Pipe Flow
(Denn Chapter 3)

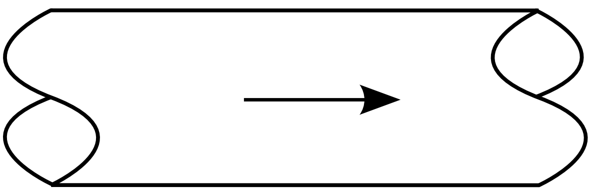
LEARNING OBJECTIVES

1. Apply dimensional analysis with physical insight and experimental data to find design equations for pipe flow.
2. Describe the laminar and turbulent regimes of fluid pipe flow both qualitatively and quantitatively.
3. Interpret the numerator and denominator of Reynold's number physically and in relation to laminar and turbulent flow.
4. Derive Poiseuille's law using dimensional analysis and physical insight.
5. Evaluate quantitatively power consumption and cost tradeoffs when designing pipe flow parameters.

DIMENSIONAL ANALYSIS

PIPE FLOW

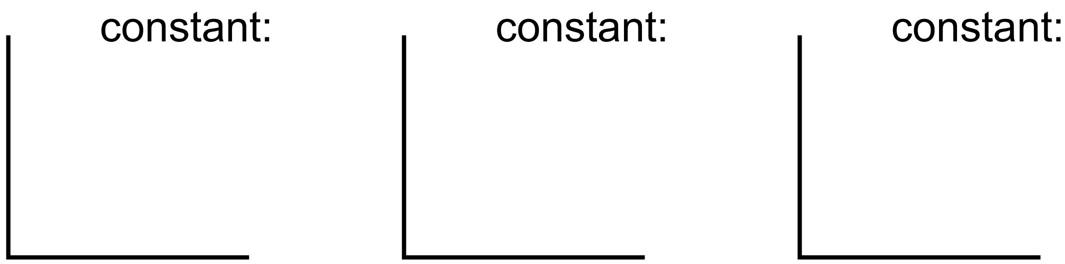
We wish to measure the pressure drop for a fluid through a pipe. The fluid is incompressible and Newtonian. The flow is fully developed. What are potential variables involved?



How do we figure out how these variables fit into a pipe flow model?

$$\Delta p = f($$

Traditional Answer:



The last experiment doesn't seem particularly feasible. In addition, we'd have to do A LOT of experiments. With six variables where we hold four variables constant for each experiment, we'd have to do ...

INTRODUCING DIMENSIONAL ANALYSIS

Dimensional analysis, along with Buckingham π theorem, allows us combine variables into dimensionless groups that characterize the process. These dimensionless groups serve as the main parameters describing our model.

Buckingham π theorem: $G = V - D$

V

D

G

Why?

Applying π theorem to our above pipe flow example, we have

$$G = V - D = 6 - 3 = \boxed{3}$$

So if we don't have a model for pipe flow, how do we find our 3 dimensionless groups?

Variables Involved?

Variable	Dimensions (Length, Time, Mass)
Δp	$\frac{M}{LT^2}$

Messing around:

We need something with Δp

Lots of groups; many more than 3! What's going on?

Which 3 should we pick?

1.

2. We wish to see how Δp depends on the other variables, so let's select only <u>one</u> group with that.

3.

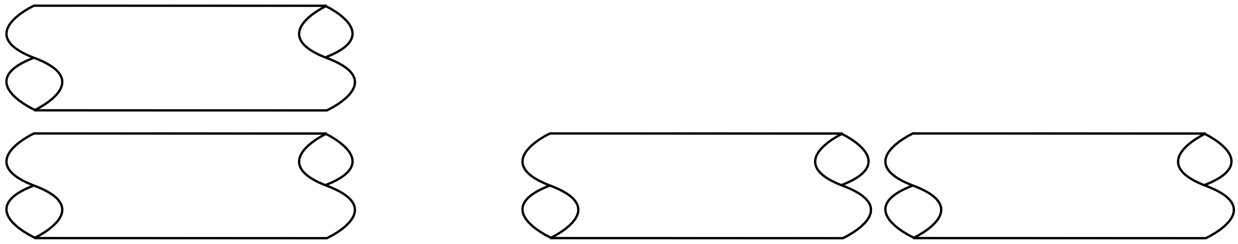
Dimensional Analysis tells us that we can go from

$$\Delta p = f($$

$$\Downarrow$$

USING PHYSICAL INSIGHT

We found appropriate dimensionless groups through dimensional analysis, now using physical insight, we can simplify further. Consider two sections of pipes.



