Pattern Recognition (2018)

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Exercise T-7.1

Problem

You are given the exponential distribution

$$fx(x|\Theta) = fX(x|\lambda) = \lambda_e^{\lambda_x}$$

defined for $x > 0$ and $\lambda > 0$. Mathematically determine the Maximum-likelihood solution for parameter λ if you are given a set of training samples $D = x1, x2, ..., xn$

Solution

$$L(\overrightarrow{x}, \lambda) = \prod_{i=1}^{n} f(x_i|\lambda) = \prod_{i=1}^{n} \lambda e^{-\lambda x_i} = \lambda^n e^{-\lambda \sum_{i=1}^{n} x_i} \tag{1}$$

log-likelihood function:

$$l(\overrightarrow{x}, \lambda) = ln(L(\overrightarrow{x}, \lambda)) = nln(\lambda) - \lambda \sum_{i=1}^{n} x_i \tag{2}$$

derivative of l

$$\frac{\sigma}{\sigma\lambda} l(\overrightarrow{x}, \lambda) = \frac{n}{\lambda} - \sum_{i=1}^{n} x_i \neq 0 \Rightarrow \hat{\lambda} = \frac{n}{\sum_{i=1}^{n}} x_i \tag{3}$$