Bootstrap Inference for Cointegrating Polynomial Regressions

Karsten Schweikert*

University of Hohenheim

[Latest update: April 14, 2021]

Abstract

This paper studies bootstrap refinements for inference in cointegrating polynomial regressions. A sieve bootstrap procedure is suggested to repeatedly draw from the autocorrelated innovation process driving the cointegrated system. Monte Carlo simulations show that the proposed procedure leads to smaller size distortions and improved size-adjusted power over the usual asymptotic approximations.

Keywords: Cointegration; Environmental Kuznets curve; Sieve bootstrap
JEL Classification: C12, C22

*Address: University of Hohenheim, Core Facility Hohenheim & Institute of Economics, Schloss Hohenheim 1 C, 70593 Stuttgart, Germany, e-mail: *karsten.schweikert@uni-hohenheim.de*

1 Introduction

Empirical studies investigating the so-called environmental Kuznets curve (EKC) hypothesis, an inverted U-shaped relationship between economic activity and measures of pollution or emissions, rely on cointegrated polynomial regressions (CPR) and are often hampered by (very) small sample sizes. For example, Wagner (2015) studies the EKC hypothesis using yearly data for 19 early industrialized countries over the period 1870–2000. Since the data on carbon dioxide and sulphur dioxide emissions are usually not available at a higher frequency, researchers are often left with less than 200 observations to estimate their models. Although the OLS estimator has very high convergence rates for coefficients in CPRs, to conduct proper inference, the FM-OLS estimator is needed and this estimator builds on long-run covariance estimators with substantially slower convergence rates. Therefore, using asymptotic approximations for inference can potentially lead to size distortions and reduced power for hypothesis testing. Wagner and Hong (2016) show that inference based on the FM-OLS reduces size distortions compared with tests based on the asymptotically invalid OLS standard errors. However, substantial size distortions are still present for moderate sample sizes. Furthermore, the size-adjustments used in Wagner and Hong (2016) for simulated data cannot be straightforwardly applied by practitioners because the severity of the size distortions depend on the regressor endogeneity which is unknown in empirical applications.

Different methods are available to bootstrap time series regressions (see Palm et al., 2008, for a recent overview). Particularly, the sieve bootstrap has been successfully applied in regressions that involve (potentially) nonstationary processes, for example in the context of unit root testing (Chang and Park, 2003; Palm et al., 2008), or VAR modelling (Swensen, 2006). For cointegrating regressions, Li and Maddala (1997) show that bootstrapping can lead to substantial improvements over asymptotic approximations and Psaradakis (2001) proposes a sieve bootstrap that outperforms their block bootstrap approach. Further applications of the bootstrap in cointegrating regressions can be found in Chang et al. (2006), Shin and Hwang (2013) and Schild and Schweikert (2019).

In this paper, we propose bootstrap refinements for inference in CPRs based on the sieve bootstrap for the innovation process driving the cointegrated system. Their finite sample performance is evaluated in Monte Carlo simulations using the same parametrization as in Wagner and Hong (2016). Assuming that the number of bootstrap replications is fixed, every added Monte Carlo iteration contributes multiplicatively to the overall computational cost when evaluating such bootstrap methods. To avoid this inefficiency, we refer to the 'Warp-speed' bootstrap described in Giacomini et al. (2013). Our results show that the bootstrap has much smaller size distortions compared with asymptotic approximations proposed in the literature. Moreover, these size distortions are in the conservative direction which makes these tests easier to handle for practitioners. The size-adjusted power is higher (lower) for tests of single hypothesis about the coefficient of the linear (quadratic) term. Differences in relative performance can be explained by the different convergence rates of the FM-OLS estimator for linear and quadratic terms in the CPR.

The remainder of this paper is structured as follows: we briefly outline the CPR methodology and describe the bootstrap algorithm in Section 2, discuss our simulation experiments in Section 3, and conclude in Section 4.

