

Dynamics of Execution Quality

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Sunday, July 02, 2023

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1 Abstract

A brokerage firm routes its orders to multiple market centers. Some of the reputed market centers are Citadel, Virtu Americas, and Jane Street. These market centers some times pay a fee to the brokerage firm for the opportunity to execute the trades. This is called payment for order flow (PFOF). As part of this project, we analyse the execution quality of the market centers which includes factors such as speed of execution, the likelihood of obtaining the best available price, minimization of price slippage, etc. We analyze multiple market centers to find which one of them provides better price improvement.

We use Rule 606 filings to understand the order routing, and Rule 605 filings of individual market centers to gather information on order execution quality. The net price improvement is calculated and the performance of individual market center is measured based on the weighted mean of net price improvement over the years.

2 Introduction

For any individual investor to purchase or sell stocks, the most convenient way is to open an online brokerage account with a reputable brokerage firm or an online trading platform (ex. Robinhood Securities, Interactive Brokers, etc.) For this project, we consider the example of Robinhood Securities, LLC. Once an individual investor places a buy or sell order with the brokerage firm, the order is routed to one of the market centers associated with the firm. Once the market center receives the order from the firm, it matches it with a corresponding order from another participant (Buy is matched will Sell and vice-versa) and executes the order. These market centers make money through the bid-ask spread, which is the difference between the buying price (bid) and selling price (ask) of a security. They earn a profit by buying securities at a slightly lower price than the market price and selling them at a slightly higher price.

2.1 Rule 606

Broker-dealers that route client orders for equity and option securities are required by Rule 606 to produce quarterly reports that give a general overview of their routing procedures. This report makes order routing procedures measurable, encourages market participants to compete, and informs the public on how broker-dealers handle orders. Broker-dealers are also required to tell clients where their specific orders were sent upon request. Each client may ask for a free written copy of the report to be mailed to them. Each market center must report their order execution details, which is discussed in *Section 2.2*.

Figure 1 shows the Robinhood Securities, LLC's rule 606 filing for January 2023 for S&P 500 stocks. We can observe that the huge proportion of orders placed with Robinhood have been routed to Virtu Americas LLC (51.51%) and CITADEL Securities LLC (25.61%), followed by G1 Execution Services LLC (11.37%) and Jane Street Capital (10.06%)

Robinhood Securities, LLC - Held NMS Stocks and Options Order Routing Public Report

Generated on Wed Apr 26 2023 12:00:10 GMT-0400 (Eastern Daylight Time)

1st Quarter, 2023

January 2023

S&P 500 Stocks

Summary

Non-Directed Orders as % of All Orders	Market Orders as % of Non-Directed Orders	Marketable Limit Orders as % of Non-Directed Orders	Non-Marketable Limit Orders as % of Non-Directed Orders	Other Orders as % of Non-Directed Orders
100.00	27.46	17.00	13.52	42.01

Venues

Venue - Non-directed Order Flow	Non-Directed Orders (%)	Market Orders (%)	Marketable Limit Orders (%)	Non-Marketable Limit Orders (%)	Other Orders (%)	Net Payment Paid/Received for Market Orders(USD)	Net Payment Paid/Received for Market Orders(cents per hundred shares)	Net Payment Paid/Received for Marketable Limit Orders(USD)	Net Payment Paid/Received for Marketable Limit Orders(cents per hundred shares)	Net Payment Paid/Received for Non-Marketable Limit Orders(USD)	Net Payment Paid/Received for Non-Marketable Limit Orders(cents per hundred shares)	Net Payment Paid/Received for Other Orders(USD)	Net Payment Paid/Received for Other Orders(cents per hundred shares)
Virtu Americas, LLC	51.51	37.79	39.74	49.96	65.74	83,675.50	38.7984	47,030.54	38.9090	6,339.53	32.3805	206,579.38	68.8164
CITADEL SECURITIES LLC	25.61	18.93	18.24	24.03	33.46	64,263.69	23.2252	30,148.53	21.0807	5,050.35	17.5312	112,922.24	44.7419
G1 Execution Services, LLC	11.37	21.55	21.02	13.25	0.21	59,085.79	34.3500	32,212.39	33.0412	4,428.76	26.4033	1,371.59	35.7185
Jane Street Capital	10.06	19.13	18.82	11.12	0.24	83,244.33	33.5575	43,266.32	33.5549	5,596.53	24.2568	1,766.08	31.5715

Figure 1: Robinhood Rule 606 filing for January 2023

2.2 Rule 605

According to **FINRA**, the Exchange Act Rule 11Ac1-5 was adopted by the SEC in November 2000. The purpose of Rule 11Ac1-5 is to enhance order execution quality disclosure to the general public. The SEC requires market centers that trade securities on the national market system to submit monthly electronic reports in accordance with Rule 11Ac1-5. These reports contain details about the execution quality at each market center on a stock-by-stock basis, including how market orders of different sizes are carried out in relation to the open quotes. These reports are required to include information on effective spreads, or the spreads that investors who route their orders to a specific market center actually pay. Market centers must also disclose the extent to which they offer investors employing limit orders executions at prices superior to the public quotes. As per **SEC**, the Rule instructed the self-regulatory organizations

(‘SROs’) that trade national market system securities to collaborate and submit to the Commission a new national market system plan (the ‘Joint-SRO Plan’) establishing procedures for market centers to follow in making their monthly reports available to the public.

