CODING THEORY

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August 12, 2025

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Chapter 1

Basics

1.1 Basic Notations and Definitions

Definition 1.1.1: Code

A *code* C is a subset of Σ^n where Σ is an alphabet, where n is the block length of C. We typically use q to denote $|\Sigma|$.

Another way to view the definition of a code to be a map $C : [M] \to \Sigma^n$, where $M = |C|$.

Definition 1.1.2: Dimension of a code

Dimension of a code C, denoted as k, is defined as the following way,

$$k := \log_q |C|.$$

Remark. *Note that,*

1. *For any $C \subseteq \Sigma^n$, $k \leq n$.*

2. *k can be non-integral.*

One way to quantify *Redundancy* in a code is via its rate.

Definition 1.1.3: Rate of a Code

Rate of a code C of block length n and dimension k, denoted as R, is defined as

$$R := \frac{k}{n}.$$

Example 1.1. *Define a code $C \subseteq \{0,1\}^5$ that maps a binary string (x_1, x_2, x_3, x_4) to $(x_1, \ldots, x_4, x_1 \oplus \ldots \oplus x_4)$.*

1.2 Formalizing Error Correction

Definition 1.2.1: Encoding & Decoding Functions

- Let $C \subseteq \Sigma^n$. An equivalent description of the code C is an injective mapping $E : [|C|] \to \Sigma^n$ called the *encoding function*.
- Let $C \subseteq \Sigma^n$ be a code. A mapping $D : \Sigma^n \to [|C|]$ is called a *decoding function*.