1. INTRODUCTION

In this section, we provide supplementary experimental results for the proposed EPNet. We compare EPNet-shared and EPNet-unshared with the component networks of LISTA [1], FirmNet [2], and SCAD-Net. For compressed sensing, we also include comparisons with FISTA [3], LAMP [4], and LISTA-CP [5]. For convolutional sparse coding, we compare the convolutional counterparts of EPNet, LISTA [1], FirmNet [2], and SCAD-Net, namely, ConvEPNet, ConvLISTA [6], ConFirmNet [7], and ConvSCADNet.

2. EXPERIMENTAL RESULTS

The R-SNR performance over different noise levels is shown in Fig. 1. The yellow line corresponds to LISTA, the blue line to FirmNet, and the red to EPNet-shared. The shaded regions showcase the variation in R-SNR by $\pm$ standard deviation. We observe that for $\sigma = 0.01$, EP-Net shared has superior performance over LISTA and FirmNet. A considerable amount of overlap of blue and red shaded regions indicates that the performance of FirmNet is on-par with that of EPNet-shared for $\sigma = 0.03$.

An analysis of the learning characteristics of ensemble weights $\{\alpha_i\}$ and network training is illustrated in Fig. 2 for $\sigma = 0.01$ and $\rho = 0.2$. The weights $\{\alpha_i\}$ are initialized as $1/3$ each. At the end of the training, α_{MCP} reached 0.899, whereas α_{ℓ_1} and α_{SCAD} settled at 0.0475 and 0.0531, respectively. This result shows that a significant portion of the ensemble network output is coming from the proximal operator of MCP. The higher contribution of the MCP results in the comparable performance of FirmNet and EPNet (cf. Fig. **??**). The reconstruction error (in red) plotted in Fig. 2 shows that both the training and validation errors (not normalized) converge with epochs. Further, we observe that the values of $\{\alpha_i\}$ start settling between epochs 0-20, and the training and validation loss also drop within the same epoch range.

Table 1) compares the performance of EPNet-unshared and EPNet-shared with the individual networks LISTA [1], FirmNet [2], and SCAD-Net, and other CS algorithms, namely, FISTA [3], LAMP [4], and LISTA-CP [5], for sparsity factor $\rho = 0.15$. EPNet-unshared and EPNet-shared outperform the compared techniques in amplitude recovery.

Table 2 presents the results of the convolutional counterparts of LISTA, FirmNet, and EPNet-shared, namely, ConvLISTA [6], ConFirm-

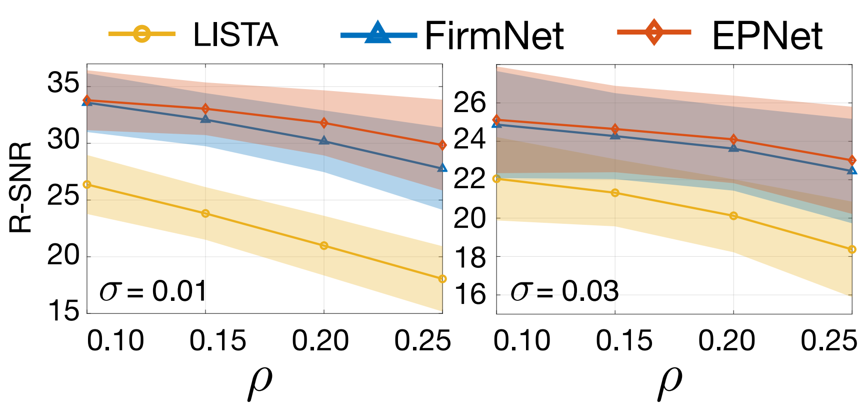

Fig. 1. R-SNR versus sparsity factor ρ for different values of σ. The shaded region indicates the mean $\pm$ standard deviation.

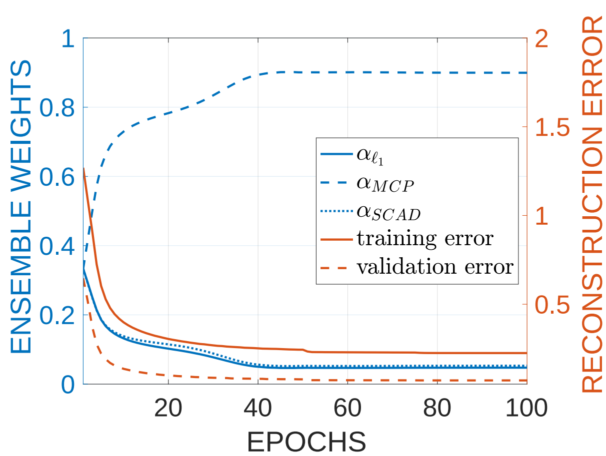

Fig. 2. Ensemble weight transition across EPNet's training.