The Joint-SRO Plan states that the rule 605 filing has 26 fields as below:

1. **Designated Participant:** The designated participant refers to an entity that engages in trading activities within a specific market. It can be a broker-dealer, market center, or any other entity involved in executing orders. The Participant identification codes are as follows: Amex - “A”; BSE - “B”; CHX - “M”; CSE - “C”; NASD - “T”; NYSE - “N”; PCX - “P”; Phlx - “X”.
2. **Market Center Code:** A market center is a venue or platform where trading takes place. It can be an exchange, electronic communication network (ECN), or other trading systems. The market center code is assigned by a Designated Participant.
3. **Month and Year:** Date refers to the specific day or time period for which the data in the Rule 605 filing is being reported. It indicates when the trading activity took place. The report shows it as a six digit code in the format “yyyymm”.
4. **Ticker Symbol:** Ticker represents the unique symbol or abbreviation used to identify a particular security or financial instrument. It is commonly used to represent the traded instrument’s name in stock exchanges or financial markets.
5. **Order Type:** Order type specifies the instructions given by an investor or trader when placing an order to buy or sell a security. The order type codes are as follows: market orders - “11”; marketable limit orders - “12”; inside-the-quote limit orders - “13”; at-the-quote limit orders - “14”; near-the-quote limit orders - “15”.
6. **Order Size** Order size refers to the quantity or volume of shares or contracts specified in an order to buy or sell a security. It indicates the number of units the investor or trader wishes to transact. The order size codes are as follows: 100-499 shares - “21”; 500-1999 shares - “22”; 2000-4999 shares - “23”; 5000 or more shares - “24”.
7. **Total Covered Orders:** Total covered orders represent the overall number of orders placed by participants during the specified period.
8. **Total Covered Shares:** Total shares represent the total volume or quantity of shares involved in the orders placed by participants during the specified period.
9. **Cancelled Shares:** Cancelled shares refer to the volume or quantity of shares that were initially part of an order but were later canceled by the participant before execution.
10. **Market Center Executed Shares:** Market center executed shares represents the cumulative number of shares of covered orders executed at the receiving market center.
11. **Away Executed Shares:** Away executed shares represent the cumulative number of shares of covered orders executed at any other venue.
12. **Shares from 0 to 9 Seconds:** It represents the cumulative number of shares of covered orders executed from 0 to 9 seconds after the time of order receipt.
13. **Shares from 10 to 29 Seconds:** It represents the cumulative number of shares of covered orders executed from 10 to 29 seconds after the time of order receipt.
14. **Shares from 30 to 59 Seconds:** It represents the cumulative number of shares of covered orders executed from 30 to 59 seconds after the time of order receipt.
15. **Shares from 60 to 299 Seconds:** It represents the cumulative number of shares of covered orders executed from 60 to 299 seconds after the time of order receipt.
16. **Shares from 5 to 30 Minutes:** It represents the cumulative number of shares of covered orders executed from 5 to 30 minutes after the time of order receipt.
17. **Average Realized Spread:** Average realized spread refers to the average difference between the execution price of a trade and the midpoint of the National Best Bid and Offer (NBBO) at the time of trade. It provides an indication of the cost or benefit associated with executing trades. It is expressed in dollars carried out to 4 decimal places.

18. **Average Effective Spread:** Average effective spread represents the average difference between the execution price of a trade and the prevailing market price at the time of trade. It considers the impact of market liquidity on the transaction's cost. It is expressed in dollars carried out to 4 decimal places.
19. **Price Improved Shares:** Price improved shares represent the volume or quantity of shares that were executed at prices better than the National Best Bid and Offer (NBBO) midpoint.
20. **Price Improved Avg Amount:** For shares executed with price improvement, it is the share-weighted average amount per share that prices were improved. The amount shall be expressed in dollars and carried out to four decimal places.
21. **Price Improved Avg Time (Secs):** For shares executed with price improvement, it is the share-weighted average period from the time of order receipt to the time of order execution. The period shall be expressed in number of seconds and carried out to one decimal place.
22. **At-the-Quote Shares:** At the quote shares represents the cumulative number of shares of covered orders executed at the quote.
23. **At-the-Quote Avg Time (Secs):** For shares executed at the quote, it is the share-weighted average period of time from the time of order receipt to the time of order execution. The period shall be expressed in number of seconds and carried out to one decimal place.
24. **Outside-the-Quote Shares:** Outside the quote shares represents the cumulative number of shares of covered orders executed outside the quote.
25. **Outside-the-Quote Avg Amount(\$):** For shares executed outside the quote, it is the share-weighted average amount per share that prices were outside the quote. The amount shall be expressed in dollars and carried out to four decimal places.
26. **Outside-the-Quote Avg Time (Secs):** For shares executed outside the quote, it is the share-weighted average period of time from the time of order receipt to the time of order execution. The period shall be expressed in number of seconds and carried out to one decimal place.

3 Evaluating a market center

When assessing the performance of a market center, there are several factors to consider. According to Fidelity Capital Markets, below are the key metrics in evaluating a market center based on the trade execution quality:

1. **Price Improvement:** It is measured as the percentage of shares executed at prices better than the prevailing National Best Bid or Offer (NBBO). That is, either below the best offer for buy orders or above the best bid for sell orders.
2. **Execution Price:** It is measured as the percentage of shares executed at or within the National Best Bid or Offer (NBBO) for a security across all exchanges and/or market centers.
3. **Execution Speed:** It is the average time taken from the order reception by the market center to the order execution by the market center.
4. **Effective Spread:** It is the average difference between the execution price of a trade and the prevailing market price at the time of trade. This amount captures both how often, and by how much, a broker-dealer improves the price of a share.

It's important to note that assessing the performance of market centers can be challenging for individual traders as some performance metrics and data may not be readily available. In such cases, it can be helpful to rely on industry reports, reviews, and insights from reputable sources to gauge the market center's performance.

This project primarily deals with analyzing and understanding the dynamics of order execution quality based on the data from rule 605 filings.