This insight allows us to characterize our model further

And now we can define the Fanning friction factor (f) and Reynold's number (Re).

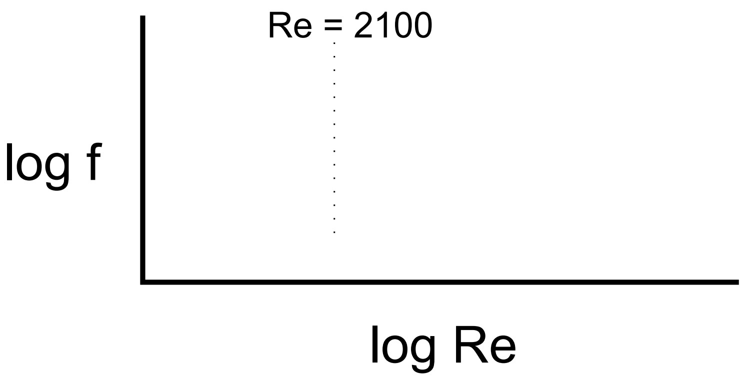
f =

Re =

$$\boxed{\mathrm{f} = f(\mathrm{Re})}$$

EXPERIMENTS

We were successfully able to apply dimensional analysis and physical insight to relate two parameters and perform only one experiment!



Data

- Different
- Different
- Different
- Different

"Dynamic Similarity" (DVD pp 534-540; 568-601)

REYNOLDS EXPERIMENT

(DVD pp 730-733)

Re < 2100

Laminar Flow

$v_z = v_z(r)$

$v_r = 0$

$v_\theta = 0$

Re > 2100

Turbulent Flow

$v_z = v_z(r,\theta,z,t)$

$v_r = v_r(r,\theta,z,t)$

$v_\theta = v_\theta(r,\theta,z,t)$

ANOTHER DIMENSIONAL ANALYSIS EXAMPLE

Liquid is slowly dripping out of a faucet of diameter D under the influence of gravity g. The liquid has density ρ and surface tension σ (dimensions force/length). Using dimensional analysis, determine how fluid mass M will relate to the other variables in the problem.

PIPE FLOW DATA CORRELATIONS

RE < 2100

Laminar flow →

$Q =$ $\Rightarrow v = \frac{Q}{A} =$

Now rearrange to

Multiply both by...

