

1. INTRODUCTION

In this section, we provide supplementary experimental results for the proposed EPNet. We compare EPNet-shared and EPNet-unshared with the component networks of LISTA [1], FirmNet [2], and SCAD-Net. For compressed sensing, we also include comparisons with FISTA [3], LAMP [4], and LISTA-CP [5]. For convolutional sparse coding, we compare the convolutional counterparts of EPNet, LISTA [1], FirmNet [2], and SCAD-Net, namely, ConvEPNet, ConvLISTA [6], ConFirmNet [7], and ConvSCADNet.

2. EXPERIMENTAL RESULTS

The R-SNR performance over different noise levels is shown in Fig. 1. The yellow line corresponds to LISTA, the blue line to FirmNet, and the red to EPNet-shared. The shaded regions showcase the variation in R-SNR by \pm standard deviation. We observe that for $\sigma = 0.01$, EP-Net shared has superior performance over LISTA and FirmNet. A considerable amount of overlap of blue and red shaded regions indicates that the performance of FirmNet is on-par with that of EPNet-shared for $\sigma = 0.03$.

Tables 1, 2, 3, and 4 compare the performance of EPNet-unshared and EPNet-shared with the individual networks LISTA [1], FirmNet [2], and SCAD-Net, and other CS algorithms, namely, FISTA [3], LAMP [4], and LISTA-CP [5], for sparsity factors $\rho \in \{0.10, 0.15, 0.20, 0.25\}$. For sparsity factor 0.15 and 0.20, EPNet-unshared and EPNet-shared outperform the compared techniques in amplitude recovery. For sparsity factor 0.10 and 0.25, EPNet-unshared and EPNet-shared are on par with or better than the benchmark methods across all noise levels. For sparsity factor 0.10, the performance of EPNet-unshared deteriorates with increasing noise levels, whereas that of EPNet-shared is robust. For sparsity factor 0.25, EPNet-unshared performs marginally worse than the benchmark methods for noise levels 0.05 and 0.07, while it is on par with or better than the benchmark methods for noise levels 0.01, 0.03, and 0.1.

Table 2 presents the results of the convolutional counterparts of LISTA, FirmNet, and EPNet-shared, namely, ConvLISTA [6],

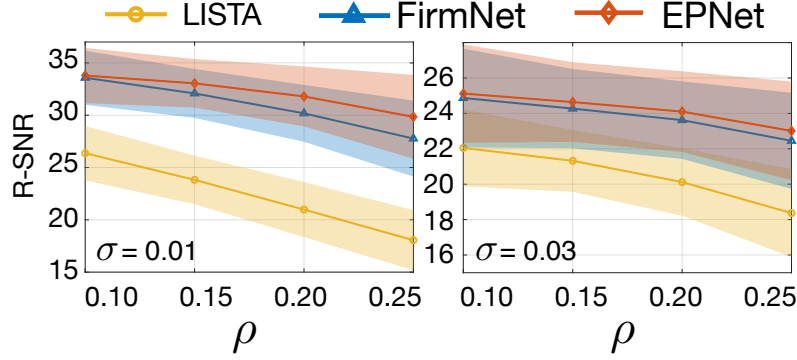


Fig. 1. R-SNR versus sparsity factor ρ for different values of σ . The shaded region indicates the mean \pm standard deviation.

Table 1. Comparison of various methods in the R-SNR (dB) metric for different noise levels with sparsity factor $\rho = 0.10$. The best entries are colored in red, and the second best in blue.

Network	Noise level (σ)				
	0.01	0.03	0.05	0.07	0.1
LISTA [1]	28.04 \pm 1.86	22.05 \pm 2.17	18.52 \pm 2.28	16.13 \pm 2.27	13.52 \pm 2.30
SCAD-Net	33.46 \pm 2.58	24.75 \pm 2.72	20.65 \pm 2.66	18.01 \pm 2.70	15.01 \pm 2.71
FirmNet [2]	33.58 \pm 2.58	24.87 \pm 2.78	20.86\pm2.74	18.16 \pm 2.79	15.14\pm2.77
EPNet-shared	33.79\pm2.64	25.12\pm2.78	21.06\pm2.74	18.33\pm2.79	15.27\pm2.78
EPNet-unshared	34.47\pm2.68	25.48\pm2.80	20.87\pm2.76	18.26\pm2.79	14.91 \pm 2.74

