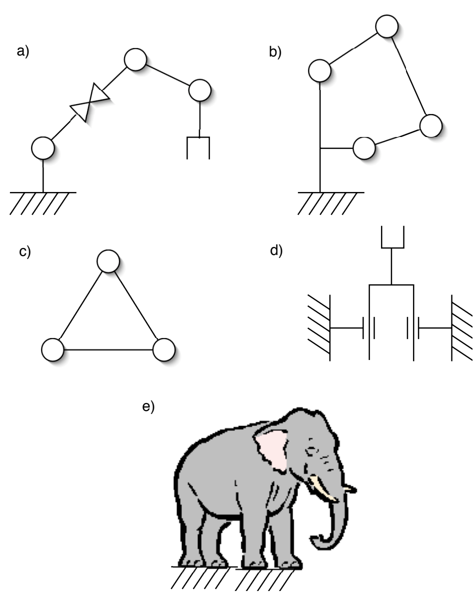
Fall 2017—16-642 Manipulation, Estimation, and Control
Problem Set 1

Due: 20 September 2017

GUIDELINES:

- You must *neatly* write up (preferably type up) your solutions and submit all required material electronically via Canvas by the start of the lecture on the due date.
- You are encouraged to work with other students in the class, however you must turn in your own *unique* solution. If you work with others, you must list their names on your submission.
- Late Policy: If you do not turn your problem set in on time, you can turn it in up to 48 hours later but you will lose half of the points. After 48 hours, you will receive a zero.

1. (15 points) Refer to the figures below. For each mechanism, give the number of DOFs. Assume each link is rigid unless otherwise specified. Assume that mechanisms are free to move in 3-space unless they are depicted as grounded. Grippers do not add any extra degrees of freedom. Assume the elephant is rigid, except for the trunk which may take on the shape of any smooth curve of the correct length.

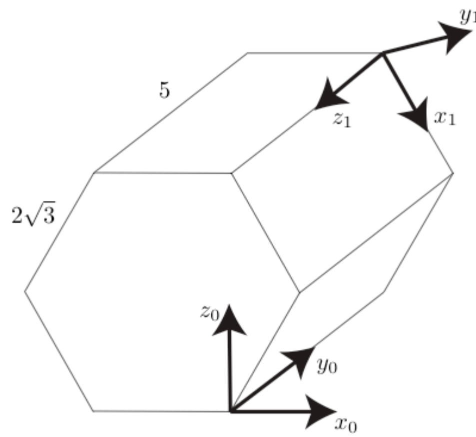


2. (10 points) Consider the homogeneous transformation matrix

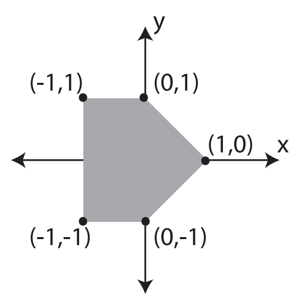
$$H_1^0 = \begin{bmatrix} & R_1^0 & & d_1^0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Express H_0^1 in terms of R_1^0 and d_1^0.

3. (10 points) Consider the 3D shape pictured below. Find H_1^0. The front and back faces are regular hexagons, the length of each side is $2\sqrt{3}$. The six side faces are all identical rectangles that have two sides with length $2\sqrt{3}$ and two sides with length 5. The interior angles of a regular hexagon are 120^o.



4. (15 points) **Fun With SE(2):** Consider the planar rigid body defined below. It is possible to "move" this body by transforming all of the corner points according to a 2D homogeneous transformation matrix and plotting the result.



Consider the two transformation matrices:

$$A = \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 0.866 & 0.500 & 0 \\ -0.500 & 0.866 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Move the rigid body according to the motions described below and plot the results. (It's probably best to write some sort of program to do this – we recommend using MATLAB, but you can use any language you want. Please include the source file of your program with your solution.)

(a) A, relative to the fixed frame.

(b) A, relative to the fixed frame, followed by B, relative to the current frame.

(c) A, relative to the fixed frame, followed by B, relative to the fixed frame.

(d) B, relative to the fixed frame.

(e) B, relative to the fixed frame, followed by A, relative to the fixed frame.

(f) B, relative to the fixed frame, followed by A, relative to the current frame.

5. (10 points) A multipurpose robot arm will be picking up a screwdriver and then trying to use it to reach a screw. Sensors on the robot's wrist will determine the pose of the tip screwdriver in the wrist's frame H_t^w. A camera mounted on the ceiling will determine the pose of the slot in the top of a screw in the camera's frame H_s^c. The location of the camera in the robot arm base frame is a known quantity H_c^b. The robot arm's wrist postition can be programmed directly by specifying the homogeneous transform that describes the desired wrist postition in the base frame H_w^b. What should H_w^b be for the the tip of the screwdriver to meet the slot in the screw? (assume that the frames on the screwdriver and screw are defined so that they are colocated when the tip of the screwdriver is inserted in the slot).

6. (15 points) Pittsburgh is at latitude 40.5^oN, longitude 80^oW. Greenwich, England is at latitude 51^oN, longitude 0^o. Both cities have frames attached to them that have the x axis pointing North and the y axis pointing East. Assume the Earth is a perfect sphere with radius 6,000 km. Find $H_{\text{Greenwich}}^{\text{Pittsburgh}}$.

   hint: It may be easiest to solve this problem using a sequence of motions, the first of which takes you to the center of the Earth.

7. (15 points) Given a quaternion $Q = [q_0, q_1, q_2, q_3]^T$, derive the rotation matrix R that corresponds to the rotation represented by Q.

8. (10 points) Consider the 3-link "spherical wrist" manipulator drawn below in the configuration where all of the joint angles are zero. The frame $\{0\}$ is rigidly attached to the ground, the frame $\{1\}$ is rigidly attached to the last link. Derive the rotation matrix R_1^0 as a function of θ_1, θ_2, and θ_3 that describes the orientation between $\{0\}$ and $\{1\}$ when the joint variables are not zero. Does this seem familiar?