Table 1. Comparison of various methods in the R-SNR (dB) metric for different noise levels with sparsity factor $\rho = 0.15$. The best entries are colored in red, and the second best in blue.

Network	**Noise level (σ)**				
	0.01	**0.03**	**0.05**	**0.07**	**0.1**
FISTA [3]	24.18±3.01	21.33±2.88	18.04±2.63	15.23±2.44	11.93±2.28
LAMP [4]	25.22±4.59	18.47±2.86	15.30±2.52	13.30±2.42	11.20±2.34
LISTA-CP [5]	27.84±4.74	19.63±2.64	17.70±7.94	13.23±2.05	10.58±1.59
LISTA [1]	25.74±1.64	21.32±1.75	17.91±1.85	15.56±1.91	13.00±1.92
SCAD-Net	31.81±2.33	24.12±2.21	20.00±2.24	17.34±2.24	14.34±2.23
FirmNet [2]	32.08±2.34	24.27±2.24	20.20±2.27	17.48±2.30	14.43±2.27
EPNet-shared	33.05±2.32	24.64±2.25	20.47±2.29	17.69±2.33	14.58±2.29
EPNet-unshared	33.88±2.48	24.90±2.25	20.89±2.27	18.00±2.33	14.87±2.32

Net [7], and ConvEPNet, for images from the Set12 dataset. The results show that ConvEPNet outperforms the benchmark methods. Fig. 3 compares the reconstructions obtained using ConvLISTA and ConvEPNet with unshared weights. The ConvEPNet reconstruction is close to the clean image in terms of contrast, while that of ConvLISTA appears to be similar to that of the noisy image. Table 3 presents results for all the images from the BSD68 dataset [8]. The results show that ConvEPNet-unshared outperforms all the benchmark methods, whereas the shared variant is on par with or better than the benchmarks.

Table 2. Comparison of various methods in the R-SNR (dB) metric ($\sigma = 0.098$) for images taken from the Set12 dataset. The best entries are in bold.

Image	ConvLISTA [6]	ConFirmNet [7]	ConvEPNet
Airplane	27.75 ± 0.040	27.73 ± 0.049	**27.85 ± 0.047**
Boat	28.79 ± 0.052	28.84 ± 0.022	**28.86 ± 0.051**
Cameraman	28.26 ± 0.041	28.32 ± 0.049	**28.45 ± 0.047**
Parrot	28.12 ± 0.044	28.18 ± 0.043	**28.24 ± 0.044**
Lena	30.14 ± 0.025	30.32 ± 0.027	**30.46 ± 0.031**
Pirate	28.93 ± 0.023	**28.97 ± 0.024**	**28.97 ± 0.023**
Monarch	28.64 ± 0.050	28.66 ± 0.047	**28.67 ± 0.049**
House	30.10 ± 0.054	30.41 ± 0.061	**30.65 ± 0.061**

Table 3: R-SNR (dB) based comparison of methods for $\sigma = 0.098$ for images from the BSD68 [8] dataset. The best entries are in bold face.