2 Methodology

The notation chosen for this section closely follows Wagner and Hong (2016). To simplify the exposition and without loss of generality, we focus on cointegrating polynomial regressions involving a single regressor variable x_t. Further, we restrict our discussion to CPRs with a constant and a linear trend:

$$y_t = c + \delta t + \beta_1 x_t + \cdots + \beta_q x_t^q + u_t, \qquad t = 1, \ldots, T, \tag{1}$$

where x_t is $I(1)$, $\Delta x_t = v_t$, q denotes the polynomial order, and u_t is a stationary error term. The $T \times (q+2)$ matrix $Z := (1, t, x_t, \ldots, x_t^q)$ contains the regressors and θ contains the corresponding coefficients. We further define the vector stochastic process $\{\xi_t\}_{t\in\mathbb{Z}} := \{(u_t, v_t)\}_{t\in\mathbb{Z}}$. Then, we can define its long-run covariance matrix

$$\Omega := \sum_{h=-\infty}^{\infty} E(\xi_0 \xi_h') = \begin{pmatrix} \Omega_{uu} & \Omega_{uv} \\ \Omega_{vu} & \Omega_{vv} \end{pmatrix}. \tag{2}$$

Analogously, we define the one-sided long-run covariance matrix $\Delta := \sum_{h=0}^{\infty} E(\xi_0 \xi_h')$ and $\omega_{u \cdot v} = \Omega_{uu} - \Omega_{uv}\Omega_{vv}^{-1}\Omega_{vu}$.

The FM-OLS estimator applied to Equation (1) is based on two transformations. The first transformation involves the dependent variable and requires a consistent estimator for the long-run covariance matrix:

$$y_t^+ = y_t - \Delta x_t \hat{\Omega}_{vv}^{-1}\hat{\Omega}_{vu}. \tag{3}$$

For the second transformation, we need the estimator $\hat{\Delta}_{vu}^+ = \hat{\Delta}_{vu} - \hat{\Omega}_{uv}\hat{\Omega}_{vv}\hat{\Delta}_{vv}$ to build the correction term

$$A^* = \hat{\Delta}^+_{vu} \begin{bmatrix} 0 \\ 0 \\ T \\ 2\sum x_t \\ \vdots \\ q\sum x_t^{q-1} \end{bmatrix}. \tag{4}$$

Finally, the FM-OLS estimator is given by $\theta^+ = (Z'Z)^{-1}(Z'y^+ - A^*)$ and a consistent estimator of its variance is $\hat{\omega}_{u\cdot v}(Z'Z)^{-1}$. The corresponding t-statistics for single coefficient hypothesis have asymptotically standard normal distribution.[1]

Next, we propose bootstrap refinements for inference in CPRs. To test null hypotheses in the form of $H_0 : \beta_i = \beta_{i,0},\ i \in \{1, \ldots, q\}$ against the two-sided alternative, we use the following sieve bootstrap algorithm:

1. Compute the residuals $\{\hat{u}^+_t = (y_t - Z\theta^+_0, \Delta x_t)'\}^T_{t=1}$, where θ^+_0 is the FM-OLS estimator under the null hypothesis, i.e., the i-th element is substituted by the coefficient under the null hypothesis.

2. Estimate the VAR(p) model for $\{\hat{u}^+_t\}^T_{t=1}$ to obtain the coefficients $\Phi_1, \ldots, \Phi_p$ and save the residuals from this regression $\{\hat{e}_t\}^T_{t=1}$.

3. Center the estimated residuals $\{\hat{e}_t\}^T_{t=1}$ and draw a random sample from the centered residuals to obtain a bootstrap sample $\{e^*_t\}^T_{t=1}$. Construct the bootstrap noise series $\{u^*_t\}^T_{t=1}$ using the recursion

$$u^*_t = \sum_{j=1}^{p} \hat{\Phi}_j u^*_{t-j} + e^*_t, \tag{5}$$

   where the initial p values are given by $u^*_t = \hat{u}^+_t$.

4. Using the partition $u^*_t = (u^*_{1t}, u^*_{2t})'$, generate the bootstrap replicate $\{x^*_t\}^T_{t=1}$ according to

$$x^*_t = x^*_{t-1} + u^*_{2t}, \qquad x^*_0 = 0, \tag{6}$$

   and construct the matrix $Z^* = (1, t, x^*_t, \ldots, x^{*q}_t)$. Build the bootstrap replicate $\{y^*_t\}^T_{t=1}$ according to

$$y^*_t = Z^*\theta^+_0 + u^*_{1t}. \tag{7}$$

5. Estimate θ^+ using the bootstrap sample $\{(x^*_t, y^*_t)\}^T_{t=1}$ and compute the associated t-statistic for $H_0 : \beta_i = \beta_{i,0}$.

