Fall 2017 – 16-642 Manipulation, Estimation, and Control
Problem Set 3

Due: 20 November 2017

GUIDELINES:

- You must *neatly* write up your solutions and submit all required material electronically via canvas by the start of the lecture on November 20th.
- You are encouraged to work with other students in the class, however you must turn in your own *unique* solution.
- Late Policy: If you do not turn your problem set in on time, you can turn it in up to 48 hours later but you will lose half of the points. After 48 hours, you will receive a zero.

1. All of the following problems refer to the following linear time invariant state space system:

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 5 & 7 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 0 & 1 & 3 \end{bmatrix} x(t).$$

   (a) (5 points) Is the system stable? Explain your answer.

   (b) (5 points) Is the system controllable? Explain your answer.

   (c) (5 points) Is the system observable? Explain your answer.

   (d) (5 points) Let the initial state vector be

$$x_0 = x(0) = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}.$$

   Using the MATLAB `expm` command, plot the output of the unforced system for $t \in [0, 2]$.

   (e) (5 points) Use the MATLAB `place` command to find the matrix K such that the matrix $A - bK$ contains the following set of eigenvalues:

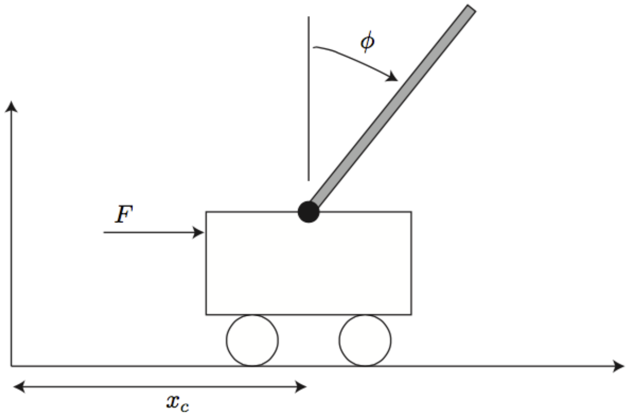
$$\{-1 + i, \, -1 - i, \, -2\}$$

   (f) (5 points) Let the initial state vector be

$$x_0 = x(0) = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}.$$

   Use the MATLAB `expm` command to plot the output of the system under the feedback law $u(t) = -Kx(t)$ for $t \in [0, 10]$. Use the K found in the previous problem.

2. Consider the "pendulum on a cart" system:



The equations of motion for this system are

$$\gamma \ddot{x}_c - \beta \ddot{\phi} \cos\phi + \beta \dot{\phi}^2 \sin\phi + \mu \dot{x}_c = F$$

$$\alpha \ddot{\phi} - \beta \ddot{x}_c \cos\phi - D \sin\phi = 0,$$

where α, β, γ, D, and μ are physical constants determined by the masses of the cart and pendulum, pendulum length, and friction, and F is an externally applied force.

(a) (10 points) Define the state vector

$$x = \begin{bmatrix} x_c \\ \phi \\ \dot{x}_c \\ \dot{\phi} \end{bmatrix}$$

and the input $u = F$. Write the cart/pendulum equations of motion as a nonlinear state space equation. *hint:* It may help to convert the system into standard mechanical form as an intermediate step, and note that the matrix

$$M = \begin{bmatrix} \gamma & -\beta \cos\phi \\ -\beta \cos\phi & \alpha \end{bmatrix}$$

is always invertible.

(b) (5 points) Describe the set of equilibrium points for the system. Describe them mathematically (i.e., with equations) and in English (i.e., what do they physically mean).

(c) (10 points) Let $\gamma = 2$, $\alpha = 1$, $\beta = 1$, $D = 1$, and $\mu = 3$. The linearized system about the equilibrium point at $x = 0$ is

$$\dot{x}(t) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & -3 & 0 \\ 0 & 2 & -3 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} u(t).$$

Assume that the entire state can be measured directly (i.e., there is no need for an observer). Letting $R = 10$ and

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix},$$

use the MATLAB lqr command to find the corresponding optimal feedback control $u(t) = -K_c x(t)$. Using a timestep of $T = 0.01$ seconds and a final time of $t_f = 30$ seconds and in intial state of

$x_0 = [0,\ 0.1,\ 0,\ 0]^T$, calculate and plot the state of the *linearized system* under the feedback control law above. You may either write your own 4th order Runge-Kutta routine to solve the state equation, or use the MATLAB function `ode45`. Repeat for $x_0 = [0,\ 0.5,\ 0,\ 0]^T$, $x_0 = [0\,1.0886\,0\,0]^T$, and $x_0 = [0,\ 1.1,\ 0,\ 0]^T$.

(d) (5 points) repeat part 2c using the full nonlinear state equations in the simulation (as opposed to the linearized state equations). Explain any differences you see in the results.

3. Given an open-loop transfer function $G(\mathrm{s}) = \frac{200}{\mathrm{s}^3+22\mathrm{s}^2+141\mathrm{s}+2}$, answer the following questions (4 points each):

   (a) (5 points) Determine the closed-loop transfer function $T(\mathrm{s}) = \frac{Y(\mathrm{s})}{R(\mathrm{s})}$ with unity negative feedback.

   (b) (5 points) Determine poles and zeros of $T(\mathrm{s})$.

   (c) (5 points) Plot $y(t)$ using MATLAB's $step$ function and discuss which poles of $T(\mathrm{s})$ dominate the response and why?

   (d) (5 points) Find the steady state value using Final Value Theorem.

4. (20 points) Implement a PID controller in MATLAB and use it to control the plant with transfer function

$$G(s) = \frac{s+10}{s^4+71s^3+1070s^2+1000s}.$$

Your design objectives are to have a rise time of 0.5 seconds, a maximum percent overshoot of less than 5%, and a steady state error of zero. Tune the gains manually to achieve these objectives. List the final gains you choose and provide a plot of the resulting closed loop step response.