theoretical exercise 4

Pattern Recognition (2018)

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Exercise T-3.1

Consider a two-category classification problem and show that - in a specific case - the decision boundary for a MAP classifier is given by setting the log-likelihood ratio to zero. What is the special condition required in that case?

Solution:

As of the formula for $g(x)$:

$$g(x) = ln\frac{P(x|\omega_1)}{P(x|\omega_2)} + ln\frac{P(\omega_1)}{P(\omega_2)} \tag{1}$$

The log-likelihood-ratio can be 0 when the ratio of the two possibilities is 1. This is only the case, when $P(\omega_1) = P(\omega_2)$.

Exercise T-3.2

Problem

We consider a two-category (ω_1, ω_2) two-dimensional (x_1, x_2) classification problem. Assume that the given 4 data points for each class

$$\begin{aligned} \omega_1 : &\quad \{(3,8),(2,6),(3,4),(4,6)\} \\ \omega_2 : &\quad \{(3,0),(3,-4),(1,-2),(5,-2)\} \end{aligned}$$

are normally distributed and that the priors of both classes are equal.

Compute the decision boundary and specify it as a function of x_1 , i.e. $x_2 = f(x_1)$. Illustrate the boundary together with the two point clouds in an appropriate diagram.

It is not allowed to use a computer (Octave, Matlab, ...) to solve this task.

Solution

We could not find a mathematically calculated formula for a function $x_2 = g(x_1)$.
As the priors of the classes are equal and therefore have the same prior propability (0.5), we assumed that the decision boundary would be a horizontal line at $x_2 = 2$ as this would be the mean between the means of the two pointclouds ω_1 and ω_2 which both are *symmetric* as shown in figure 1. Intuitively, for every x_1, the equation $p(x|\omega_1) \cdot p(\omega_1) = p(x|\omega_2) \cdot p(\omega_2)$ would be fulfilled on $x_2 = 2$.

The means of ω_1 and ω_2 are:

$$\mu_1 = \begin{pmatrix} 3 \\ 6 \end{pmatrix}$$

$$\mu_2 = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$$

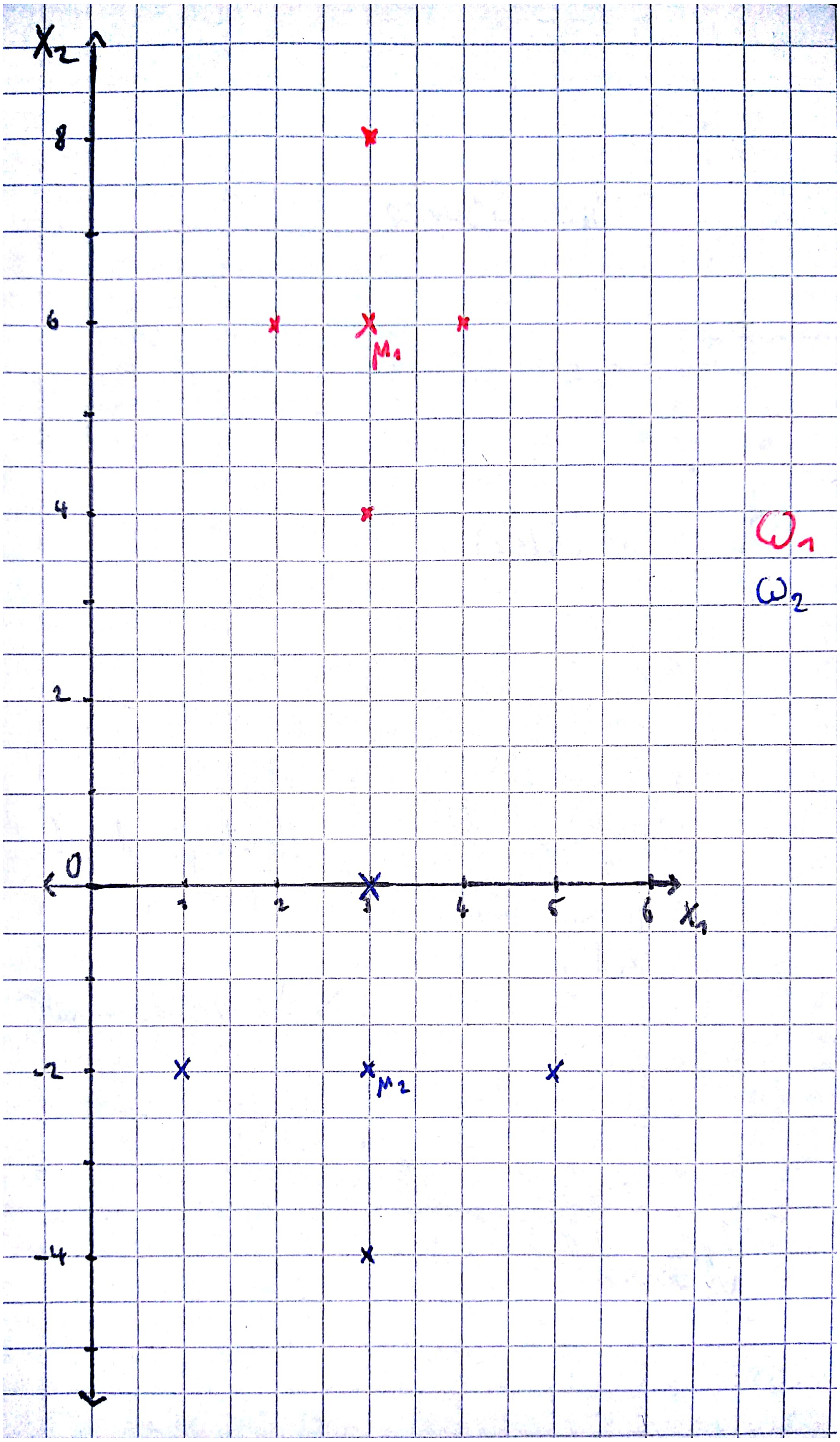

Figure 1: text