[1] Wald tests for joint hypothesis in the form of $R\theta = r$ are also discussed in Wagner and Hong (2016). The outlined bootstrap algorithm can, in principle, be used to test joint hypothesis. However, we found that the size-adjusted power of the bootstrap refinement was inferior to those of the asymptotic approximations.

6. Repeat Steps 3-5 sufficiently often to obtain a bootstrap sample of t-statistics.
7. Obtain a bootstrap estimate of the critical values at the α significance level from the $(\alpha/2)$-th and $(1-\alpha/2)$-th quantiles of the empirical bootstrap distribution of t-statistics.

The bootstrap critical value can then be used as a bootstrap refinement of the usual asymptically standard normal t-test approximations. Selecting the unknown lag order p in the sieve bootstrap can be based on the familiar Akaike information criterion (Psaradakis, 2001). The long-run covariance matrices needed for the computation of the FM-OLS estimator and test statistics based upon it are obtained using the Bartlett and Quadratic Spectral (QS) kernels. For each kernel, we consider two bandwidth choices: (i) the data-dependent rule of Andrews (1991), and (ii) the sample size dependent rule of Newey and West (1994), i.e., $\lfloor 4(T/100)^{2/9} \rfloor$.

3 Finite sample performance

We adopt the same data-generating process for the quadratic cointegrating polynomial regression model that is used in Wagner and Hong (2016):

$$y_t = c + \delta t + \beta_1 x_t + \beta_2 x_t^2 + u_t, \tag{8}$$

where $v_t = \Delta x_t$ and u_t are generated as:

$$\begin{aligned} u_t &= \rho_1 u_{t-1} + e_{1,t} + \rho_2 e_{2,t}, \qquad u_0 = 0, \\ v_t &= e_{2,t} + 0.5\, e_{2,t-1}, \end{aligned}$$

and $(e_{1,t}, e_{2,t})' \sim \mathcal{N}(0, 1)$. The parameter ρ_1 controls the autocorrelation in the error term u_t and the parameter ρ_2 determines the degree of regressor endogeneity. The coefficient values are set to $c = \delta = 1$, $\beta_1 = 5$, and $\beta_2 = -0.3$.

First, we conduct experiments to determine the empirical size of the t-tests. Our results are reported in Table 1. The values for the (asymptotically invalid) OLS estimator and those of both asymptotic approximations, using bandwidth choice (i) and (ii), for the FM-OLS estimator are largely identical to the values reported in Table C4 in Wagner and Hong (2016). The small differences can be explained by the larger number of replications (10,000 instead of 5,000) we employ to ensure that these values can be compared with the 'Warp-speed' bootstrap results.

We find that our sieve bootstrap has better size properties than the asymptotic approximations. For example, in case of $T = 200$ and $\rho_1 = \rho_2 = 0.8$, the asymptotic approximations show no improvement over the $T = 100$ case and exhibit size distortions of more than five times their nominal value. For all parameter configurations, the asymptotic approximation responds only minimally to an increased sample size from $T = 100$ to $T = 200$. A finding that is also reported in Wagner and Hong (2016). The bootstrap for β_1 tends to be too conservative for $T = 100$, but improves for $T = 200$. Its size properties are largely independent of kernel and bandwidth choices across parameterizations and across estimators.

Second, we depict the size-adjusted power curves for $\rho_1 = \rho_2 = 0.6$ in Figure 1. It turns out that the bootstrap has better size-adjusted power for β_1 (roughly 10 percentage points more power for most of the analyzed range of values) but it performs worse for β_2. This finding can be explained by the higher convergence rates of the FM-OLS estimator for β_2 making asymptotic approximation a more natural choice. The bootstrap of t-test for hypotheses involving β_2 is still an interesting alternative for practitioners, because size distortions are minimal at moderate levels of regressor endogeneity and it tends to be conservative. Hence, size-adjustments are not necessarily needed to avoid Type-I errors.

4 Conclusion

This paper demonstrated that using the sieve bootstrap algorithm can improve the finite sample size properties and the size-adjusted power of hypothesis tests in cointegrated polynomial regressions. While size distortions are reduced for the coefficients of the linear and quadratic terms, we only improve the size-adjusted power for tests involving the linear coefficient. Still, we recommend that practitioners use these bootstrap tests to avoid the need for size-adjustments that depend on the (unknown) regressor endogeneity.

