\therefore k < -2$, $k > 2$ ///

10. $P(1,2)$ 代入 $\Rightarrow 2^2+4^2 = 20 \Rightarrow P$ 在圓上.

$\therefore (x+1)(1+1) + (y+2)(2+2) = 20.$

$2x+2+4y+8-20 = 0$

$\therefore L: x+2y-5 = 0$ ///