# Optimization of Shifting Schedule for Vehicle with Automated Mechanical Transmission based on Dynamic Programming Algorithm

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Abstract—The dynamics model of the AMT system was established. Optimal shifting schedule is proposed by dynamic programming algorithm, which use speed and power requirements as control parameters, and the minimum fuel consumption as the optimization goal. The results showed that the optimal shifting schedule could reduce the shift frequency and improve the fuel economy on the premise of no loss of power.

Keywords- AMT; shifting schedule; dynamic programming algorithm.

### I. Introduction

Automated mechanical transmission (AMT) is based on manual mechanical transmission. AMT is equipped with automated shifting mechanism to achieve vehicle starting and shifting, which has the advantages of simple structure, low manufacturing cost, high transmission efficiency, etc.

Shifting schedule is one of the key points of AMT control, and affects the vehicle power and economy directly. Optimal shifting schedule is proposed by dynamic programming algorithm, maintain both the power performance and fuel economy. The results showed that shifting schedule can keep the power and improve the fuel economy.

# II. DYNAMICS ANALYSIS OF TRANSMISSION SYSTEM

# A. Engine model

In the steady state, the engine torque could be expressed as:

$$T_e = f(\alpha, n_e) \tag{1}$$

Where  $\alpha$  is throttle opening(%),  $n_e$  is engine speed (rad/s).

Engine torque under steady state is [1]:

$$T_e = a_0 + a_1 n_e^3 + a_2 n_e^2 \alpha + a_3 n_e \alpha^2 + a_4 \alpha^3 + a_5 n_e^2 + a_6 n_e \alpha + a_7 \alpha^2 + a_8 n_e + a_9 \alpha$$
(2)

Where  $a_i$  is constant,  $i=0,1,\ldots,9$ .

## B. Transmission model

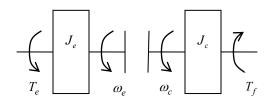


Figure 1. Simplified AMT driveline

During the shifting, the clutch engagement process is shown in figure 1. The formula can be expressed as:

$$J_e \dot{\omega}_e = T_e - c_e \omega_c - T_c \tag{3}$$

$$J_c \dot{\omega}_c = T_c - c_v \omega_c - T_f \tag{4}$$

$$\omega_e = i_0 \cdot i_g \cdot \omega_w \tag{5}$$

Where  $T_c$  is clutch friction torque(N·m),  $T_f$  is resistance torque(N·m),  $J_e$  and  $J_c$  are inertia moments(kg·m²),  $\omega_e$  is engine speed(rad/s),  $\omega_c$  is clutch output speed(rad/s),  $\omega_w$  is wheel rotation speed(rad/s),  $\dot{\omega}_e$  is engine angular acceleration(rad/s²),  $\dot{\omega}_c$  is clutch angular acceleration(rad/s²),  $c_e$  and  $c_v$  are damp coefficients(N·m/(rad/s)),  $i_0$  is main retarder ratio,  $i_g$  is transmission ratio.

# C. Vehicle dynamics model

According to the longitudinal mechanical analysis, the external resistance ( $F_t$ ) mainly includes air resistance, ramp

resistance and rolling friction resistance, which can be expressed as:

$$F_t = mgf\cos\beta + mg\sin\beta + \frac{C_D A}{21.15}v^2 \tag{6}$$

Where m is vehicle mass (kg),  $\beta$  is slope angle (°),  $C_D$  is air resistance coefficient, f is rolling resistance coefficient, A is frontal area, V is vehicle speed.

# D. Dynamic Programming Algorithm

Dynamic Programming Algorithm is a global optimization method for multi stage decision problems [2]. The decision principle is if a decision-making process is optimal, then any phase of the state that determine the next decision must be optimal. A decision is only related to the current state, its future decision must constitute to be the optimal strategy [3].

First, the decision-making process is divided into N stages. In the stage k ( $k = 0,1,2\cdots N$ ), the state variables are expressed as x(k), the decision variables are expressed as u(k). The state transition equation is used to describe the transfer law:

$$x(k+1) = f(x(k), u(k)) \tag{7}$$

The expression of the index function is:

$$J = G_N(x(N)) + \sum_{k=0}^{N-1} L_k(x(k), u(k))$$
 (8)

Where  $G_N(x(N))$  is the index function of the final state,  $L_k(x(k),u(k))$  is index function when the system is transferred to phase k. When the index function takes the maximum or minimum value, the optimal control can be achieved. The optimal control of the whole process is obtained from the optimal value of the optimal decision of each stage and the optimal value of the variable.

