Flow Past Objects
(Denn Chapter 4)

LEARNING OBJECTIVES

1. Relate and calculate the Drag Coefficient (c_D) to fluid and object parameters for different Reynold's number regions.
2. Derive and apply terminal (settling) velocity as a function of fluid and object parameters.
3. Derive and apply a model for fluid flow through packed bed systems.

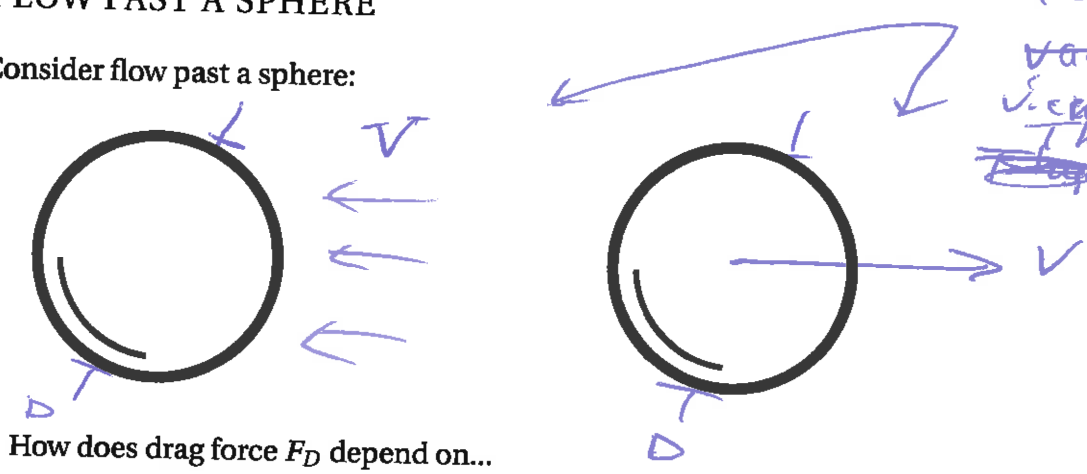
INTRODUCTION

Why do engineers care about flow past an object?

- drag force   - settling
- separations

FLOW PAST A SPHERE

Consider flow past a sphere:



consider from ~~vant~~ point of view of object There ~~there~~ are the same!

How does drag force F_D depend on...

V, D, ρ_{fluid}, η

DIMENSIONAL ANALYSIS

Variable	Dimensions (Length, Time, Mass)
F_D	$\frac{ML}{T^2}$
v	L/T
D	L
ρ	M/L^3
η	$M/L \cdot T$

Π theorem:

$5 - 3 = 2$ groups

Many options; conventions dictate:

$$Re = \frac{Dv\rho}{\eta}$$

$$c_D = \frac{8}{\pi} \frac{F_D}{\rho v^2 D^2}$$

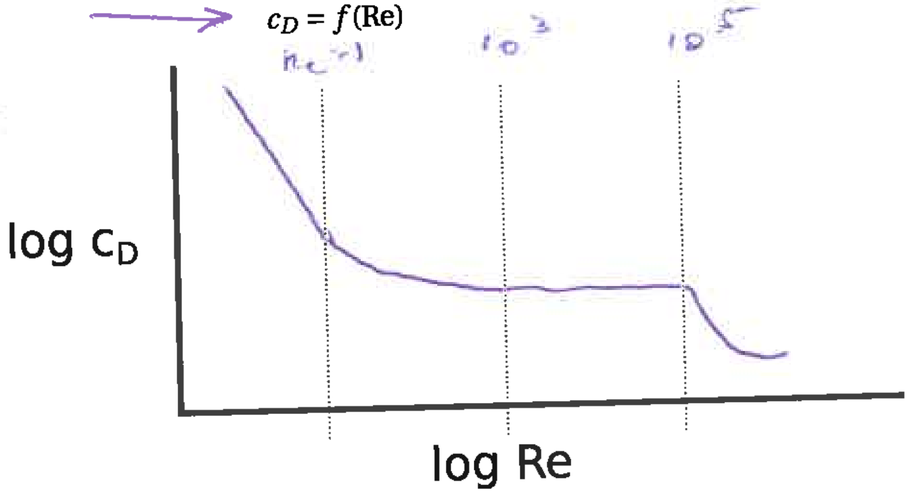
For the drag coefficient c_D, the general definition is

$$\frac{F_D}{(\frac{1}{2}\rho v^2)(\text{projected area})}$$

(for sphere, projected area $= \frac{\pi D^2}{4}$)

$\longrightarrow$ $c_D = f(\text{Re})$

Data



lots of fluids, viscosities, diameters

↓

~~one~~ groups

"dynamic ~~sy~~ similarity"

