theoretical exercise 7

Pattern Recognition (2018)

Prof. Dr. Hauke Schramm

Marc Engelmann
Lasse B. Petersen
Jan N. Steeg

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Exercise T-7.1

Problem

You are given the exponential distribution

$$f_X(x|\Theta) = f_X(x|\lambda) = \lambda e^{-\lambda x}$$

defined for $x > 0$ and $\lambda > 0$. Mathematically determine the Maximum-likelihood solution for parameter λ if you are given a set of training samples $D = x_1, x_2, ..., x_n$.

Solution

Likelihood function:

$$L(\overrightarrow{x}, \lambda) = \prod_{i=1}^{n} f_X(x_i|\lambda) = \prod_{i=1}^{n} \lambda e^{-\lambda x_i} = \lambda^n e^{-\lambda \sum_{i=1}^{n} x_i}$$

Log-likelihood function:

$$l(\overrightarrow{x}, \lambda) = \ln L(\overrightarrow{x}, \lambda) = n \ln(\lambda) - \lambda \sum_{i=1}^{n} x_i$$

Derivative of l:

$$\frac{\partial}{\partial \lambda} l(\overrightarrow{x}, \lambda) = \frac{n}{\lambda} - \sum_{i=1}^{n} x_i \neq 0 \Longrightarrow \hat{\lambda} = \frac{n}{\sum_{i=1}^{n} x_i}$$