4 Input Parameters

Below is the input configuration file (config.yml) which allows the user to provide the working directory for the project along with order sizes, order types, tickers, and market center IDs to be considered for analysis.

```
## #####
## # This is the configuration file that lets the user to select
## # order sizes, order types, ticker list, market center IDs for analysis.
## # User can also select the project working directory.
## # If multiple ranges or types are desired, use comma to separate values
## # If no specific value is desired, use "".
## # Automatic SEC Rule 605 file downloads are currently supported
## # using download_rule605_filings.R for CDRG, JNST, NITE.
## # -----
## # Order Size Mapping
## # -----
## # 100-499 shares - 21
## # 500-1999 shares - 22
## # 2000-4999 shares - 23
## # 5000 or more shares - 24
## # -----
## # Order Type Mapping
## # -----
## # market orders - 11
## # marketable limit orders - 12
## # inside-the-quote limit orders - 13
## # at-the-quote limit orders - 14
## # near-the-quote limit orders - 15
## # -----
## # If the project folder is saved in C drive as "DEQ", give project_dir as "C:/DEQ"
## #####
## # Provide Input Values below:
##
## default:
##   project_dir: "C:/new_DEQ"
##   order_type: "11"
##   order_size: "21"
##   tickers: "AAPL,AMZN,MSFT,GOOG"
##   market_center_ids: "CDRG,JNST,NITE"
##
```

5 Data Collection and Pre-processing

Rule 605 filing is done every month, which means we have to look at 12 different filings to understand and evaluate the performance of a market center based on most recent data. For this project, we chose to consider at least 4 years of historical data for each market center to fairly evaluate its order execution quality. The files can be downloaded in either *.dat* or *.txt* formats.

5.1 Step 1: Collect the data

Table 1 shows various market centers along with their IDs and URLs to download the latest rule 605 filings available on their respective websites.

Table 1: Market Centers and URLs to download Rule 605 filings

mc_id	mc_name	mc_url
ADAM	A.G.P. / ALLIANCE GLOBAL PARTNERS	https://vrs.vista-one-solutions.com/sec605rule.aspx
ALNC	A.G.P. / ALLIANCE GLOBAL PARTNERS	https://vrs.vista-one-solutions.com/sec605rule.aspx
BARD	ROBERT W. BAIRD & CO. INCORPORATED	https://public.s3.com/rule605/bard/
BBOK	BARCLAYS CAPITAL INC.	https://vrs.vista-one-solutions.com/sec605rule.aspx
BERN	BERNSTEIN SANFORD C & CO. LLC	https://vrs.vista-one-solutions.com/sec605rule.aspx
BMOC	BMO CAPITAL MARKETS	https://vrs.vista-one-solutions.com/sec605rule.aspx
BTIG	BTIG LLC	https://vrs.vista-one-solutions.com/sec605rule.aspx
CANT	CANTOR FITZGERALD & CO.	https://vrs.vista-one-solutions.com/sec605rule.aspx
CDRG	CITADEL SECURITIES LLC	http://www.citadelsecurities.com/rule-605-606-statements/
CODA	CODA Markets Inc.	https://public.s3.com/rule605/coda/
COWN	COWEN AND COMPANY LLC	http://disclosures.bestxstats.com/cown/
CSTI	Canaccord Genuity Inc.	https://vrs.vista-one-solutions.com/sec605rule.aspx
DADA	DAVIDSON (D.A.) & CO. INC.	https://vrs.vista-one-solutions.com/sec605rule.aspx
DAWA	DAIWA CAPITAL MARKETS AMERICA INC.	https://ecsdashboard.jandj.com/public/d4568e59-8f68-d4d7-9c90-b6a0d29ab1c6
DOWL	DOWLING & PARTNERS SECURITIES LLC	http://private.bestxstats.com/605_public/?bxs_mpid=dowling
EBXL	LEVEL ATS	https://www.levelats.com/order-execution-disclosure
EDJO	Edward Jones	https://vrs.vista-one-solutions.com/sec605rule.aspx
ETMM	G1 Execution Services	https://vrs.vista-one-solutions.com/sec605rule.aspx
FBCO	CREDIT SUISSE SECURITIES (USA) LLC	https://bestex.credit-suisse.com/index.html
GSCO	GOLDMAN SACHS AND COMPANY	https://www.goldmansachs.com/compliance-rule605/index.html
GTSH	GTS Securities LLC	http://finra-605.s3-website-us-east-1.amazonaws.com/
GUGS	GUGGENHEIM SECURITIES LLC	http://www.guggenheimpartners.com/services/securities/sales-trading
HRTF	HRT Financial LP	https://external.s3.com/rule605/hrtf/
IATS	Interactive Brokers LLC	https://www.interactivebrokers.com/en/index.php?f=2333
ICBX	Instinet LLC	https://www.instinnet.com/order-disclosures
INTL	StoneX Financial Inc.	https://vrs.vista-one-solutions.com/sec605rule.aspx
JEFF	Jefferies LLC	mta.ihsmarket.com/app-v2/public-report-library/public-report-library-view/Jefferies Group LLC/287
JNST	JANE STREET CAPITAL LLC	https://www.janestreet.com/execution-quality-reports
JPBX	J.P. Morgan Securities LLC	https://vrs.vista-one-solutions.com/sec605rule.aspx
JPMS	J.P. MORGAN SECURITIES LLC	https://vrs.vista-one-solutions.com/sec605rule.aspx
JPMX	J.P. MORGAN SECURITIES LLC	https://vrs.vista-one-solutions.com/sec605rule.aspx
JSCA	Jane Street Capital LLC	http://www.janestreet.com/execution-quality-reports
JSJX	JANE STREET CAPITAL LLC	https://www.janestreet.com/execution-quality-reports
KBWI	KEEFE BRUYETTE & WOODS INC.	http://disclosures.bestxstats.com/kbwi/605/
KEYB	KEYBANC CAPITAL MARKETS INC.	https://www.fisglobal.com/en/ptc/rule-605
KING	C. L. KING & ASSOCIATES INC.	https://vrs.vista-one-solutions.com/sec605rule.aspx
LATS	BARCLAYS CAPITAL INC.	https://vrs.vista-one-solutions.com/sec605rule.aspx
LEER	LEERINK Partners LLC	https://vrs.vista-one-solutions.com/sec605rule.aspx
LEHM	BARCLAYS CAPITAL INC.	https://vrs.vista-one-solutions.com/sec605rule.aspx
LQNA	Liquidnet H2O	http://www.liquidnet.com/pages/monthly-execution-reports
LQNT	LIQUIDNET	http://www.liquidnet.com/pages/monthly-execution-reports
MACQ	MACQUARIE CAPITAL (USA) INC.	http://vrs.vista-one-solutions.com/sec605rule.aspx
MAXM	MAXIM GROUP LLC	https://www.maximgrp.com/public-disclosures/
MLCO	BofA Securities Inc.	https://www.bofamf.com/en-us/content/best_execution.html
MLIX	Instinct X	https://www.bofamf.com/en-us/content/best_execution.html
MSCO	MORGAN STANLEY & CO. LLC	https://external.s3.com/rule605/msco/
MZHO	MIZUHO SECURITIES USA INC.	http://public.s3.com/rule605/mzho/
NFSC	NATIONAL FINANCIAL SERVICES LLC	https://vrs.vista-one-solutions.com/sec605rule.aspx
NITE	Virtu Americas LLC	https://www.virtu.com/about/transparency/rule-605-and-606-reporting/
OPCO	OPPENHEIMER & CO. INC.	http://external.s3.com/rule605/opco
OTAA	OTA LLC	https://www.otallc.com/OTA_605.html
PAUL	PAULSON INVESTMENT CO. INC.	https://abelnoser.com/605-paulson.html
PIPR	PIPER JAFFRAY & CO.	https://vrs.vista-one-solutions.com/sec605rule.aspx
PPFD	Pershing LLC	https://nms605.karngroup.com/48d68ad95546424752413d3d
RAJA	RAYMOND JAMES & ASSOCIATES INC.	https://www.fisglobal.com/ptc/rule-605
RBCM	RBC CAPITAL MARKETS LLC	http://disclosure.bestxstats.com/rbcm/605/
RHCO	TRUIST SECURITIES INC.	https://vrs.vista-one-solutions.com/sec605rule.aspx