# E. Optimization of Shifting Schedule

In order to facilitate the realization of the optimal control of shift schedule, the model of the transmission system is discretized and simplified [5].

The transmission model is as follows by the formula (6) discretization:

$$\omega_e(k) = i_g(k) \cdot i_0 \cdot \omega_w(k) \tag{9}$$

$$T_{w}(k) = \eta \cdot i_{g}(k) \cdot i_{0} \cdot T_{e}(k) \tag{10}$$

Where  $\omega_e(k)$  is engine speed in stage k(r/min),  $\eta$  is transmission efficiency,  $T_w(k)$  is Wheel drive torque (N·m).

Formula (11) and (12) are the vehicle dynamics model:

$$\omega_{w}(k+1) = \omega_{w}(k) + \frac{1}{J_{w}}(T_{w}(k) - F_{t}(k)r_{w})\Delta t$$
 (11)

$$J_{v} = ((J_{e} + J_{c} + J_{in})i_{g}^{2} + J_{out})i_{0}^{2} + mr^{2}$$
 (12)

Where  $J_{v}$   $J_{in}$   $J_{out}$  are the inertia moments of wheel, Transmission input shaft, Transmission output shaft, r is wheel radius.

By the automobile theory [6], the car's dynamic performance can be reflected by the reserve power, the reserve power reflects the climbing and acceleration performance of the vehicle, the reserve power is expressed as:

$$\Delta P = P_e - \frac{P_f + P_w}{n} \tag{13}$$

Where  $P_e$  is the engine output power,  $P_f$  is the rolling resistance power,  $P_w$  is the air resistance power (kW).

Fuel consumption is:

$$Q = \sum_{t=0}^{N-1} Q_t \Delta t \tag{14}$$

$$Q_t = \frac{P_e b_e}{367.1\rho g} \tag{15}$$

Where Q and  $Q_t$  are fuel consumption under constant working condition and fuel consumption per unit time(g),  $b_e$  is Engine fuel consumption rate(g/(kW·h)), P is fuel density(kg/m<sup>3</sup>), g is acceleration of gravity(9.8 m/s<sup>2</sup>).

The optimal shift schedule can be obtained by calculating the maximum value of the backup power  $\Delta P_{MAX}$ , and then using the fuel consumption as the index function.

Select the fuel consumption as the evaluation index function, and the minimum value of the index function  $J^*$  is expressed as:

$$J^* = \min Q = \min(\sum_{k=1}^{N} Q_k \Delta t)$$
 (16)

According to the analysis of the transmission system model, the current gear position G(k) and the angular velocity of the wheel  $\omega_w(k)$  are selected as the state

variables, gear adjustment  $G_s(k)$  and engine torque  $T_e(k)$  are decision variables.

State variables and decision variables are expressed as follows:

$$x(k) = \left[\omega_{w}(k), G(k)\right]^{T} \tag{17}$$

$$u(k) = [G_s(k), T_e(k)]^T$$
(18)

Where  $G_s(k)$  is gear adjustment, range of values  $\{-1,0,1\}$ , -1 is downshift, 0 is not changed, 1 is up-shift.

By formula (11), (17), (18) the state transition equation (7) can be transformed into:

$$\begin{bmatrix} \omega_{w}(k+1) \\ G(k+1) \end{bmatrix} = \begin{bmatrix} \omega_{w}(k) \\ G(k) \end{bmatrix} + \begin{bmatrix} 0 & \frac{i_{g}(k)i_{0}\eta\Delta t}{J_{v}} \\ 1 & 0 \end{bmatrix} \begin{bmatrix} G_{s}(k) \\ T_{e}(k) \end{bmatrix} + \begin{bmatrix} -\frac{F_{t}(k)r\Delta t}{J_{v}} \\ 0 \end{bmatrix}$$
(19)

Constraint conditions are:

$$\omega_{w-\min}(k) \le \omega_w(k) \le \omega_{w-\max}(k)$$
 (20)

$$1 \le G(k) \le 6 \tag{21}$$

$$T_{e-\min}(k) \le T_e(k) \le T_{e-\max}(k)$$
 (22)

The optimization strategy used a recursive solution [4], first step: k = N,  $J_N^* = 0$ ; the second step: k = N - 1, the calculation formula is  $J_k^* = \min[Q(x(k))\Delta t + J_{k+1}^*(x(k+1))]$ ; the second step is repeated until k = 0.