REPRESENTATION OF DATA

SMALL RE (<1) — Stokes regime

Find that $c_D \propto \frac{1}{\text{Re}}$. In fact, $\boxed{c_D = 24/Re}$

$$\frac{8}{\pi}\frac{F_D}{\rho v^2 D^2} = \frac{24\eta}{Dv\rho} \Rightarrow \boxed{F_D = 3\pi\eta D v}$$

"Stokes law" → may derived from first principles

INTERMEDIATE REGION (1<RE<10^3)

$c_D =$ 18 $Re^{-0.6}$ (curve fit)

LARGE RE (10^3<RE<2×10^5) "Newton" regime

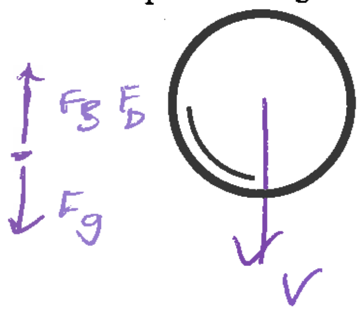
$c_D \approx$ const $= 0.44 \Rightarrow F \sim v^2$

For $\text{Re} = 2\times10^5$, sharp drop... "drag crisis"

TERMINAL VELOCITY

settling velocity

Sphere falling/rising in fluid by gravitational force.



steady state; const v $\Rightarrow$ forces balance

$$\underbrace{(\rho_s-\rho)\frac{\pi D^3}{6}g}_{\text{net gravitational force}} = \underbrace{\frac{\pi}{8}\rho v^2 D^2 C_D}_{\text{Drag force}}$$

$$\Rightarrow v^2 = \frac{4}{3}\left(\frac{\rho_s-\rho}{\rho}\right)\frac{gD}{C_D}$$

Stokes regime:

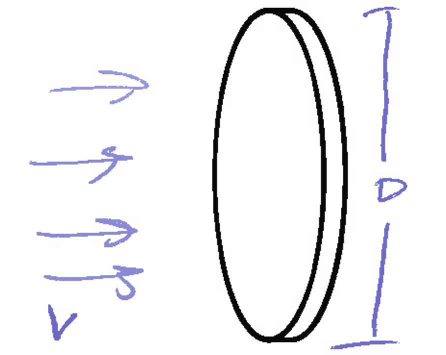
$$C_D = 24/Re \Rightarrow v = \frac{gD^2(\rho_s-\rho)}{18\eta} \qquad Re < 1$$

Newton regime:

$$C_D = 0.44 \Rightarrow v = \left[3Dg\left(\frac{\rho_s-\rho}{\rho}\right)\right]^{1/2}$$

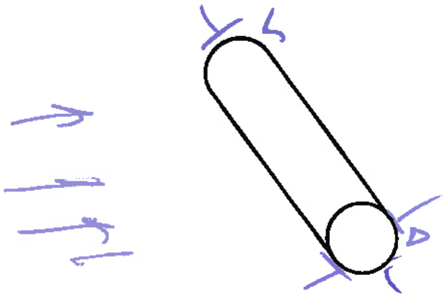
$$10^3 < Re < 10^5$$

Other Shapes



circular disk

$$C_D = \frac{F_D}{(\frac{1}{2}\rho V^2)\left(\frac{\pi D^2}{4}\right)}$$



cylinder, length L

$$C_D = \frac{F_D}{(\frac{1}{2}\rho V^2)(DL)}$$

Denn Fig. 4-8 ... for cylinders at low Re:

$$C_D = \frac{8\pi}{Re} \frac{1}{\quad}$$

Handouts

Table 9.4 from Munson, Young, and Okiishi. Fundamentals of Fluid Mechanics, 5th edition.

Comments

- for 3D objects, always take form $C_D \sim \frac{c}{Re}$ as $Re \to 0$ — not true for 2D!
- ~~commonplace~~ usage of C_D as const → Newton regime (e.g. cylinder
- role of streamlining

Flow Patterns/Discussion — DVD-ROM

- symmetry at low Re
- recirculating wakes as Re increases ("separation
- Karman vortex sheet for cylinders
- drag crisis

EXAMPLE

How much energy does the 1992 version of Professor Burghardt expend to overcome aerodynamic drag while running a complete marathon race on a day with no wind? Assume that he completed a marathon in 3 hours.