Acknowledgements

I thank Konstantin Kuck for helpful comments and Domenic Franjic for excellent research assistance.

Table 1: Empirical Null Rejection Probabilities (5% Significance Level)

Panel A: t-tests for $H_0 : \beta_1 = 5$

$T = 100$

		Bartlett Kernel				QS Kernel			
		Asymp.		Bootstrap		Asymp.		Bootstrap	
ρ_1, ρ_2	OLS	AND	NW	AND	NW	AND	NW	AND	NW
0.0	.0476	.1053	.0923	.0383	.0352	.1210	.1096	.0409	.0416
0.3	.1241	.1184	.1215	.0524	.0542	.1164	.1179	.0500	.0587
0.6	.2770	.1466	.1603	.0329	.0371	.1378	.1473	.0291	.0368
0.8	.5193	.2743	.2926	.0239	.0199	.2625	.2822	.0192	.0211

$T = 200$

		Bartlett Kernel				QS Kernel			
		Asymp.		Bootstrap		Asymp.		Bootstrap	
ρ_1, ρ_2	OLS	AND	NW	AND	NW	AND	NW	AND	NW
0.0	.0493	.0833	.0747	.0401	.0380	.0878	.0885	.0423	.0430
0.3	.1286	.0900	.0962	.0716	.0814	.0839	.0890	.0706	.0764
0.6	.3073	.1220	.1309	.0696	.0771	.1142	.1207	.0712	.0759
0.8	.5643	.2706	.2840	.0570	.0483	.2584	.2744	.0563	.0447

Panel B: t-tests for $H_0 : \beta_2 = -0.3$

$T = 100$

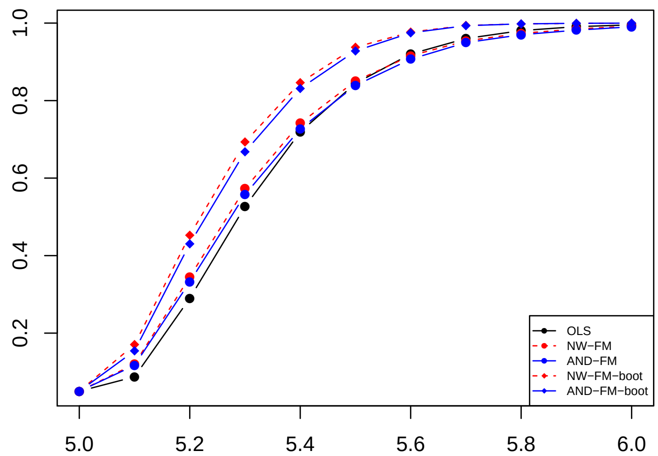
		Bartlett Kernel				QS Kernel			
		Asymp.		Bootstrap		Asymp.		Bootstrap	
ρ_1, ρ_2	OLS	AND	NW	AND	NW	AND	NW	AND	NW
0.0	.0509	.1009	.0885	.0421	.0400	.1158	.1060	.0421	.0455
0.3	.1234	.1065	.1100	.0386	.0427	.1052	.1071	.0367	.0419
0.6	.2364	.1319	.1431	.0408	.0430	.1244	.1326	.0398	.0426
0.8	.3761	.1871	.1986	.0481	.0471	.1787	.1878	.0481	.0474