Figure 2 are optimal trajectories of engine speed, vehicle speed, throttle opening by using Dynamic Programming Algorithm. The power demand is 30%.

Shifting schedule is shown in figure 3 by solving the fixed power of the shift sequence respectively and combining with the calculation results.

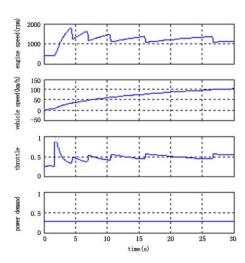


Figure 2. Result of 30% power demand

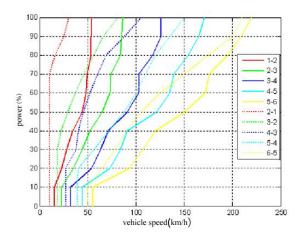


Figure 3. Shift schedule based on optimal shifting schedule

# III. SIMULATION AND EXPERIMENT ANALYSIS

A model of AMT vehicle is established by using Matlab /Simulink. The main parameters of the vehicle are shown in table 1.

Figure 4 is the simulation curve of engine speed, throttle, gear, vehicle speed under C-WTVC conditions by using normal shifting schedule and optimal schedule.

TABLE I. MAIN PARAMETER OF TEST VEHICLE

Main parameter of test vehicle	
parameter	value
mass m/kg	6500
wheelbase L/mm	4250
Power rating/speed n(kW/(r /min))	101/2500
$\begin{array}{ccc} \text{maxmum} & \text{torque} & T_m/\text{speed} \\ \text{n(N•m/(r/min))} & \end{array}$	430/1500
transmission ratio $i_g$	6.11,5.22,3.39,2.05,1.32,1.00
$\begin{array}{ccc} \text{maxmum} & \text{vehicle} & \text{speed} & V_{\text{max}} \\ / (\text{km/h}) & & \end{array}$	99
wheel radius r/mm	406
frontal area A/ m <sup>2</sup>	6.5
air resistance coefficient C <sub>D</sub>	0.8
rolling resistance coefficient f	0.008

In the first 50-70 seconds of normal shifting schedule, shifting occurred frequently, engine speed changed obviously. Fuel consumption was 8.85L/100km. In contrast, when using optimal shifting schedule, the shifting process is stable, the engine speed fluctuation was small, the throttle opening was lower, the engine working efficiency was improved. Fuel consumption was 8. 54L/100km. Optimal shifting schedule increased the fuel economy by 3.5%.

## IV. CONCLUSION

A dynamic model of AMT transmission system is established. An optimal shifting schedule is proposed, which

depends on dynamic programming algorithm that maintain the power performance, keep fuel economy at the same time.

Through large amount of simulation by using general shifting schedule and optimal shifting schedule, some valuable conclusions are obtained:

Optimal shifting schedule ensures the power of the vehicle. It has the advantages of low shifting frequency and high fuel economy compared with the normal shift schedule.

### ACKNOWLEDGMENT

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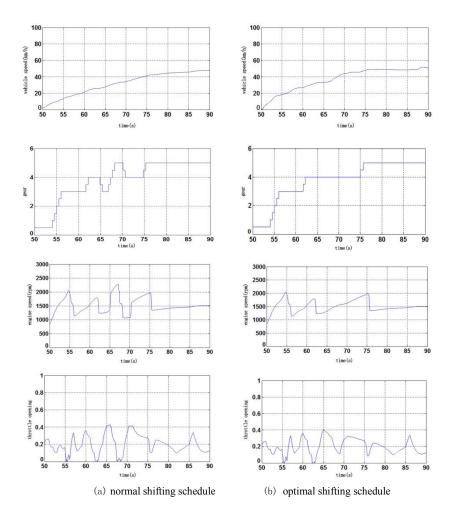


Figure 4. Simulation curve of C-WTVC conditions.