theoretical exercise 8

Pattern Recognition (2018)

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Exercise T-8.1: MAP parameter estimation

Problem

In this exercise, we consider an image region with constant but unknown gray value c. This constant c is assumed to be a normally distributed random variable with zero mean and variance σ_c^2.

The region is observed by a sensor in the presence of Gaussian noise e, which has zero mean and variance σ_c^2. These n *measured* pixel intensities, which constitute the training corpus, are given by

$$\{x_k \quad | \quad x_k = c + e_k, \quad k = 1, ..., n\}$$

Use MAP parameter estimation to determine the constant gray value $\hat{c}$ of the noisy image region. Compare your result **theoretically and practically** (Octave experiments) with the Maximum-likelihood solution and discuss the differences. Which influence does the number of training samples have?

Hint: The mean of the a-posteriori distribution $p(x_k|c)$ is c since the random variable e has the mean 0.

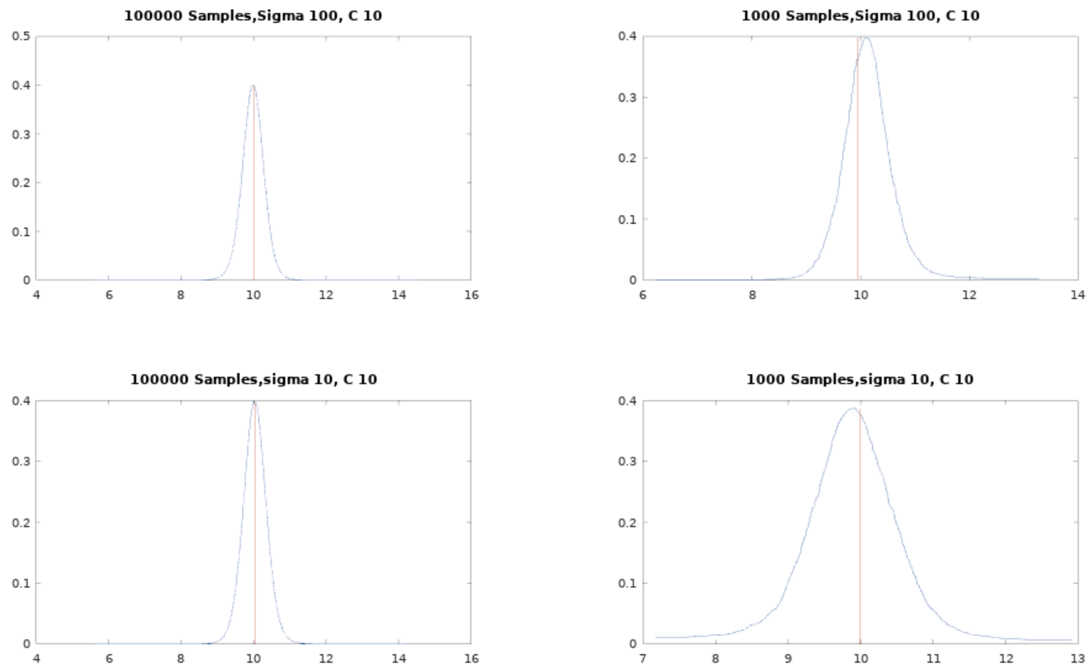

Figure 1: Distributions with different sample sizes and variance.

Solution

$$
\begin{aligned}
p(c)_{MAP} &= \arg\max_c (p(c) \cdot \prod_{i=1}^{k} p(c+e|c)) \\
&\Rightarrow \arg\max_c \log p(c) + \sum_{i=1}^{k} p(c+e_i|c) \\
&\Rightarrow \arg\max_c \log p(c) + \sum_{i=1}^{k} c + p(e_i)
\end{aligned}
$$

With k becoming very large, it gets irrelevant and can be ignored.

$$
\begin{aligned}
&\arg\max_c \sum_{i=1}^{k} c + p(e_i) \\
&p(e) = \frac{1}{\sqrt{2\pi\sigma}} \cdot \exp -\frac{1}{2}(\frac{e_i-\mu}{\sigma})^2
\end{aligned}
$$