$=$

$f =$

RE > 2100

Turbulent flow →

EMPIRICAL EQUATIONS

Blasius Equation

$f =$

von Karman-Nikuradse correlation

$\frac{1}{\sqrt{f}} =$

2100 < RE <4000

Chart

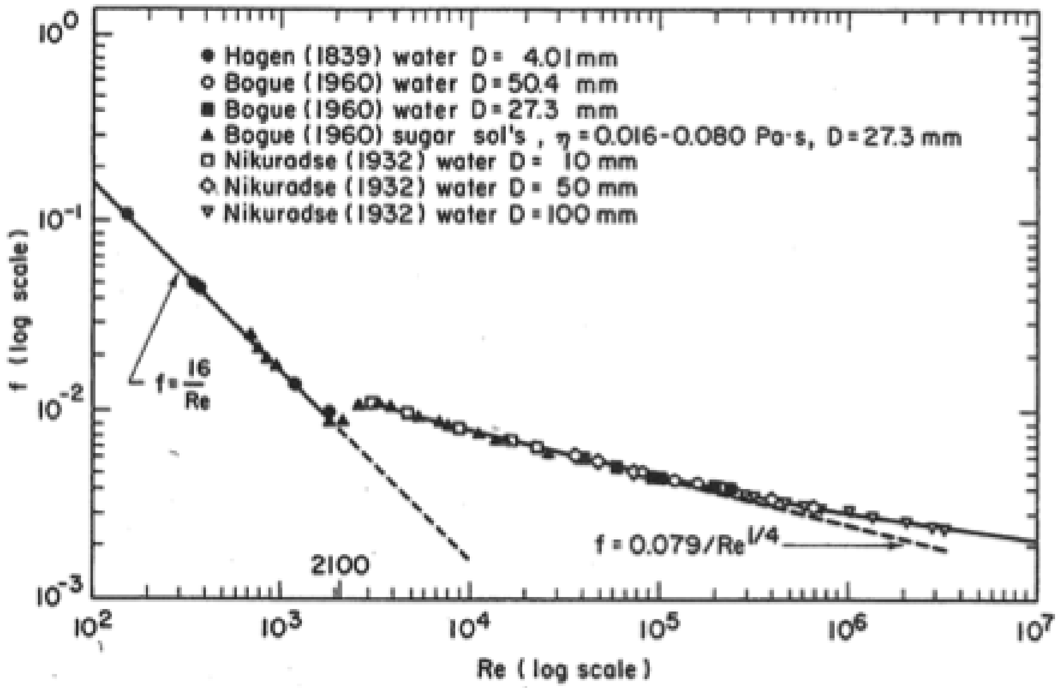


Figure 3-1. Friction factor as a function of Reynolds number for incompressible Newtonian fluids.

Example Problem

Water is pumped through 50m of a smooth pipe with an inside diameter of 5cm at a volumetric flow rate of 4 liters/second. What is the pressure drop?

Physical Interpretation of Reynold's Number

Reynold's number can be thought of the ratio between inertial forces and viscous forces. (DVD pp. 496-508).

$$\text{Re} = \frac{\text{Inertial Forces}}{\text{Viscous Forces}}$$

Inertial forces

Inertia is what must be overcome to change speed/direction of a fluid. Consider a blob of fluid impinging on a wall with mass M, moving at velocity V, with a distance Δl to the wall.

$$\text{Inertial Force} = F_I =$$

Viscous Forces

Viscous forces are associated with deformation of a fluid. Consider a blog of fluid moving parallel to the wall.

$$\text{Shear rate } \Gamma_s \approx$$

$$\text{Shear stress } \tau_s \approx$$

$$\text{Viscous Force } F_V \approx$$

RATIO

$$\frac{F_I}{F_V} =$$

REVISITING DIMENSIONAL ANALYSIS FOR LAMINAR PIPE FLOW

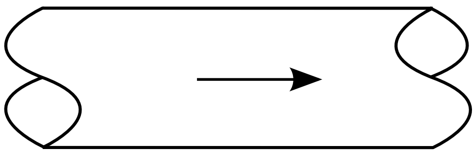
Streamlines are straight, there is constant velocity (no acceleration), so therefore ...

$$Q \propto$$

We obtained a mathematical form of Poiseuile's law without a detailed solution using only dimensional analysis and physical insight!

COROLLARY

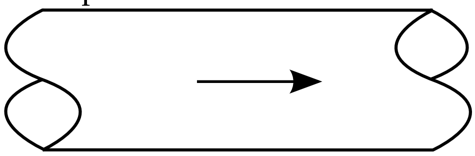
$$f =$$



Power Requirement for Pumping

Power =

For Pipe flow:



Work =

Over time Δt, the fluid moves distance Δl

Power =

Power =

Optimal Pipe Diameter

We want to pump fluid distance L with volumetric flow rate Q. What pipe diameter do we choose? How do we choose?

