theoretical exercise 2

Pattern Recognition (2018)

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Exercise T-2.1

Problem

Consider a sample space X comprising three possible outcomes $X = v_1, v_2, v_3$.

We define the events

$$E = \{v_1, v_2\}$$
$$F = \{v_1, v_3\}$$

and denote by E^c the complement of E.

Compute $P(F|E^c)$, the conditional probability of F given E^c , using the conditional probability formula for the events A and B:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Solution

$$
\begin{aligned}
P(F|E^c) &= \frac{P(F \cap E^c)}{P(E^c)} \\
&= \frac{P(F) - P(F \cap E)}{1 - P(E)} \\
&= \frac{P(\{v_1\} \cup \{v_3\}) - P((\{v_1\} \cup \{v_3\}) \cap (\{v_1\} \cup \{v_2\}))}{1 - P(\{v_1\} \cup \{v_2\})} \\
&= \frac{P(\{v_1\} \cup \{v_3\}) - P(\{v_2\} \cap \{v_3\})}{1 - P(\{v_1\} \cup \{v_2\})} \\
&= \frac{(P(v_1) + P(v_3)) - P(v_2) \cdot P(v_3)}{1 - (P(v_1) + P(v_2))}
\end{aligned}
$$

Exercise T-2.2

Problem

The Census Bureau has estimated the following survival probabilities for men:

1. probability that a man lives at least 70 years: 80 %
2. probability that a man lives at least 80 years: 50 %

What is the conditional probability that a man lives at least 80 years given that he has just celebrated his 70th birthday?

Solution

Given probabilities:

$$
\begin{aligned}
p(70) &= 0.8 : \text{(Chance a man lives at least 70 years)} \\
p(80) &= 0.5 : \text{(Chance a man lives at least 80 years)}
\end{aligned}
$$

Calculation of $p(80|70)$:

$$
\begin{aligned}
p(70|80) &= 1 \text{ (no man can live 80 years if he died before living 70 years)} \\
p(80|70) &= \frac{p(70|80) \cdot p(80)}{p(70)} = \frac{1 \cdot 0.5}{0.8} = 0.625
\end{aligned}
$$

Solution: The conditional probability that a man lives at least 80 years given that he has just celebrated his 70th birthday is at 62.5%.