mc_id	mc_name	mc_url
RILY	B. Riley Securities Inc.	https://brileyfin.com/disclosures#securities
SBSH	CITIGROUP GLOBAL MARKETS INC.	icg.citi.com/rcs/icgPublic/storage/public/disclosures/SEC-Rule-605-Disclosure.pdf
SGAS	SG Americas Securities LLC	https://www.sgasdisclosure.com/en/order-handling/order-execution-routing-practices-sec-rule-605-606/
SGMT	Goldman Sachs & Co LLC	https://www.goldmansachs.com/compliance-rule605/index.html
SOHO	TWO SIGMA SECURITIES LLC	https://vrs.vista-one-solutions.com/sec605rule.aspx
SPHN	STEPHENS INC.	http://disclosures.bxstech.com/sphn
STFL	STIFEL NICOLAUS & COMPANY INC	http://disclosures.bestxstats.com/stfl/605/
SUFI	SUSQUEHANNA FINANCIAL GROUP LLP	https://www.sig.com/disclosures/sfg-sec-rule-605-report/
UBSA	UBS ATS	https://vrs.vista-one-solutions.com/sec605rule.aspx
UBSS	UBS SECURITIES LLC	https://vrs.vista-one-solutions.com/sec605rule.aspx
VERT	THE VERTICAL TRADING GROUP LLC	http://disclosures.bxstech.com/vert/605/
VIRT	Virtu Americas LLC	https://www.virtu.com/about/transparency/rule-605-and-606-reporting/
VNDM	Wall Street Access	https://public.s3.com/rule605/vndm/
WABR	WALL STREET ACCESS	https://public.s3.com/rule605/vndm
WBLR	WILLIAM BLAIR & COMPANY LLC	https://www.abelnoser.com/605-william-blair.html
WBSI	Wedbush Securities Inc.	http://public.s3.com/rule605/wedb/
XTXE	XTX Execution Services LLC	https://www.txmarkets.com/execution-quality-reports/

The user can either manually download the rule 605 filings for the chosen market centers or run the “download_rule605_filings.R” file to automatically download all the available filings from the respective URLs.

```
file_path <- paste0(project_dir, "/data/f605_data")
file_name <- list.files(file_path, recursive = TRUE)[1]
rule605_file <- file.path(file_path, file_name)
file_content <- readLines(rule605_file, n = 5)

# Add a new line for better readability

rows <- ""
for (line in file_content) {
  rows <- paste(rows, line, "\n", sep = "\n")
}
cat(rows)
```

Read the downloaded rule 605 filings (.txt or .dat files) of each market center, to which the orders are routed by the broker. The list of market centers to be considered is given by the broker's rule 606 filing. Combine the data into a data frame (table) with relevant column names for ease of analysis and readability. The structure of the data frame reflects that of the rule 605 filing where in each row corresponds to the monthly orders for a given ticker.