Index	ConvLISTA [6]	ConFirmNet [7]	ConvSCADNet	ConvEPNet-shared	ConvEPNet-unshared
1	30.49 ± 0.04	30.44 ± 0.02	30.16 ± 0.03	30.81 ± 0.03	**30.89 ± 0.04**
2	25.73 ± 0.02	25.74 ± 0.02	25.72 ± 0.02	25.66 ± 0.02	**25.98 ± 0.02**
3	28.78 ± 0.02	28.82 ± 0.03	28.73 ± 0.02	28.77 ± 0.03	**28.89 ± 0.03**
4	29.14 ± 0.02	29.24 ± 0.03	29.09 ± 0.03	29.17 ± 0.03	**29.30 ± 0.02**
5	26.67 ± 0.02	26.69 ± 0.02	26.60 ± 0.03	26.52 ± 0.03	**26.90 ± 0.02**
6	25.58 ± 0.02	25.55 ± 0.02	25.60 ± 0.02	25.52 ± 0.01	**25.82 ± 0.02**
7	26.13 ± 0.02	26.13 ± 0.02	26.11 ± 0.02	26.04 ± 0.03	**26.43 ± 0.02**
8	27.03 ± 0.01	26.99 ± 0.02	27.00 ± 0.02	27.07 ± 0.01	**27.17 ± 0.01**
9	23.72 ± 0.02	23.57 ± 0.02	23.69 ± 0.02	23.49 ± 0.02	**23.96 ± 0.02**
10	29.56 ± 0.03	29.93 ± 0.04	29.68 ± 0.04	29.84 ± 0.02	**30.11 ± 0.03**
11	30.22 ± 0.04	30.42 ± 0.03	30.01 ± 0.03	30.60 ± 0.04	**31.18 ± 0.04**
12	30.60 ± 0.04	30.65 ± 0.04	30.09 ± 0.04	30.85 ± 0.03	**31.42 ± 0.04**
13	27.68 ± 0.02	27.73 ± 0.04	27.65 ± 0.02	27.69 ± 0.02	**27.93 ± 0.02**
14	26.15 ± 0.02	25.99 ± 0.02	26.04 ± 0.02	25.97 ± 0.02	**26.26 ± 0.02**
15	30.10 ± 0.03	30.18 ± 0.03	29.90 ± 0.02	30.26 ± 0.01	**30.34 ± 0.03**
16	27.65 ± 0.02	27.72 ± 0.02	27.62 ± 0.02	27.61 ± 0.04	**28.01 ± 0.03**
17	25.29 ± 0.01	25.26 ± 0.02	25.25 ± 0.02	25.17 ± 0.01	**25.49 ± 0.02**
18	27.67 ± 0.03	27.72 ± 0.04	27.58 ± 0.02	27.66 ± 0.02	**28.02 ± 0.03**
19	24.25 ± 0.01	24.19 ± 0.02	24.19 ± 0.02	24.08 ± 0.02	**24.43 ± 0.01**

