theoretical exercise 8

Pattern Recognition (2018)

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Exercise T-8.1: MAP parameter estimation

Problem

In this exercise, we consider an image region with constant but unknown gray value c. This constant c is assumed to be a normally distributed random variable with zero mean and variance σ_c^2.

The region is observed by a sensor in the presence of Gaussian noise e, which has zero mean and variance σ_c^2. These n *measured* pixel intensities, which constitute the training corpus, are given by

$$\{x_k \quad | \quad x_k = c + e_k, \quad k = 1, ..., n\}$$

Use MAP parameter estimation to determine the constant gray value $\hat{c}$ of the noisy image region. Compare your result **theoretically and practically** (Octave experiments) with the Maximum-likelihood solution and discuss the differences. Which influence does the number of training samples have?

Hint: The mean of the a-posteriori distribution $p(x_k|c)$ is c since the random variable e has the mean 0.

Solution

$$p(c|c+e) = \frac{p(c) \cdot P(c+e|c)}{P(e)} \Rightarrow \log(p(c)) + \log(p(c+e|e))$$

The probability of e can be ignored as it is a constant.