8


```

rule605_files <- list.files(rule605_path, recursive = TRUE)

rule605_col_names <- c(
  "participant", "market_center", "date",
  "ticker", "order_type", "order_size",
  "total_orders", "total_shrs", "cancelled_shrs",
  "mc_exec_shrs", "away_exec_shrs", "shrs_0to9sec",
  "shrs_10to29sec", "shrs_30to59sec", "shrs_60to299sec",
  "shrs_5to30min", "avg_realzd_spread", "avg_effec_spread",
  "px_improved_shrs", "px_improved_avg_amt", "px_improved_avg_secs",
  "at_quote_shrs", "at_quote_avg_secs", "outside_quote_shrs",
  "outside_quote_avg_amt", "outside_quote_avg_sec"
)

rule605_col_types <- c(
  "character", "character", "character",
  "character", "character", "character",
  "numeric", "numeric", "numeric",
  "numeric", "numeric", "numeric",
  "numeric", "numeric", "numeric",
  "numeric", "numeric", "numeric",
  "numeric", "numeric", "numeric",
  "numeric", "numeric", "integer",
  "numeric", "numeric"
)

rule605_all <- process_rule605_files(rule605_files, rule605_col_names, rule605_col_types)

```

5.2 Step 2: Add listing information as new columns

We add five new columns to the data frame namely - in_sp500, in_r1000, on_nyse, on_nasdaq, on_amex. For each row, based on the corresponding ticker listing, these columns will be marked either "TRUE" or "FALSE".

```

constituent_data_dir <- paste0(project_dir, "/data/constituent_data")

setwd(constituent_data_dir)

rule605_all <- add_listing_data(rule605_all, "sp500constituents.csv", "in_sp500")
rule605_all <- add_listing_data(rule605_all, "russell1000_constituents.csv", "in_r1000")
rule605_all <- add_listing_data(rule605_all, "tickers_NYSE.csv", "on_nyse")
rule605_all <- add_listing_data(rule605_all, "tickers_NASDAQ.csv", "on_nasdaq")
rule605_all <- add_listing_data(rule605_all, "tickers_AMEX.csv", "on_amex")

```

5.3 Step 3: Filter the data frame based on user input

The data frame is filtered based on the order type and order size chosen by the user in configuration file.

```

rule605_df <- rule605_all %>%
  filter(order_type %in% order_type_ip) %>%
  filter(order_size %in% order_size_ip) %>%
  filter(mc_exec_shrs != 0)

rule605_df[is.na(rule605_df)] <- 0

```

We make the below updates to the data frame:

1. order_type and order_size codes are translated to their respective descriptions based on the

mapping provided in the configuration file.

2. A new column `total_exec_shrs` is added as the sum of `mc_exec_shrs` and `away_exec_shrs`.

Table 2 shows sample rows and columns from the updated data frame `rule605_df`.

Table 2: Sample rows and columns from the data frame

market_center	date	ticker	order_type	order_size	px_improved_shrs	px_improved_avg_amt
NITE	2022 Oct	OMFL	mkt_ordr	100-499	40704	0.0075
TCDRG	2020 Jun	USOI	mkt_ordr	100-499	103931	0.0150
TCDRG	2019 Oct	EXPE	mkt_ordr	100-499	68063	0.0227
TCDRG	2022 Feb	FST	mkt_ordr	100-499	1715	0.0067
TJNST	2023 Mar	BITF	mkt_ordr	100-499	95906	0.0030

6 Calculation of Net Price Improvement

Net price improvement refers to the overall benefit obtained by a trader or investor when executing trades. It is calculated by considering the difference between the execution prices achieved and the prevailing market prices at the time of trade.

In the context of trading, price improvement occurs when a trade is executed at a better price than the prevailing market price. It can be achieved by executing at prices better than the National Best Bid and Offer (NBBO) or by obtaining more favorable execution prices compared to the midpoint of the NBBO.

Net price improvement takes into account both positive and negative price improvements. Positive price improvement occurs when trades are executed at prices better than the prevailing market price, resulting in cost savings or additional gains for the trader. On the other hand, negative price improvement occurs when trades are executed at prices worse than the prevailing market price, resulting in additional costs or losses for the trader.

By calculating the net price improvement, traders can evaluate the overall impact of their execution quality. It provides a measure of how effectively trades are executed in terms of obtaining better prices compared to the prevailing market conditions. Positive net price improvement indicates that the trader achieved better prices overall, while negative net price improvement suggests that the trader experienced worse prices on average compared to the market.

Net price improvement is an important metric for assessing execution quality, especially when evaluating the performance of market centers, brokers, or trading algorithms. It helps traders and investors analyze the impact of execution decisions and identify opportunities to improve trading strategies and outcomes.

Below is how we calculate the net price improvement per ticker:

(px_improved_shrs * px_improved_avg_amt): This calculates the total value of price improvement for the shares that were executed at better prices than the NBBO midpoint.

(outside_quote_shrs * outside_quote_avg_amt): This calculates the total value of price deviation for the shares executed at prices outside the NBBO quote.

((px_improved_shrs * px_improved_avg_amt) - (outside_quote_shrs * outside_quote_avg_amt)): This calculates the net value improvement by summing the value of price improvements and subtracting the value of price deviations.

we multiply the value by 100 to convert the value into cents from dollars. Let us call it **net_pi_numerator**.

we divide the `net_pi_numerator` value by the total number of shares executed to get the net price improvement per ticker value.

Hence, the formula we use to calculate the net price improvement is:

net_pi_numerator/total_exec_shrs

Table 3 shows the sample rows from the data frame along with the `net_pi` column

Table 3: Sample rows and columns from the data frame

market_center	date	ticker	order_type	order_size	net_pi_numerator	total_exec_shrs	net_pi
NITE	2021 Sep	JAX	mkt_ordr	100-499	238.76	623	0.383
NITE	2020 May	FSEA	mkt_ordr	100-499	840.00	100	8.400
TETMM	2023 Apr	REGL	mkt_ordr	100-499	125325.53	67019	1.870
TETMM	2022 Apr	RDWR	mkt_ordr	100-499	9084.20	4453	2.040
TJNST	2023 Apr	MFLX	mkt_ordr	100-499	1331.46	569	2.340

7 Weighted-Mean & Weighted-Median of Net Price Improvement

Weighted mean and median of net price improvement are commonly used measures to evaluate the execution quality of a market maker or trading strategy. Using weighted mean and median allows for the consideration of trade sizes or volumes. This is important because larger trades may have a greater impact on the overall execution quality, and assigning weights based on trade sizes helps reflect this influence.