Index	ConvLISTA [6]	ConFirmNet [7]	ConvSCADNet	ConvEPNet-shared	ConvEPNet-unshared
20	23.37 $\pm$ 0.01	23.50 $\pm$ 0.02	23.54 $\pm$ 0.02	23.33 $\pm$ 0.01	**23.78 $\pm$ 0.02**
21	29.14 $\pm$ 0.03	29.38 $\pm$ 0.04	29.16 $\pm$ 0.02	29.33 $\pm$ 0.03	**29.67 $\pm$ 0.03**
22	27.41 $\pm$ 0.02	27.38 $\pm$ 0.02	27.36 $\pm$ 0.02	27.37 $\pm$ 0.02	**27.65 $\pm$ 0.02**
23	23.19 $\pm$ 0.02	23.23 $\pm$ 0.02	23.26 $\pm$ 0.02	23.03 $\pm$ 0.02	**23.47 $\pm$ 0.01**
24	26.13 $\pm$ 0.02	26.20 $\pm$ 0.02	26.11 $\pm$ 0.02	26.05 $\pm$ 0.01	**26.33 $\pm$ 0.02**
25	28.05 $\pm$ 0.02	28.26 $\pm$ 0.02	28.09 $\pm$ 0.03	28.15 $\pm$ 0.02	**28.54 $\pm$ 0.02**
26	28.79 $\pm$ 0.04	28.97 $\pm$ 0.03	28.80 $\pm$ 0.03	28.94 $\pm$ 0.03	**29.14 $\pm$ 0.04**
27	30.40 $\pm$ 0.04	30.86 $\pm$ 0.03	30.52 $\pm$ 0.03	30.69 $\pm$ 0.03	**31.08 $\pm$ 0.05**
28	27.30 $\pm$ 0.02	27.31 $\pm$ 0.03	27.23 $\pm$ 0.03	27.18 $\pm$ 0.03	**27.65 $\pm$ 0.02**
29	28.35 $\pm$ 0.03	28.56 $\pm$ 0.03	28.37 $\pm$ 0.04	28.54 $\pm$ 0.04	**28.88 $\pm$ 0.03**
30	26.62 $\pm$ 0.03	26.68 $\pm$ 0.02	26.64 $\pm$ 0.03	26.62 $\pm$ 0.02	**27.02 $\pm$ 0.03**
31	31.07 $\pm$ 0.04	30.95 $\pm$ 0.04	30.67 $\pm$ 0.03	31.51 $\pm$ 0.03	**31.47 $\pm$ 0.05**
32	27.72 $\pm$ 0.02	27.70 $\pm$ 0.02	27.70 $\pm$ 0.03	27.69 $\pm$ 0.03	**27.95 $\pm$ 0.02**
33	31.00 $\pm$ 0.04	31.19 $\pm$ 0.04	30.83 $\pm$ 0.05	31.47 $\pm$ 0.04	**31.62 $\pm$ 0.04**
34	27.65 $\pm$ 0.03	27.64 $\pm$ 0.02	27.58 $\pm$ 0.03	27.63 $\pm$ 0.03	**27.95 $\pm$ 0.03**
35	28.63 $\pm$ 0.01	28.61 $\pm$ 0.02	28.41 $\pm$ 0.03	28.72 $\pm$ 0.03	**28.92 $\pm$ 0.02**
36	25.40 $\pm$ 0.02	25.39 $\pm$ 0.03	25.36 $\pm$ 0.03	25.32 $\pm$ 0.02	**25.74 $\pm$ 0.02**
37	30.74 $\pm$ 0.02	31.22 $\pm$ 0.03	30.84 $\pm$ 0.05	31.22 $\pm$ 0.03	**31.49 $\pm$ 0.03**
38	27.02 $\pm$ 0.01	27.00 $\pm$ 0.02	26.98 $\pm$ 0.02	27.01 $\pm$ 0.02	**27.31 $\pm$ 0.02**
39	28.08 $\pm$ 0.03	28.02 $\pm$ 0.02	27.94 $\pm$ 0.02	28.09 $\pm$ 0.03	**28.35 $\pm$ 0.03**
40	27.39 $\pm$ 0.03	27.35 $\pm$ 0.02	27.37 $\pm$ 0.02	27.37 $\pm$ 0.02	**27.67 $\pm$ 0.02**
41	28.02 $\pm$ 0.02	28.09 $\pm$ 0.04	27.99 $\pm$ 0.02	28.05 $\pm$ 0.03	**28.33 $\pm$ 0.03**
42	30.07 $\pm$ 0.02	30.11 $\pm$ 0.03	29.87 $\pm$ 0.04	30.39 $\pm$ 0.02	**30.50 $\pm$ 0.03**
43	27.81 $\pm$ 0.02	27.65 $\pm$ 0.03	27.64 $\pm$ 0.02	27.86 $\pm$ 0.01	**27.99 $\pm$ 0.02**
44	30.46 $\pm$ 0.03	31.01 $\pm$ 0.04	30.63 $\pm$ 0.05	30.91 $\pm$ 0.03	**31.24 $\pm$ 0.04**
45	30.57 $\pm$ 0.03	30.96 $\pm$ 0.03	30.58 $\pm$ 0.02	30.96 $\pm$ 0.03	**31.37 $\pm$ 0.03**
46	29.72 $\pm$ 0.02	29.83 $\pm$ 0.02	29.67 $\pm$ 0.04	30.06 $\pm$ 0.03	**30.05 $\pm$ 0.03**
47	26.12 $\pm$ 0.02	26.02 $\pm$ 0.02	26.04 $\pm$ 0.02	25.99 $\pm$ 0.02	**26.21 $\pm$ 0.02**
48	28.77 $\pm$ 0.03	29.01 $\pm$ 0.03	28.84 $\pm$ 0.03	28.96 $\pm$ 0.03	**29.35 $\pm$ 0.04**
49	28.29 $\pm$ 0.03	28.40 $\pm$ 0.03	28.24 $\pm$ 0.02	28.46 $\pm$ 0.03	**28.74 $\pm$ 0.03**
50	28.76 $\pm$ 0.03	28.88 $\pm$ 0.03	28.79 $\pm$ 0.03	28.87 $\pm$ 0.03	**29.12 $\pm$ 0.03**
51	26.17 $\pm$ 0.01	26.01 $\pm$ 0.02	26.08 $\pm$ 0.02	25.97 $\pm$ 0.03	**26.43 $\pm$ 0.01**
52	27.68 $\pm$ 0.02	27.75 $\pm$ 0.04	27.68 $\pm$ 0.01	27.80 $\pm$ 0.02	**28.16 $\pm$ 0.03**
53	28.15 $\pm$ 0.03	28.33 $\pm$ 0.03	28.15 $\pm$ 0.02	28.28 $\pm$ 0.03	**28.65 $\pm$ 0.03**
54	25.15 $\pm$ 0.02	25.09 $\pm$ 0.02	25.08 $\pm$ 0.01	25.09 $\pm$ 0.02	**25.43 $\pm$ 0.02**
55	28.09 $\pm$ 0.03	28.10 $\pm$ 0.02	28.06 $\pm$ 0.02	28.05 $\pm$ 0.03	**28.32 $\pm$ 0.03**
56	26.32 $\pm$ 0.02	26.23 $\pm$ 0.01	26.24 $\pm$ 0.02	26.18 $\pm$ 0.01	**26.43 $\pm$ 0.02**
57	27.91 $\pm$ 0.03	27.93 $\pm$ 0.02	27.88 $\pm$ 0.03	27.94 $\pm$ 0.03	**28.26 $\pm$ 0.03**
58	25.78 $\pm$ 0.02	25.76 $\pm$ 0.02	25.77 $\pm$ 0.02	25.66 $\pm$ 0.02	**25.99 $\pm$ 0.02**
59	29.09 $\pm$ 0.03	29.21 $\pm$ 0.02	29.07 $\pm$ 0.02	29.34 $\pm$ 0.02	**29.45 $\pm$ 0.03**
60	32.40 $\pm$ 0.04	33.43 $\pm$ 0.04	32.64 $\pm$ 0.05	33.43 $\pm$ 0.06	**33.84 $\pm$ 0.06**
61	27.23 $\pm$ 0.02	27.28 $\pm$ 0.03	27.08 $\pm$ 0.03	27.17 $\pm$ 0.02	**27.66 $\pm$ 0.02**
62	26.90 $\pm$ 0.02	26.89 $\pm$ 0.03	26.86 $\pm$ 0.02	26.85 $\pm$ 0.03	**27.22 $\pm$ 0.02**
63	28.16 $\pm$ 0.03	28.27 $\pm$ 0.03	28.09 $\pm$ 0.02	28.27 $\pm$ 0.03	**28.52 $\pm$ 0.03**
64	28.81 $\pm$ 0.03	29.07 $\pm$ 0.03	28.88 $\pm$ 0.04	29.01 $\pm$ 0.03	**29.31 $\pm$ 0.03**
65	26.54 $\pm$ 0.02	26.53 $\pm$ 0.03	26.54 $\pm$ 0.02	26.44 $\pm$ 0.02	**26.83 $\pm$ 0.02**
66	26.88 $\pm$ 0.03	26.78 $\pm$ 0.02	26.79 $\pm$ 0.02	26.82 $\pm$ 0.03	**27.08 $\pm$ 0.03**
67	26.93 $\pm$ 0.03	26.87 $\pm$ 0.03	26.82 $\pm$ 0.02	26.91 $\pm$ 0.02	**27.25 $\pm$ 0.03**
68	29.36 $\pm$ 0.03	29.59 $\pm$ 0.03	29.39 $\pm$ 0.03	29.53 $\pm$ 0.03	**29.71 $\pm$ 0.04**

