Linear Forms

11^{th} Maths - Chapter 10

The following problem is question 13 from exercise 10.3:

1. Find the equation of the right bisector of the line segment joining the points (3, 4) and (–1, 2).

Solution:
Given that

$$(x_1, y_1) = (3, 4) \tag{1}$$
$$(x_2, y_2) = (-1, 2) \tag{2}$$

The midpoint (x , y) is given by.

$$m(x, y) = \left(\frac{x1 + x2}{2}, \frac{y1 + y2}{2}\right) \tag{3}$$
$$= \left(\frac{3 - 1}{2}, \frac{4 + 2}{2}\right) \tag{4}$$
$$= (1, 3) \tag{5}$$

The direction vector of a line containing two points (1) and (2) is given by.

$$\mathbf{V} = \begin{pmatrix} x2 & -x1 \\ \\ y2 & -y1 \end{pmatrix} \tag{6}$$

$$= \begin{pmatrix} -1 & -3 \\ \\ 2 & 4 \end{pmatrix} \tag{7}$$

$$= \begin{pmatrix} -4 \\ \\ -2 \end{pmatrix} \tag{8}$$

The direction vector of right bisector is given by.

$$\mathbf{V_{perpendicular}} = \begin{pmatrix} 2 \\ \\ -4 \end{pmatrix} \tag{9}$$

The position vector $\mathbf{P}$ at (5) of line is given by.

$$\begin{pmatrix} x & -1 \\ \\ y & -3 \end{pmatrix} \tag{10}$$

The equation of line in vector form is given by.

$$\mathbf{V}.\mathbf{P} = \mathbf{V_{perpendicular}}.M \tag{11}$$

$$\begin{pmatrix} -4 \\ \\ -2 \end{pmatrix} \begin{pmatrix} x & -1 \\ \\ y & -3 \end{pmatrix} = \begin{pmatrix} 2 \\ \\ -4 \end{pmatrix} \begin{pmatrix} 1 \\ \\ 3 \end{pmatrix} \tag{12}$$

By simplifying this, we get

$$2x + y = 5 \tag{13}$$

Therefore, the above equation can be written as

$$\begin{pmatrix} 2 & 1 \end{pmatrix} \mathbf{x} = 5 \tag{14}$$

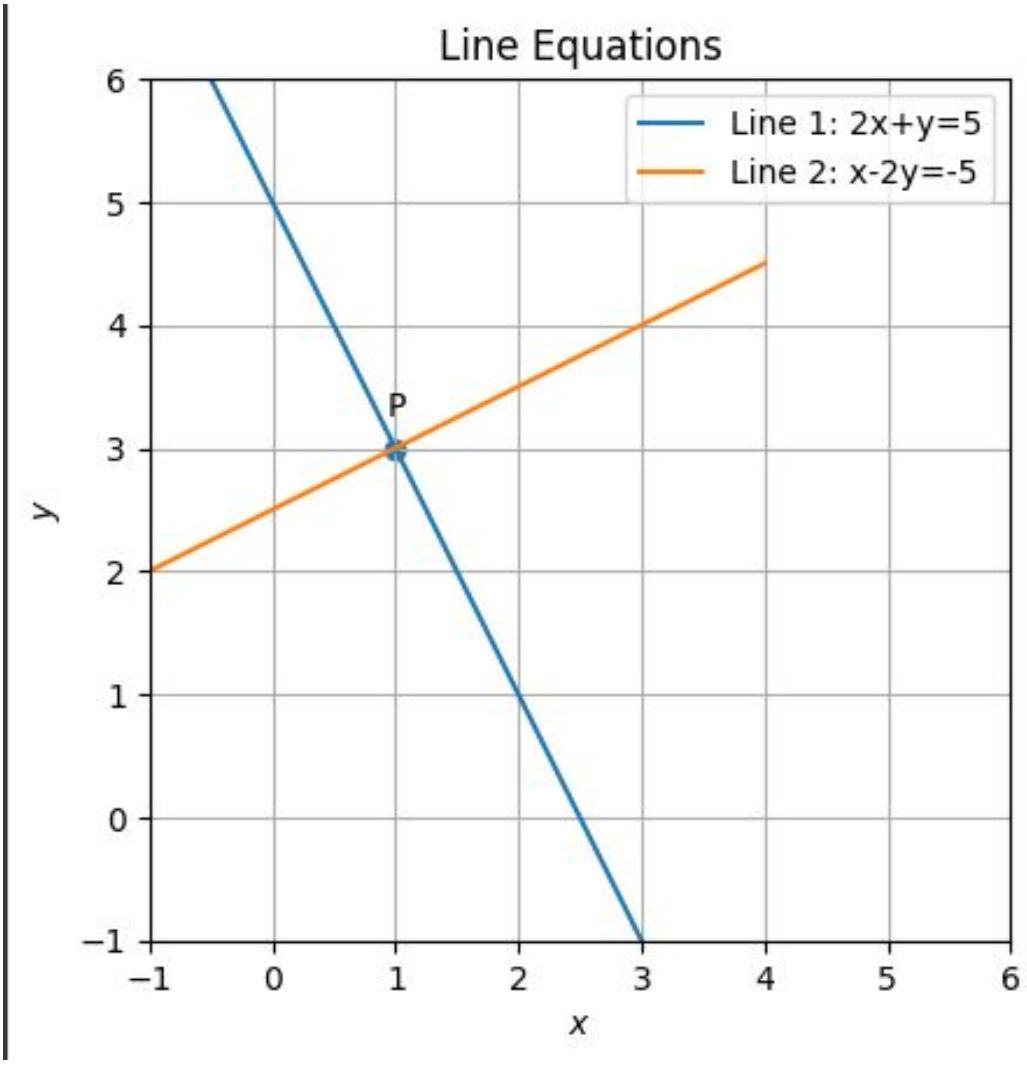

Figure 1: Graph