$T = 200$

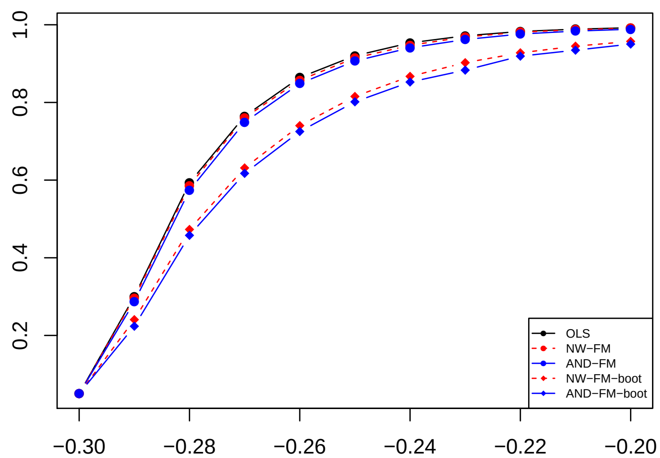
		Bartlett Kernel				QS Kernel			
		Asymp.		Bootstrap		Asymp.		Bootstrap	
ρ_1, ρ_2	OLS	AND	NW	AND	NW	AND	NW	AND	NW
0.0	.0516	.0838	.0736	.0433	.0429	.0884	.0867	.0436	.0463
0.3	.1315	.0851	.0917	.0472	.0509	.0792	.0855	.0487	.0513
0.6	.2619	.1137	.1221	.0651	.0671	.1083	.1123	.0679	.0702
0.8	.4222	.1855	.1957	.0865	.0859	.1799	.1882	.0835	.0853

Note: We draw $R = 10,000$ replications from the DGP described in Equation (8) and apply the "Warp-speed" bootstrap algorithm described in Giacomini et al. (2013) to obtain bootstrap distributions. AND and NW denote the the data-dependent rule of Andrews (1991) and the sample size dependent rule of Newey and West (1994), respectively.

Figure 1: These figures depict the size-corrected power curves of the OLS estimator (black), FM-OLS estimator with data-dependent bandwidth selection (blue) and sample size dependent bandwidth selection (red). The corresponding bootstrap refinements are marked with a diamond.



(a) Size-corrected power curves for β_1 $(\rho_1, \rho_2 = 0.6)$



(b) Size-corrected power curves for β_2 $(\rho_1, \rho_2 = 0.6)$

References

Andrews, D.W.K., 1991. Heteroskedasticity and Autocorrelation Consistent Covariance Matrix Estimation. Econometrica 59, 817–858. doi:10.2307/2938229.

Chang, Y., Park, J.Y., 2003. A Sieve bootstrap for the test of a unit root. Journal of Time Series Analysis 24, 379–400. doi:10.1111/1467-9892.00312.

Chang, Y., Park, J.Y., Song, K., 2006. Bootstrapping cointegrating regressions. Journal of Econometrics 133, 703–739. doi:10.1016/j.jeconom.2005.06.011.

Giacomini, R., Politis, D.N., White, H., 2013. A Warp-Speed Method for Conducting Monte Carlo Experiments Involving Bootstrap Estimators. Econometric Theory 29, 567–589. doi:10.1017/S0266466612000655.

Li, H., Maddala, G.S., 1997. Bootstrapping cointegrating regressions. Journal of Econometrics 80, 297–318. doi:10.1016/S0304-4076(97)00043-2.

Newey, W.K., West, K.D., 1994. Automatic lag selection in covariance matrix estimation. The Review of Economic Studies 61, 631–653. doi:10.2307/2297912.

Palm, F.C., Smeekes, S., Urbain, J.P., 2008. Bootstrap unit-root tests: Comparison and extensions. Journal of Time Series Analysis 29, 371–401. doi:10.1111/j.1467-9892.2007.00565.x.

Psaradakis, Z., 2001. On bootstrap inference in cointegrating regressions. Economics Letters 72, 1–10. doi:10.1016/S0165-1765(01)00410-4.

Schild, K.H., Schweikert, K., 2019. On the Validity of Tests for Asymmetry in Residual-Based Threshold Cointegration Models. Econometrics 7, 1–13. doi:10.3390/econometrics7010012.

Shin, D.W., Hwang, E., 2013. Stationary bootstrapping for cointegrating regressions. Statistics & Probability Letters 83, 474–480. doi:10.1016/j.spl.2012.10.007.

Swensen, A.R., 2006. Bootstrap Algorithms for Testing and Determining the Cointegration Rank in VAR Models. Econometrica 74, 1699–1714. doi:10.1111/j.1468-0262.2006.00723.x.

Wagner, M., 2015. The Environmental Kuznets Curve, Cointegration and Nonlinearity. Journal of Applied Econometrics 30, 948–967. doi:10.1002/jae.2421.

Wagner, M., Hong, S.H., 2016. Cointegrating Polynomial Regressions: Fully Modified OLS Estimation and Inference. Econometric Theory 32, 1289–1315. doi:10.1017/S0266466615000213.