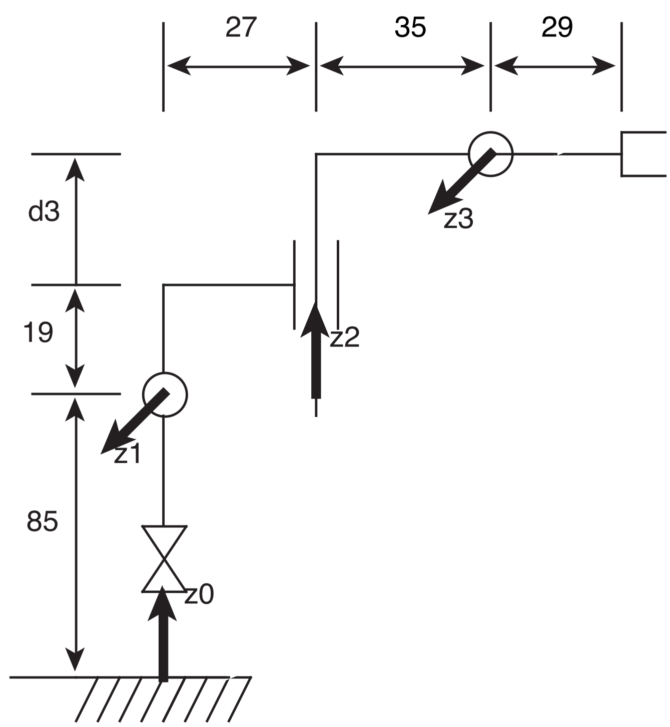
Fall 2017—16-642 Manipulation, Mobility, and Estimation
Problem Set 2

Due: 16 October 2017

GUIDELINES:

- You must *neatly* write up (preferably type up) your solutions and submit all required material electronically via canvas by the start of the lecture on the due date.
- You are encouraged to work with other students in the class, however you must turn in your own *unique* solution. If you work with others, you must list their names on your submission.
- Late Policy: If you do not turn your problem set in on time, you can turn it in up to 48 hours later but you will lose half of the points. After 48 hours, you will receive a zero.

1. (15 points) For the manipulator drawn below, draw the location of the DH frames and create a table of DH parameters. The positive direction of each joint is depicted by the z axis associated for that joint, which has conveniently been included for you. For each frame, explain whether the frame was uniquely defined by the DH convention. If it was not, describe the choices you made in defining it. (and don't forget to include the last frame!)

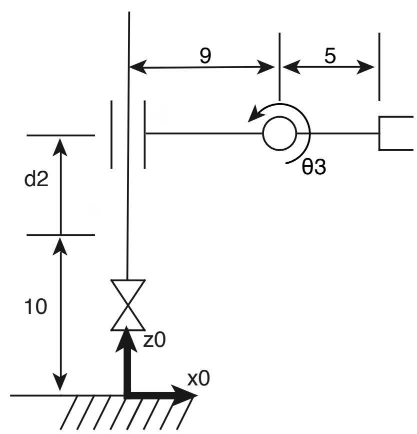


2. (10 points) Given the DH parameters in the table below, draw the manipulator that they describe. Assume that frame 0 is oriented with the z-axis pointing up, x-axis pointing right, and y-axis pointing into the page.

i	θ_i	d_i	a_i	α_i
1	θ_1	5	7	90^o
2	θ_2	0	2	-90^o
3	0	ℓ_3	0	90^o
4	θ_4	0	3	0

hint: First use the parameters to determine where all of the frames should be, then determine where the joints should go, then draw in the links connecting them.

3. Consider the manipulator drawn below in a configuration where $\theta_1 = \theta_3 = 0$ and the task space is assumed to be the position only of the end effector (i.e., $\mathbb{X} = \mathbb{R}^3$). The positive direction of the first joint is given by the z_0 axis.



- (10 points) Find the Jacobian $J(\Theta)$ using the direct differentiation method.
- (10 points) Find the Jacobian $J(\Theta)$ using the column-by-column building method. Make sure to explain your answer.
- (5 points) Are there any singular configurations? If so list them. You can use whatever method you want to find them, but make sure to explain your answer.

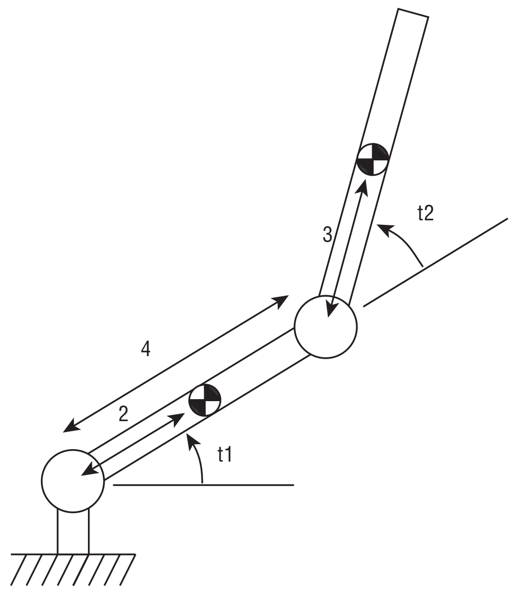
4. **tee-ball robot:** Consider using a 3 link RRR planar manipulator to hit a baseball off of a tee. The third link of the robot will be used as the bat. The ball is placed at the position $(x, y) = (2, 0)$. In order to get a perfect hit, the manipulator should strike the ball with the center of the third link, the third link should be aligned with the x axis, and the point that strikes the ball should be moving with a velocity of 10 m/s in the y direction and 0 m/s in the x direction. Each link is 1 m long, and you can assume that the width of the link and the radius of the ball are both negligible.

   Note: this problems assumes that the 2D coordinate frame is placed with its origin at the center of the first joint, with the x-axis pointing to the right and the y-axis pointing up.

   (a) (10 points) At the moment the manipulator strikes the ball, what joint angles are required for a perfect hit? Please state all possible solutions. *hint:* you might find the discussion of the inverse kinematics of the planar RR arm contained in the notes useful here.

(b) (10 points) At the moment the manipulator strikes the ball, what joint velocities are required for a perfect hit? If there are multiple solutions to part a. then state a set of joint velocities for each solution.

5. Consider the two-link planar RR manipulator depicted below, where m_1 and m_2 are the masses of the first and second links, respectively, and I_1 and I_2 are the moments of inertia of the first and second links, respectively, defined at the link center of mass. Assume that the acceleration due to gravity g acts downward, and assume that a torque can be applied to each joint. Note that in the picture below, the joint angles θ_1 and θ_2 are denoted by t_1 and t_2, respectively.



   (a) (10 points) Find the kinetic energy $K(q, \dot{q})$.
   (b) (5 points) Find the potential energy $P(q)$.
   (c) (5 points) Write the Lagrangian $L(q, \dot{q})$.

6. (10 points) A certain robotic system has generalized configuration and input variables, respectively,

$$q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \qquad \text{and} \qquad \tau = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}.$$

The kinetic energy of this system is

$$K(q, \dot{q}) = \dot{q}_1^2 + 3\dot{q}_1\dot{q}_2 + 2\dot{q}_2^2$$

and the potential energy is

$$P(q) = 10q_2.$$

Use Lagrange's method to find the equations of motion and write them in standard matrix form.