Assumptions

(i) Newton's regime, const C_D

(ii) $C_D A = 9 \text{ ft}^2 = 0.836 \text{ m}^2$

$\rho_{air} = 1.2 \text{ kg/m}^3$; $\mu = 1.8 \times 10^{-5} \text{ Pa·s}$

constant velocity

$$\frac{26.2 \text{ miles}}{3 \text{ hours}} \cdot \frac{1 \text{ hr}}{3600 \text{ s}} \cdot \frac{5280 \text{ ft}}{1 \text{ mile}} \cdot \frac{0.3048 \text{ m}}{1 \text{ ft}}$$

$$V = 3.9 \text{ m/s}$$

$$\text{Drag Force} = C_D\left(\tfrac{1}{2}\rho v^2\right)(A) = \tfrac{1}{2}(1.2)(3.9)^2(.836)$$

$$= 7.63 \text{ N}$$

$$\text{Energy} = F_D \cdot \text{distance}$$

$$26.2 \text{ miles} = 42{,}165 \text{ m}$$

$$\text{Energy} = (7.63 \text{ N})(42{,}165)$$

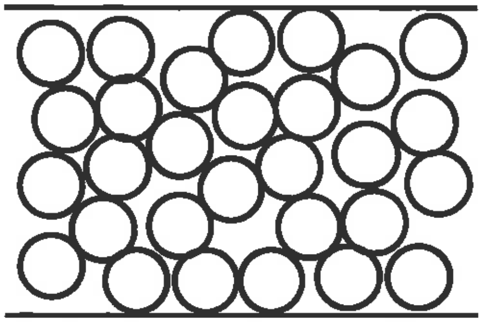
$$= \boxed{3.22 \times 10^5 \text{ J}}$$

Now suppose that Professor Burghardt had used that energy instead to power a light bulb (100 watts). How long could he generate light for a family in India who doesn't have access to electricity?

$$\text{Time} = \frac{\text{Energy}}{\text{Power}} = \frac{322{,}000 \text{ J}}{100 \text{ J/s}} = 3220 \text{ s} =$$

$$0.89 \text{ hrs}$$

PACKED BEDS



D_p - particle diameter
A - cross sectional area
L - length
Q - vol. flow rate

Uses:

- adsorbant columns (adsorbant particles)
- packed bed reactor (catalyst particles)
- gas/liquid contacting for mass transfer (2-phase; more complex)

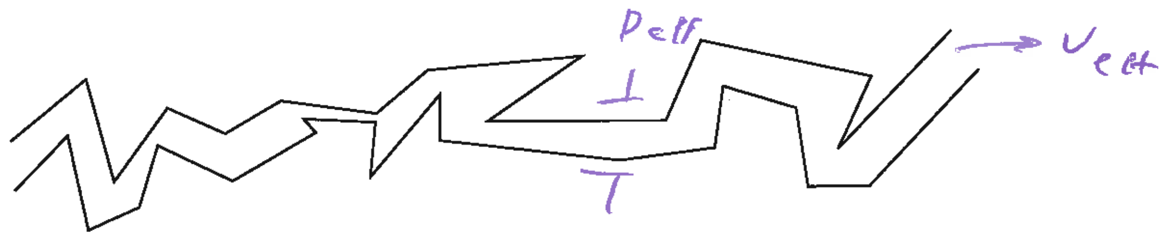
"Void Fraction" = ε = $\frac{\text{vol. empty space}}{\text{Total volume}}$

"Superficial velocity" = v_∞ = Q/A (if packing weren't there)

How does Δp depend on...

$v_\infty, L, D_p, \varepsilon, \rho, \mu$?

Approach: Think of fluid's path through packing as a complicated "pipe"



For pipe flow, correlate data:

$$\frac{\Delta p}{\rho v^2}\,\frac{D}{L} = f\left(\frac{D v \rho}{\eta}\right)$$

turn into $\longrightarrow$ $$\frac{\Delta p}{\rho v_{eff}^2}\,\frac{D_{eff}}{L} = f\left(\frac{D_{eff}\, v_{eff}\, \rho}{\eta}\right)$$

Problem: How do D_{eff} and v_{eff} relate to measurable variables?

1. Actual cross sectional area available for flow is εA

$$\boxed{v_{eff} = \frac{Q}{\varepsilon A} = \frac{V_\infty}{\varepsilon}}$$

2. Set $D_{eff} = D_H$ (hydraulic diameter)

$$= \frac{4 \cdot \text{fluid volume}}{\text{wetted surface area}}$$

particle not spherical? let $D_p = 6 \cdot \frac{\text{volume}}{\text{area}}$

V_{fluid} - Volume of the fluid $\quad$ V_{solids} - Volume of the solid particles