Total Cost = Operating Cost + Capital Costs

Capital Costs

Capital Costs =

OPERATING COSTS

Operating Costs =

TOTAL COSTS

Total Cost =

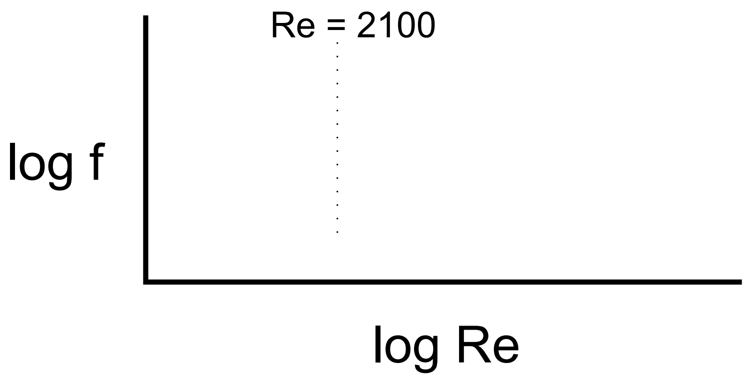
Solving for optimal diameter

Rough Pipe

Dimensional Analysis... add a new variable

$$\mathrm{f} = f($$

Experiments, Nikuradse (glue sand to pipes)



Estimating f in rough pipes: Colebrook formula

$$\frac{1}{\sqrt{\mathrm{f}}} =$$

1. Calculate
2. Calculate
3. Use the formula or chart to determine f

Commercial Pipes

Non-circular Cross Sections

For turbulent flow...

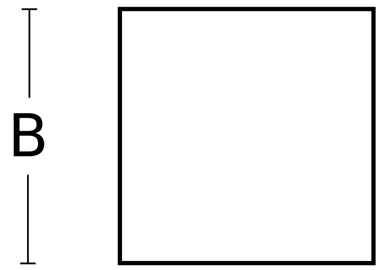
1. Define the hydraulic diameter: $D_H =$

2. Use D_H in place of D in calculating

3. Use correlations for circular tubes

Example Problem

Water is pumped through 20 m of a channel with a square cross section (see below) with B=50 mm. The channel is made from commercial steel with a surface roughness of $k = 0.05$ mm. What will be pressure drop assuming the average fluid velocity is 4 m/s. Assume $\rho = 1000 \frac{\text{kg}}{\text{m}^3}$ and $\eta = 0.001$ Pa s



Chapter Summary

Aim: To develop a fluid model for pipe flow as a function of known, involved parameters without a detailed flow solution (macroscopically).

Challenge:

Solution: Apply Dimensional Analysis and

Additionally, dimensional analysis allows us to

- Simplify the our model by reducing number of parameters (and thus experiments)
-
-

For simple pipe flow, we found two dimensionless groups to characterize flow through a pipe, Reynold's Number (Re) and Fanning Friction factor (f).

Reynold's Number reveals the following about fluid flow:

- The balance of viscous and inertial forces (laminar vs turbulent regime).
- The correlation/equation to be used with friction factor.

From experiments and derivations we can relate friction factor to Reynold's number.
Using pipe flow models we can

- Calculate the power requirement
- Find optimal sizing for our process
- Account for
- Account for

APPENDIX A

Standard Pipe Sizes

Nominal pipe size (in.)	Outside diameter (in.)	Schedule no.	Wall thickness (in.)	Inside diameter (in.)	Cross-sectional area of metal (in.^{2})	Inside sectional area (ft^2)
$\frac{1}{3}$	0.405	40	0.068	0.269	0.072	0.00040
		80	0.095	0.215	0.093	0.00025
$\frac{1}{4}$	0.540	40	0.088	0.364	0.125	0.00072
		80	0.119	0.302	0.157	0.00050
$\frac{3}{8}$	0.675	40	0.091	0.493	0.167	0.00133
		80	0.126	0.423	0.217	0.00098
$\frac{1}{2}$	0.840	40	0.109	0.622	0.250	0.00211
		80	0.147	0.546	0.320	0.00163
		160	0.187	0.466	0.384	0.00118
$\frac{3}{4}$	1.050	40	0.113	0.824	0.333	0.00371
		80	0.154	0.742	0.433	0.00300
		160	0.218	0.614	0.570	0.00206
1	1.315	40	0.133	1.049	0.494	0.00600
		80	0.179	0.957	0.639	0.00499
		160	0.250	0.815	0.837	0.00362
$1\frac{1}{2}$	1.900	40	0.145	1.610	0.799	0.01414
		80	0.200	1.500	1.068	0.01225
		160	0.281	1.338	1.429	0.00976
2	2.375	40	0.154	2.067	1.075	0.02330
		80	0.218	1.939	1.477	0.02050
		160	0.343	1.689	2.190	0.01556
$2\frac{1}{2}$	2.875	40	0.203	2.469	1.704	0.03322
		80	0.276	2.323	2.254	0.02942
		160	0.375	2.125	2.945	0.02463
3	3.500	40	0.216	3.068	2.228	0.05130
		80	0.300	2.900	3.016	0.04587
		160	0.437	2.626	4.205	0.03761

