

## theoretical exercise 2

# Pattern Recognition (2018)

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### Exercise T-2.1

#### Problem

Consider a sample space  $X$  comprising three possible outcomes  $X = v_1, v_2, v_3$ .

We define the events

$$E = \{v_1, v_2\}$$

$$F = \{v_1, v_3\}$$

and denote by  $E^c$  the complement of  $E$ .

Compute  $P(F|E^c)$ , the conditional probability of  $F$  given  $E^c$ , using the conditional probability formula for the events  $A$  and  $B$ :

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

### Solution

$$\begin{aligned}
 P(F|E^c) &= \frac{P(F \cap E^c)}{P(E^c)} \\
 &= \frac{P(F) - P(F \cap E)}{1 - P(E)} \\
 &= \frac{P(\{v_1\} \cup \{v_3\}) - P((\{v_1\} \cup \{v_3\}) \cap (\{v_1\} \cup \{v_2\}))}{1 - P(\{v_1\} \cup \{v_2\})} \\
 &= \frac{P(\{v_3\}) - P(\{v_2\} \cap \{v_3\})}{1 - P(\{v_1\} \cup \{v_2\})} \\
 &= \frac{P(\{v_2\} \cup \{v_3\} \setminus (\{v_2\} \cap \{v_3\}))}{1 - P(\{v_1\} \cup \{v_2\})}
 \end{aligned}$$

### Exercise T-2.2

#### Problem

The Census Bureau has estimated the following survival probabilities for men:

1. probability that a man lives at least 70 years: 80 %
2. probability that a man lives at least 80 years: 50 %

What is the conditional probability that a man lives at least 80 years given that he has just celebrated his 70th birthday?

#### Solution

Given probabilities:

$$p(70) = 0.8 : (\text{Chance a man lives at least 70 years})$$

$$p(80) = 0.5 : (\text{Chance a man lives at least 80 years})$$

Calculation of  $p(80|70)$ :

$$p(70|80) = 1 \text{ (no man can live 80 years if he died before living 70 years)}$$

$$p(80|70) = \frac{p(70|80) \cdot p(80)}{p(70)} = \frac{1 \cdot 0.5}{0.8} = 0.625$$

Solution: The conditional probability that a man lives at least 80 years given that he has just celebrated his 70th birthday is at 62.5%.