V_{bed} - Volume of the bed ($V_{bed} = V_{fluid} + V_{solids}$)

$$V_{fluid} = \varepsilon V_{bed} \qquad \cancel{V_{solids}}\; V_{solids} = (1-\varepsilon) V_b$$

$$V_{fluid} = \frac{\varepsilon}{1-\varepsilon} V_{solids}$$

Let N_p be the number of particles in the bed

$$V_{solids} = N_p \frac{\pi D_p^3}{6}$$

surface area $= N_p \pi D_p^2$

$$V_{fluid} = \left(\frac{\varepsilon}{1-\varepsilon}\right) N_p \frac{\pi D_p^3}{6}$$

$$\therefore D_{eff} = \frac{4 \cdot \left(\frac{\varepsilon}{1-\varepsilon}\right) \cancel{N_p} \frac{\cancel{\pi} D_p^{\cancel{3}}}{6}}{\cancel{N_p}\, \cancel{\pi}\, \cancel{D_p^2}}$$

$$\boxed{D_{eff} = \frac{2}{3} D_p \frac{\varepsilon}{1-\varepsilon}}$$

Plug back into the correlation:

$$\frac{2}{3}\frac{\Delta p\, D_p}{\rho v_\infty^2 L}\frac{\varepsilon^3}{1-\varepsilon} = f\left(\frac{2}{3}\frac{D_p v_\infty \rho}{\eta(1-\varepsilon)}\right)$$

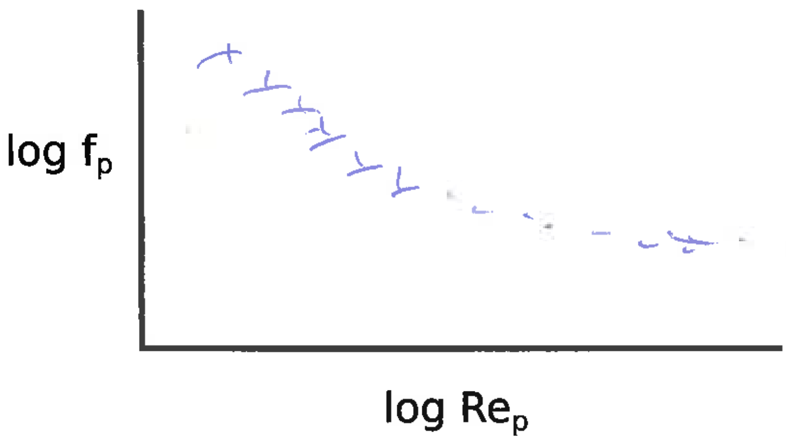
Normally drop numerical factors, and represent data according to:

$$f_p = f_p(Re_p)$$

f_p = packed bed friction factor = $\dfrac{D_p \varepsilon^3 \Delta p}{\rho v_\infty^2 (1-\varepsilon) L}$

Re_p = packed bed reynolds # = $\dfrac{D_p v_\infty \rho}{\eta(1-\varepsilon)}$

Experimental Data



Data correlation. Ergun Equation: $f_p = \dfrac{150}{Re_p} + 1.75$

note: no drag, low Re

For low Re_p, we can neglect the constant term

(slow, small pores, viscous)

$$f_p \sim 150/Re_p$$

$$\frac{\Delta p}{L}\frac{D_p \varepsilon^3}{\rho v_\infty^2 (1-\varepsilon)} = \frac{150\,\eta(1-\varepsilon)}{D_p v_\infty \cancel{\rho}}$$

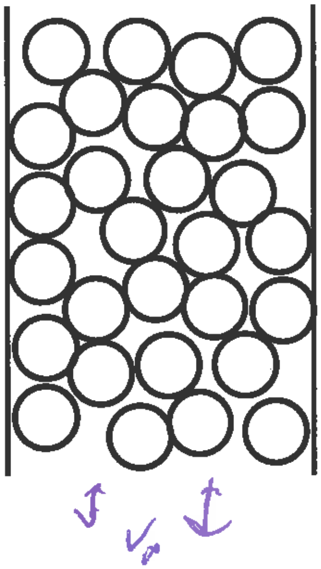
no ρ

$$\rightarrow v_\infty = \frac{\kappa}{\eta}\frac{\Delta p}{L} \qquad \kappa = \frac{D_p^2 \varepsilon^3}{150(1-\varepsilon)^2}$$

"D'arcy's law" → commonly observed for flow in porous media

κ: "permeability"

FLUIDIZED BEDS



vertical flow in packed bed

For sufficiently large v_∞, flow suspends particles, which bounce around chaotically

- excellent mixing
- improved heat transfer

CHAPTER SUMMARY

In this chapter we consider flow past objects in technically relevant areas for engineers

- drag force
- settling
- packed beds

We developed a model to relate drag force F_D to fluid and object parameters. In particular, we use the dimensionless group c_D against Reynold's number (Re).

- using many tables & correlations, can perform drag calculations on many different topologies

- terminal (settling) velocity occurs when gravitational and other forces balance

- packed bed systems
  - use pipe flow correlation
    - find v_{eff}, D_{eff} based on spacing and particle parameters