(continued)

Nominal pipe size (in.)	Outside diameter (in.)	Schedule no.	Wall thickness (in.)	Inside diameter (in.)	Cross-sectional area of metal (in.^{2})	Inside sectional area (ft^2)
4	4.500	40	0.237	4.026	3.173	0.08840
		80	0.337	3.826	4.407	0.07986
		120	0.437	3.626	5.578	0.07170
		160	0.531	3.438	6.621	0.06447
5	5.563	40	0.258	5.047	4.304	0.1390
		80	0.375	4.813	6.112	0.1263
		120	0.500	4.563	7.963	0.1136
		160	0.625	4.313	9.696	0.1015
6	6.625	40	0.280	6.065	5.584	0.2006
		80	0.432	5.761	8.405	0.1810
		120	0.562	5.501	10.71	0.1650
		160	0.718	5.189	13.32	0.1469
8	8.625	20	0.250	8.125	6.570	0.3601
		30	0.277	8.071	7.260	0.3553
		40	0.322	7.981	8.396	0.3474
		60	0.406	7.813	10.48	0.3329
		80	0.500	7.625	12.76	0.3171
		100	0.593	7.439	14.96	0.3018
		120	0.718	7.189	17.84	0.2819
		140	0.812	7.001	19.93	0.2673
		160	0.906	6.813	21.97	0.2532
10	10.75	20	0.250	10.250	8.24	0.5731
		30	0.307	10.136	10.07	0.5603
		40	0.365	10.020	11.90	0.5475
		60	0.500	9.750	16.10	0.5158
		80	0.593	9.564	18.92	0.4989
		100	0.718	9.314	22.63	0.4732
		120	0.843	9.064	26.34	0.4481
		140	1.000	8.750	30.63	0.4176
		160	1.125	8.500	34.02	0.3941
12	12.75	20	0.250	12.250	9.82	0.8185
		30	0.330	12.090	12.87	0.7972
		40	0.406	11.938	15.77	0.7773
		60	0.562	11.626	21.52	0.7372
		80	0.687	11.376	26.03	0.7058
		100	0.843	11.064	31.53	0.6677
		120	1.000	10.750	36.91	0.6303
		140	1.125	10.500	41.08	0.6013
		160	1.312	10.126	47.14	0.5592