Weighted mean provides an average net price improvement, taking into account the relative importance of each trade based on their weights (e.g., trade size or executed shares). It represents the average improvement achieved across all trades, giving more weight to larger trades. The weighted mean of the net price improvement is calculated using the below formula

wtd.mean(net_pi, total_exec_shrs)

Weighted median represents the middle value of the net price improvement when sorted in ascending order, considering the weights. It is a measure that is less affected by extreme values and provides insight into the typical or representative net price improvement. The weighted median of the net price improvement is calculated using the below formula

wtd.quantile(net_pi, total_exec_shrs , probs= 0.5)

Table 4 shows the overall weighted mean and weighted median of net price improvement for each market center.

Table 4: W-Mean and W-Median of Net Price Improvement by Ticker

market_center	ticker	net_pi_wmean	net_pi_wmed
NITE	AAPL	0.679	0.467
NITE	AMZN	11.440	0.557
NITE	GOOG	17.648	0.474
NITE	MSFT	0.970	0.968
TCDRG	AAPL	0.816	0.566
TCDRG	AMZN	15.981	0.613
TCDRG	GOOG	20.156	0.610
TCDRG	MSFT	1.088	0.923
TETMM	AAPL	0.768	0.573
TETMM	AMZN	10.933	0.725
TETMM	GOOG	17.997	0.619
TETMM	MSFT	1.236	1.236
TJNST	AAPL	0.577	0.523
TJNST	AMZN	7.606	0.631
TJNST	GOOG	11.189	0.524
TJNST	MSFT	1.201	1.169
TVIRT	AAPL	-0.020	0.000
TVIRT	AMZN	-3.566	0.000
TVIRT	GOOG	0.000	0.000
TVIRT	MSFT	-0.640	0.000
VIRT	AAPL	0.000	0.000
VIRT	AMZN	0.000	0.000
VIRT	GOOG	0.000	0.000
VIRT	MSFT	-0.333	-0.250

Table 5 shows the weighted mean and weighted median of net price improvement for each market center, grouped by order size and order type.

Table 5: W-Mean and W-Median of Net Price Improvement by Order Type and Order Size

market_center	order_type	order_size	net_pi_wmean	net_pi_wmed
NITE	mkt_ordr	100-499	1.924	0.594
TCDRG	mkt_ordr	100-499	2.059	0.648
TETMM	mkt_ordr	100-499	2.366	0.747
TJNST	mkt_ordr	100-499	1.927	0.578
TVIRT	mkt_ordr	100-499	-0.153	0.000
VIRT	mkt_ordr	100-499	-0.005	0.000

7.1 W-Mean & W-Median of Net PI by Ticker Listing

We calculate the weighted mean and weighted median of net price improvement for each market center based on listing of the executed stocks in S&P 500, Russell 1000, New York Stock Exchange, NASDAQ, and AMEX.

7.1.1 By S&P 500 listing

Table 6 shows the weighted mean and weighted median of net price improvement for various market centers based on the ticker listing in S&P 500.

Table 6: W-Mean and W-Median of Net Price Improvement by SP500 Listing

market_center	in_sp500	net_pi_wmean	net_pi_wmed
NITE	FALSE	1.673	0.534
NITE	TRUE	2.576	0.799
TCDRG	FALSE	1.790	0.587
TCDRG	TRUE	2.766	0.858
TETMM	FALSE	2.038	0.670
TETMM	TRUE	3.177	1.034
TJNST	FALSE	1.623	0.501
TJNST	TRUE	2.767	0.883
TVIRT	FALSE	-0.118	0.000
TVIRT	TRUE	-0.483	0.000
VIRT	FALSE	-0.003	0.000
VIRT	TRUE	-0.018	0.000

7.1.2 By Russell 1000 listing

Table 7 shows the weighted mean and weighted median of net price improvement for various market centers based on the ticker listing in Russell 1000.

Table 7: W-Mean and W-Median of Net Price Improvement by Russell 1000 Listing

market_center	in_r1000	net_pi_wmean	net_pi_wmed
NITE	FALSE	1.445	0.492
NITE	TRUE	2.709	0.839
TCDRG	FALSE	1.538	0.564
TCDRG	TRUE	2.941	0.883
TETMM	FALSE	1.734	0.610
TETMM	TRUE	3.347	1.066
TJNST	FALSE	1.393	0.471
TJNST	TRUE	2.840	0.897
TVIRT	FALSE	-0.091	0.000
TVIRT	TRUE	-0.386	0.000
VIRT	FALSE	-0.008	0.000
VIRT	TRUE	0.008	0.000

7.1.3 By New York Stock Exchange listing

Table 8 shows the weighted mean and weighted median of net price improvement for various market centers based on the ticker listing on New York Stock Exchange.

Table 8: W-Mean and W-Median of Net Price Improvement by NYSE Listing

market_center	on_nyse	net_pi_wmean	net_pi_wmed
NITE	FALSE	1.970	0.584
NITE	TRUE	1.815	0.617
TCDRG	FALSE	2.092	0.627
TCDRG	TRUE	1.986	0.681
TETMM	FALSE	2.429	0.729
TETMM	TRUE	2.218	0.774
TJNST	FALSE	1.914	0.563
TJNST	TRUE	1.958	0.622
TVIRT	FALSE	-0.174	0.000
TVIRT	TRUE	-0.119	0.000
VIRT	FALSE	-0.006	0.000
VIRT	TRUE	0.001	0.000

7.1.4 By NASDAQ listing

Table 9 shows the weighted mean and weighted median of net price improvement for various market centers based on the ticker listing on NASDAQ.