ConFirmNet [7], and ConvEPNet, for images from the Set12 dataset. The results show that ConvEPNet outperforms the benchmark methods. Fig. 2 compares the reconstructions obtained using ConvLISTA and ConvEPNet with unshared weights. The ConvEPNet reconstruction is close to the clean image in terms of contrast, while that of ConvLISTA appears to be similar to that of the noisy image. Tables 5 to 7 present results for all the images from the BSD68 dataset [8] for multiple noise levels. We report comparisons between BM3D [9], ConvLISTA [6], ConFirmNet [7], ConvSCADNet, CoRNet [10], ConvEPNet-shared, and ConvEPNet-unshared. The results show that ConvEPNet-unshared outperforms all the benchmark methods across all noise levels, whereas the shared variant is on par with or better than the benchmarks for $\sigma = 0.196, 0.392$. For $\sigma = 0.098$, the performance of ConvEPNet-shared is below par as compared to the benchmark methods.

Table 2. Comparison of various methods in the R-SNR (dB) metric for different noise levels with sparsity factor $\rho = 0.15$. The best entries are colored in red, and the second best in blue.

Network	Noise level (σ)				
	0.01	0.03	0.05	0.07	0.1
FISTA [3]	24.18 \pm 3.01	21.33 \pm 2.88	18.04 \pm 2.63	15.23 \pm 2.44	11.93 \pm 2.28
LAMP [4]	25.22 \pm 4.59	18.47 \pm 2.86	15.30 \pm 2.52	13.30 \pm 2.42	11.20 \pm 2.34
LISTA-CP [5]	27.84 \pm 4.74	19.63 \pm 2.64	17.70 \pm 7.94	13.23 \pm 2.05	10.58 \pm 1.59
LISTA [1]	25.74 \pm 1.64	21.32 \pm 1.75	17.91 \pm 1.85	15.56 \pm 1.91	13.00 \pm 1.92
SCAD-Net	31.81 \pm 2.33	24.12 \pm 2.21	20.00 \pm 2.24	17.34 \pm 2.24	14.34 \pm 2.23
FirmNet [2]	32.08 \pm 2.34	24.27 \pm 2.24	20.20 \pm 2.27	17.48 \pm 2.30	14.43 \pm 2.27
EPNet-shared	33.05 \pm 2.32	24.64 \pm 2.25	20.47 \pm 2.29	17.69 \pm 2.33	14.58 \pm 2.29
EPNet-unshared	33.88 \pm 2.48	24.90 \pm 2.25	20.89 \pm 2.27	18.00 \pm 2.33	14.87 \pm 2.32

Table 3. Comparison of various methods in the R-SNR (dB) metric for different noise levels with sparsity factor $\rho = 0.20$. The best entries are colored in red, and the second best in blue.

Network	Noise level (σ)				
	0.01	0.03	0.05	0.07	0.1
LISTA [1]	23.26 \pm 1.99	20.11 \pm 1.89	17.03 \pm 1.90	14.75 \pm 1.91	12.25 \pm 1.87
SCAD-Net	29.84 \pm 2.65	23.43 \pm 2.15	19.31 \pm 2.13	16.55 \pm 2.19	13.52 \pm 2.19
FirmNet [2]	30.17 \pm 2.72	23.62 \pm 2.19	19.45 \pm 2.16	16.67 \pm 2.20	13.57 \pm 2.22
EPNet-shared	31.79 \pm 2.86	24.09 \pm 2.28	19.74 \pm 2.17	16.87 \pm 2.22	13.72 \pm 2.24
EPNet-unshared	33.10 \pm 2.88	24.34 \pm 2.25	20.12 \pm 2.22	17.07 \pm 2.28	13.90 \pm 2.27

Table 4. Comparison of various methods in the R-SNR (dB) metric for different noise levels with sparsity factor $\rho = 0.25$. The best entries are colored in red, and the second best in blue.