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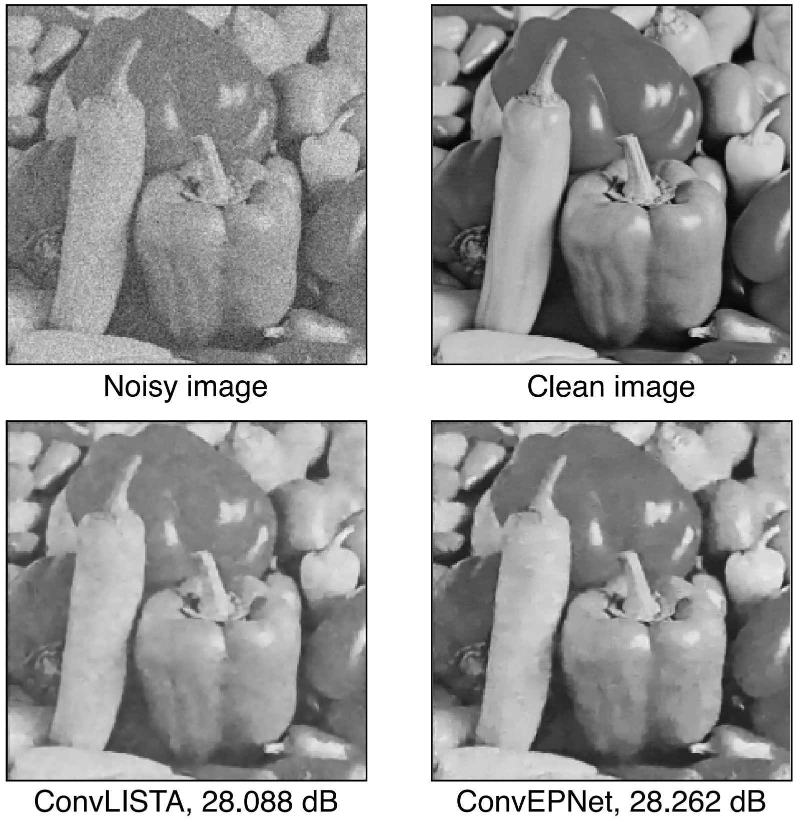

Fig. 3. Performance comparison of ConvLISTA [6] and ConvEPNet for image denoising ($\sigma = 0.118$) for an image from the Set12 dataset.

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