Table 9: W-Mean and W-Median of Net Price Improvement by NASDAQ Listing

market_center	on_nasdaq	net_pi_wmean	net_pi_wmed
NITE	FALSE	1.446	0.496
NITE	TRUE	2.707	0.857
TCDRG	FALSE	1.586	0.575
TCDRG	TRUE	2.928	0.875
TETMM	FALSE	1.733	0.610
TETMM	TRUE	3.432	1.081
TJNST	FALSE	1.464	0.490
TJNST	TRUE	2.719	0.809
TVIRT	FALSE	-0.098	0.000
TVIRT	TRUE	-0.256	0.000
VIRT	FALSE	-0.002	0.000
VIRT	TRUE	-0.008	0.000

8 Time Series of Weighted Mean of net price improvement

We plot the weighted mean of net price improvement for each market maker as a time series to analyze and evaluate which market maker has been consistently performing better than the others in terms of order execution quality and net price improvement.

```
# Use the net pi per month for each market center
```

```
net_pi_monthly_wmean$date <- as.Date(paste0(  
  substr(net_pi_monthly_wmean$date, 1, 4), "-",  
  substr(net_pi_monthly_wmean$date, 5, 6), "-01"  
))
```

```
net_pi_monthly_wmean <- net_pi_monthly_wmean %>%  
  distinct() %>%  
  mutate(date = yearmonth(date))
```

```
net_pi_monthly_wmean <- net_pi_monthly_wmean %>%  
  as_tsibble(key = market_center, index = date)
```

```
# Plot the data
```

```
autoplot(net_pi_monthly_wmean) + labs(x = "Month-Year", y = "Weighted Mean")
```

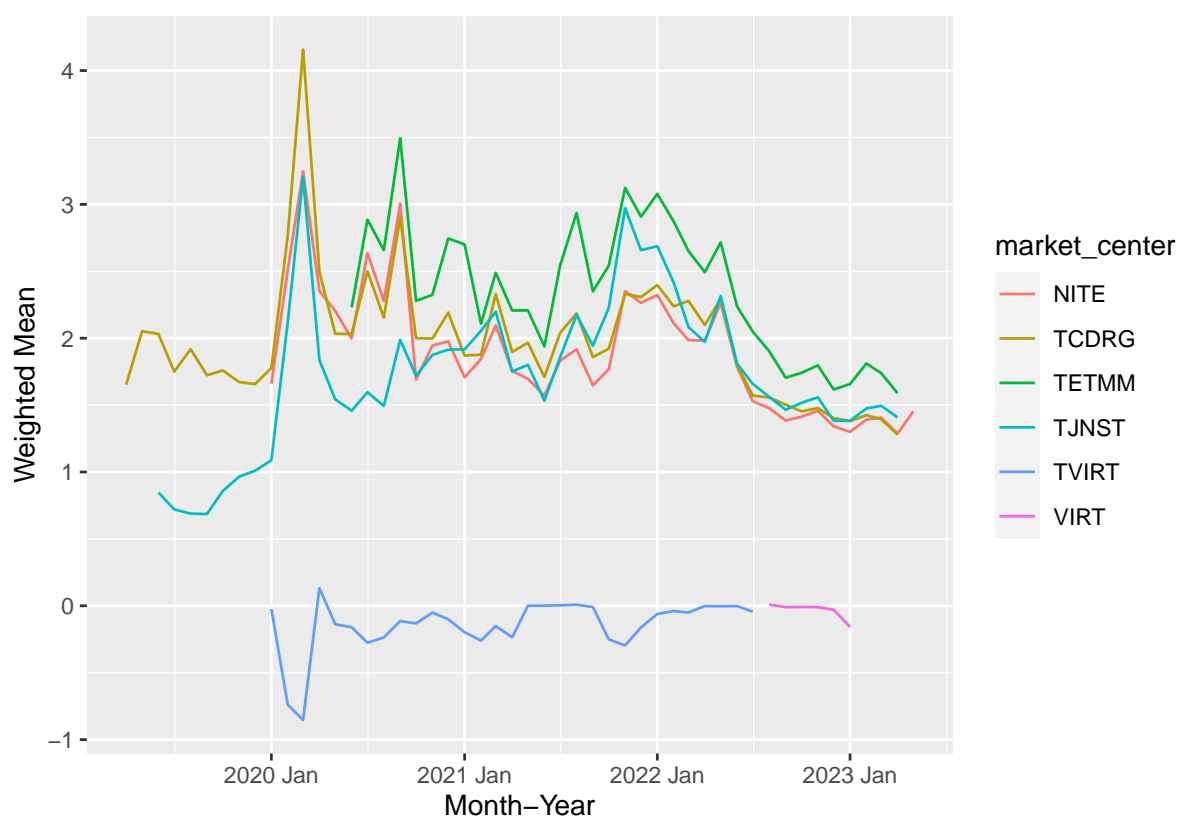


Table 10 shows which market maker has achieved the highest weighted mean of net price improvement for each month.