Network	Noise level (σ)				
	0.01	0.03	0.05	0.07	0.1
LISTA [1]	20.61 \pm 2.70	18.37 \pm 2.49	15.77 \pm 2.28	13.68 \pm 2.15	11.36 \pm 2.00
SCAD-Net	27.36 \pm 3.56	22.23 \pm 2.68	18.30 \pm 2.47	15.47 \pm 2.42	12.50 \pm 2.27
FirmNet [2]	27.76 \pm 3.62	22.44 \pm 2.71	18.38 \pm 2.49	15.52 \pm 2.43	12.53 \pm 2.26
EPNet-shared	29.84 \pm 4.00	23.01 \pm 2.78	18.66 \pm 2.52	15.69 \pm 2.45	12.65 \pm 2.27
EPNet-unshared	30.48 \pm 3.77	22.92 \pm 2.67	18.29 \pm 2.48	15.38 \pm 2.43	12.73 \pm 2.29

Table 5. R-SNR (dB) based comparison of methods for $\sigma = 0.196$ for images from the BSD68 [8] dataset. The best entries are in red, and the second best are in blue.

Index	BM3D [9]	ConvLISTA [6]	ConFirmNet [7]	ConvSCADNet	CoRNet [10]	ConvEPNet-shared	ConvEPNet-unshared
1	28.40 \pm 0.04	28.43 \pm 0.06	28.41 \pm 0.05	27.84 \pm 0.03	28.23 \pm 0.05	28.68 \pm 0.04	28.97 \pm 0.06
2	21.92 \pm 0.03	22.59 \pm 0.03	22.62 \pm 0.03	22.53 \pm 0.03	22.59 \pm 0.05	22.58 \pm 0.03	22.80 \pm 0.03
3	25.68 \pm 0.04	26.16 \pm 0.02	26.05 \pm 0.05	25.96 \pm 0.03	26.06 \pm 0.04	26.15 \pm 0.04	26.25 \pm 0.04
4	25.78 \pm 0.04	26.45 \pm 0.03	26.38 \pm 0.04	26.21 \pm 0.03	26.27 \pm 0.04	26.44 \pm 0.04	26.61 \pm 0.04
5	23.79 \pm 0.02	24.14 \pm 0.02	24.09 \pm 0.02	24.09 \pm 0.04	24.19 \pm 0.03	24.14 \pm 0.02	24.28 \pm 0.03
Average	25.01 \pm 2.58	25.19 \pm 2.26	25.17 \pm 2.25	24.95 \pm 2.08	25.14 \pm 2.22	25.31 \pm 2.38	25.54 \pm 2.43

Table 6. R-SNR (dB) based comparison of methods for $\sigma = 0.294$ for images from the BSD68 [8] dataset. The best entries are in red, and the second best are in blue.

Index	BM3D [9]	ConvLISTA [6]	ConFirmNet [7]	ConvSCADNet	CoRNet [10]	ConvEPNet-shared	ConvEPNet-unshared
1	26.79 \pm 0.06	27.01 \pm 0.04	26.74 \pm 0.07	25.32 \pm 0.08	27.22 \pm 0.05	26.94 \pm 0.08	27.65 \pm 0.09
2	20.38 \pm 0.03	20.82 \pm 0.03	20.85 \pm 0.04	19.85 \pm 0.01	20.88 \pm 0.04	20.87 \pm 0.03	21.07 \pm 0.03
3	24.23 \pm 0.02	24.87 \pm 0.03	24.68 \pm 0.04	24.35 \pm 0.05	24.82 \pm 0.05	24.76 \pm 0.04	24.93 \pm 0.04
4	24.35 \pm 0.06	25.02 \pm 0.07	24.84 \pm 0.05	24.22 \pm 0.04	25.00 \pm 0.05	24.88 \pm 0.04	25.15 \pm 0.04
5	22.56 \pm 0.02	22.9 \pm 0.05	22.84 \pm 0.03	22.54 \pm 0.03	22.91 \pm 0.03	22.87 \pm 0.05	23.03 \pm 0.03
Average	23.49 \pm 2.45	23.7 \pm 2.29	23.61 \pm 2.21	22.63 \pm 1.94	23.81 \pm 2.40	23.72 \pm 2.29	24.06 \pm 2.47

Table 7. R-SNR (dB) based comparison of methods for $\sigma = 0.392$ for images from the BSD68 [8] dataset. The best entries are in red, and the second best are in blue.