APPENDIX B

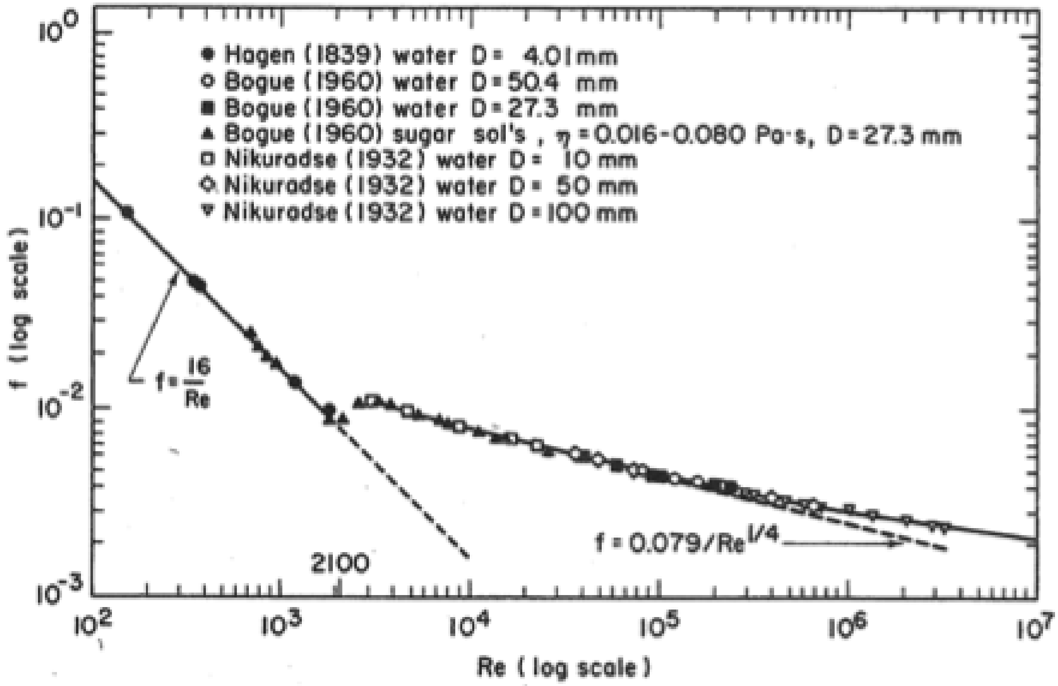


Figure 3-1. Friction factor as a function of Reynolds number for incompressible Newtonian fluids.

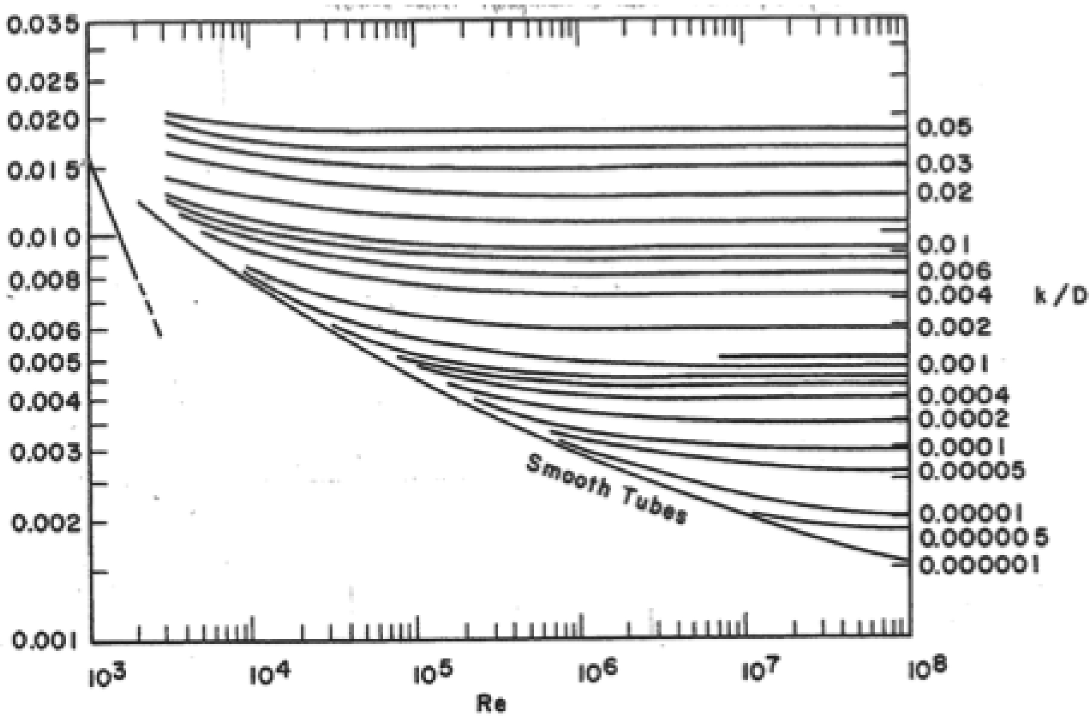


Figure 3-7. Friction factor as a function of Reynolds number for rough pipe. The lines are a graphical representation of the empirical Colebrook formula, Eq. (3.37).