Table 10: Highest W-mean of Net PI per month

date	market_center	mc_wmean
2019 Apr	TCDRG	1.65
2019 May	TCDRG	2.05
2019 Jun	TCDRG	2.03
2019 Jul	TCDRG	1.75
2019 Aug	TCDRG	1.92
2019 Sep	TCDRG	1.72
2019 Oct	TCDRG	1.76
2019 Nov	TCDRG	1.67
2019 Dec	TCDRG	1.66
2020 Jan	TCDRG	1.78
2020 Feb	TCDRG	2.77
2020 Mar	TCDRG	4.16
2020 Apr	TCDRG	2.49
2020 May	NITE	2.20
2020 Jun	TETMM	2.23
2020 Jul	TETMM	2.89
2020 Aug	TETMM	2.66
2020 Sep	TETMM	3.49
2020 Oct	TETMM	2.28
2020 Nov	TETMM	2.32
2020 Dec	TETMM	2.75
2021 Jan	TETMM	2.70
2021 Feb	TETMM	2.11
2021 Mar	TETMM	2.49
2021 Apr	TETMM	2.21

Table 10: Highest W-mean of Net PI per month (*continued*)

date	market_center	mc_wmean
2021 May	TETMM	2.21
2021 Jun	TETMM	1.94
2021 Jul	TETMM	2.54
2021 Aug	TETMM	2.94
2021 Sep	TETMM	2.35
2021 Oct	TETMM	2.54
2021 Nov	TETMM	3.12
2021 Dec	TETMM	2.91
2022 Jan	TETMM	3.08
2022 Feb	TETMM	2.87
2022 Mar	TETMM	2.65
2022 Apr	TETMM	2.49
2022 May	TETMM	2.72
2022 Jun	TETMM	2.24
2022 Jul	TETMM	2.05
2022 Aug	TETMM	1.90
2022 Sep	TETMM	1.70
2022 Oct	TETMM	1.74
2022 Nov	TETMM	1.80
2022 Dec	TETMM	1.62
2023 Jan	TETMM	1.66
2023 Feb	TETMM	1.81
2023 Mar	TETMM	1.74
2023 Apr	TETMM	1.59
2023 May	NITE	1.45

Below are the version details

```
print(sessionInfo(), locale = FALSE)
```

```
## R version 4.3.0 (2023-04-21 ucrt)
## Platform: x86_64-w64-mingw32/x64 (64-bit)
## Running under: Windows 11 x64 (build 22621)
##
## Matrix products: default
##
##
## attached base packages:
## [1] stats      graphics  grDevices utils      datasets  methods   base
##
## other attached packages:
## [1] httr_1.4.6      rvest_1.0.3      xtable_1.8-4      data.table_1.14.8
## [5] tsibbledata_0.4.1 feasts_0.3.1      fable_0.3.3      fabletools_0.3.3
## [9] tsibble_1.1.3    caret_6.0-94      lattice_0.21-8    ggfortify_0.4.16
## [13] ggplot2_3.4.2    lubridate_1.9.2    tidyr_1.3.0      dplyr_1.1.2
## [17] tibble_3.2.1     formatR_1.14      knitr_1.43        config_0.3.1
## [21] yaml_2.3.7       tidytext_0.4.1    kableExtra_1.3.4  weights_1.0.4
## [25] Hmisc_5.1-0
##
## loaded via a namespace (and not attached):
## [1] pROC_1.18.2      gridExtra_2.3      rlang_1.1.1
## [4] magrittr_2.0.3    compiler_4.3.0      gdata_2.19.0
## [7] systemfonts_1.0.4 vctrs_0.6.2         reshape2_1.4.4
## [10] stringr_1.5.0     pkgconfig_2.0.3     shape_1.4.6
```

## [13] fastmap_1.1.1	ellipsis_0.3.2	backports_1.4.1
## [16] labeling_0.4.2	utf8_1.2.3	rmarkdown_2.22
## [19] prodlim_2023.03.31	anytime_0.3.9	nloptr_2.0.3
## [22] purrr_1.0.1	xfun_0.39	glmnet_4.1-7
## [25] jomo_2.7-6	recipes_1.0.6	highr_0.10
## [28] SnowballC_0.7.1	pan_1.6	broom_1.0.4
## [31] parallel_4.3.0	cluster_2.1.4	R6_2.5.1
## [34] stringi_1.7.12	parallelly_1.36.0	boot_1.3-28.1
## [37] rpart_4.1.19	Rcpp_1.0.10	iterators_1.0.14
## [40] future.apply_1.11.0	base64enc_0.1-3	Matrix_1.5-4
## [43] splines_4.3.0	nnet_7.3-18	timechange_0.2.0
## [46] tidyselect_1.2.0	rstudioapi_0.14	timeDate_4022.108
## [49] codetools_0.2-19	listenv_0.9.0	plyr_1.8.8
## [52] withr_2.5.0	evaluate_0.21	foreign_0.8-84
## [55] future_1.32.0	survival_3.5-5	xml2_1.3.4
## [58] pillar_1.9.0	mice_3.16.0	janeaustenr_1.0.0
## [61] stats4_4.3.0	checkmate_2.2.0	foreach_1.5.2
## [64] distributional_0.3.2	generics_0.1.3	munsell_0.5.0
## [67] scales_1.2.1	minqa_1.2.5	gtools_3.9.4
## [70] globals_0.16.2	class_7.3-21	glue_1.6.2
## [73] tools_4.3.0	tokenizers_0.3.0	lme4_1.1-33
## [76] ModelMetrics_1.2.2.2	gower_1.0.1	webshot_0.5.4
## [79] grid_4.3.0	ipred_0.9-14	colorspace_2.1-0
## [82] nlme_3.1-162	htmlTable_2.4.1	Formula_1.2-5
## [85] cli_3.6.1	rappdirs_0.3.3	fansi_1.0.4
## [88] viridisLite_0.4.2	svglite_2.1.1	lava_1.7.2.1
## [91] gtable_0.3.3	digest_0.6.31	farver_2.1.1
## [94] htmlwidgets_1.6.2	htmltools_0.5.5	lifecycle_1.0.3
## [97] hardhat_1.3.0	mitml_0.4-5	MASS_7.3-58.4