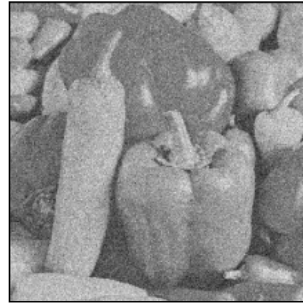
Index	BM3D [9]	ConvLISTA [6]	ConFirmNet [7]	ConvSCADNet	CoRNet [10]	ConvEPNet-shared	ConvEPNet-unshared
1	25.68 \pm 0.04	25.86 \pm 0.07	24.78 \pm 0.05	24.86 \pm 0.08	25.01 \pm 0.04	26.39 \pm 0.04	26.60 \pm 0.07
2	19.43 \pm 0.03	19.77 \pm 0.04	19.11 \pm 0.01	19.08 \pm 0.02	19.31 \pm 0.02	19.82 \pm 0.04	19.92 \pm 0.03
3	23.22 \pm 0.06	23.89 \pm 0.05	23.57 \pm 0.02	23.63 \pm 0.06	23.51 \pm 0.04	23.95 \pm 0.04	24.01 \pm 0.02
4	23.35 \pm 0.05	23.96 \pm 0.07	23.48 \pm 0.05	23.57 \pm 0.03	23.35 \pm 0.03	24.08 \pm 0.05	24.16 \pm 0.05
5	21.68 \pm 0.03	22.07 \pm 0.04	21.86 \pm 0.06	21.90 \pm 0.04	22.00 \pm 0.04	22.13 \pm 0.02	22.26 \pm 0.03
Average	22.43 \pm 2.31	22.68 \pm 2.19	21.91 \pm 1.99	21.93 \pm 2.03	22.07 \pm 1.95	22.86 \pm 2.35	23.06 \pm 2.44

Table 8. R-SNR (dB) based comparison of methods for $\sigma = 0.490$ for images from the BSD68 [8] dataset. The best entries are in red, and the second best are in blue.

Index	BM3D [9]	ConvLISTA [6]	ConFirmNet [7]	ConvSCADNet	CoRNet [10]	ConvEPNet-shared	ConvEPNet-unshared
1	24.81 \pm 0.07	25.08 \pm 0.09	24.45 \pm 0.09	24.46 \pm 0.06	25.02 \pm 0.05	26.39 \pm 0.04	26.60 \pm 0.07
2	18.74 \pm 0.02	18.92 \pm 0.03	18.54 \pm 0.02	18.58 \pm 0.02	18.96 \pm 0.05	19.06 \pm 0.04	19.08 \pm 0.04
3	22.4 \pm 0.07	23.23 \pm 0.08	23.1 \pm 0.05	23.06 \pm 0.05	23.24 \pm 0.06	23.26 \pm 0.05	23.37 \pm 0.04
4	22.64 \pm 0.06	23.25 \pm 0.07	23.03 \pm 0.03	22.98 \pm 0.06	23.21 \pm 0.06	23.25 \pm 0.04	23.5 \pm 0.05
5	21.0 \pm 0.04	21.54 \pm 0.05	21.38 \pm 0.03	21.38 \pm 0.02	21.52 \pm 0.04	21.54 \pm 0.05	21.64 \pm 0.05
Average	21.60 \pm 2.19	21.88 \pm 2.17	21.41 \pm 2.07	21.40 \pm 2.04	21.86 \pm 2.15	21.97 \pm 2.18	22.22 \pm 2.38

Table 9. Comparison of various methods in the R-SNR (dB) metric ($\sigma = 0.098$) for images taken from the Set12 dataset. The best entries are in red.

Image	ConvLISTA [6]	ConFirmNet [7]	ConvEPNet
Airplane	27.75 \pm 0.040	27.73 \pm 0.049	27.85 \pm 0.047
Boat	28.79 \pm 0.052	28.84 \pm 0.022	28.86 \pm 0.051
Cameraman	28.26 \pm 0.041	28.32 \pm 0.049	28.45 \pm 0.047
Parrot	28.12 \pm 0.044	28.18 \pm 0.043	28.24 \pm 0.044
Lena	30.14 \pm 0.025	30.32 \pm 0.027	30.46 \pm 0.031
Pirate	28.93 \pm 0.023	28.97 \pm 0.024	28.97 \pm 0.023
Monarch	28.64 \pm 0.050	28.66 \pm 0.047	28.67 \pm 0.049
House	30.10 \pm 0.054	30.41 \pm 0.061	30.65 \pm 0.061



Noisy image



Clean image



ConvLISTA, 28.088 dB



ConvEPNet, 28.262 dB

Fig. 2. Performance comparison of ConvLISTA [6] and ConvEPNet for image denoising ($\sigma = 0.118$) for an image from the Set12 dataset.

3. REFERENCES

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