Image reconstruction by combination of lattices

Tomographic image reconstruction

- *Tomography* is an imaging technique which helps to visualize the internal structures of the body

- Cross-sectional images of the body are generated from projection data using Radon transform (sinogram).

- Tomographic imaging modalities includes x-ray CT (computed tomography) magnetic resonance and nuclear-imaging (SPECT and PET) techniques.

Radon transform and Back projection

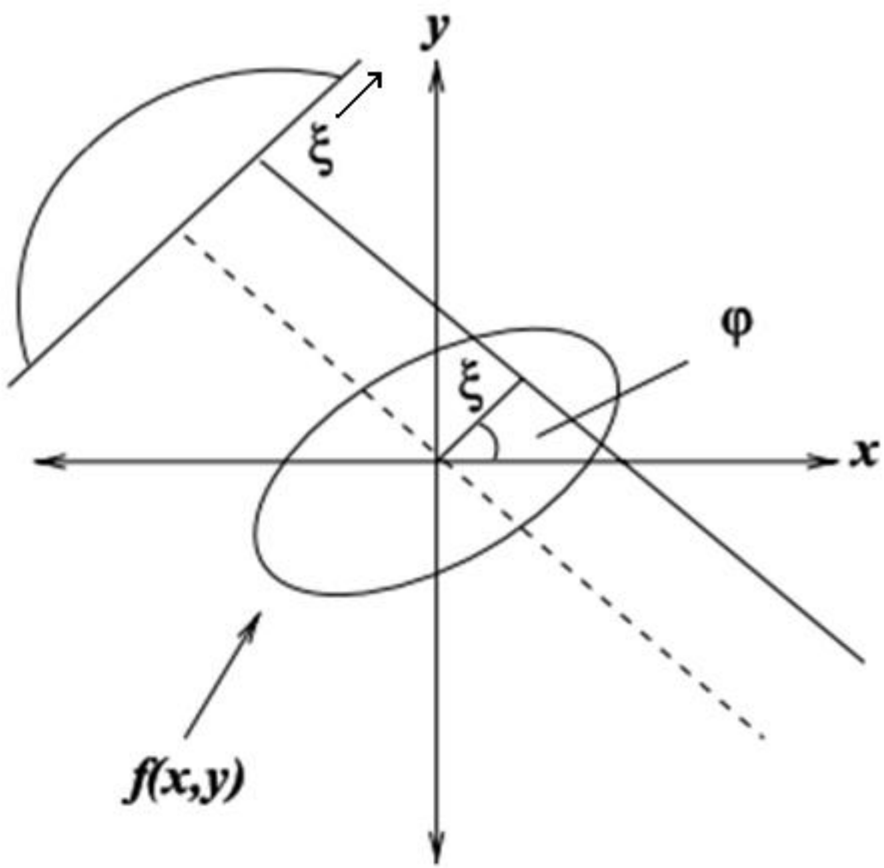
- The Radon transform is an integral transform whose inverse is used to reconstruct images

- The Radon transform (RT) of a cross section *f(x, y)* is given as

$$L(p,\theta) = \int\int f(x,y)\,\delta(p - x\cos\theta - y\sin\theta)\,dx\,dy$$

- The back projection operator is given as

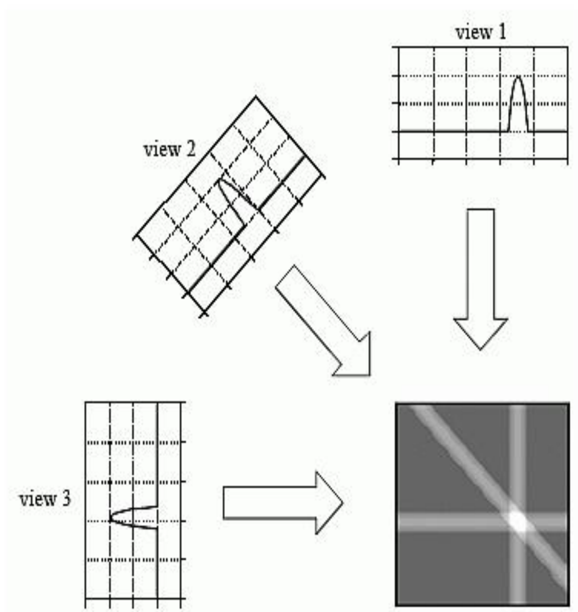
$$f_{BP}(x,y) = \int_{0}^{\pi} L(x\cos\theta + y\sin\theta, \theta)\,d\theta$$



Projection of *f(x,y)* using Radon transform

2D Tomographic Reconstruction Using Filtered Back Projection

Object (continuous) → Radon transform → Sinogram $p(\varphi,\xi)$ → DFT(1D) (in ξ) → Filtering → Inverse DFT (1D) → Back Projection (on a lattice) → Reconstructed digital image



Problem motivation and definition

Motivation

- Nuclear imaging modalities such as PET (positron emission tomography) and SPECT (Single photon emission computed tomography) produce images of poor resolution (typically 128x128).

  - Higher resolution images are often required
    - to view details
    - for multimodal registration

Problem

- To derive higher resolution tomographic images of good quality

State of the art

Options for improving resolution

Direct upsampling using a suitable kernel

Computationally inexpensive but image quality deteriorates (image definition is lost)

Super resolution techniques

(i) projection data is obtained after applying shifts and rotations to the object [kennedy]

(ii) reconstruction by union of low resolution images on shifted lattices [chang]

Super resolution technique 1 [Kennedy]

- Multiple low resolution images are acquired sequentially by translating and/or rotating an object by a subpixel distance with respect to the previous acquisition

- In practice, this has two major disadvantages :

  - increased scan duration which can increase the probability of patient motion

  - Difficulty in implementing rotation of patient

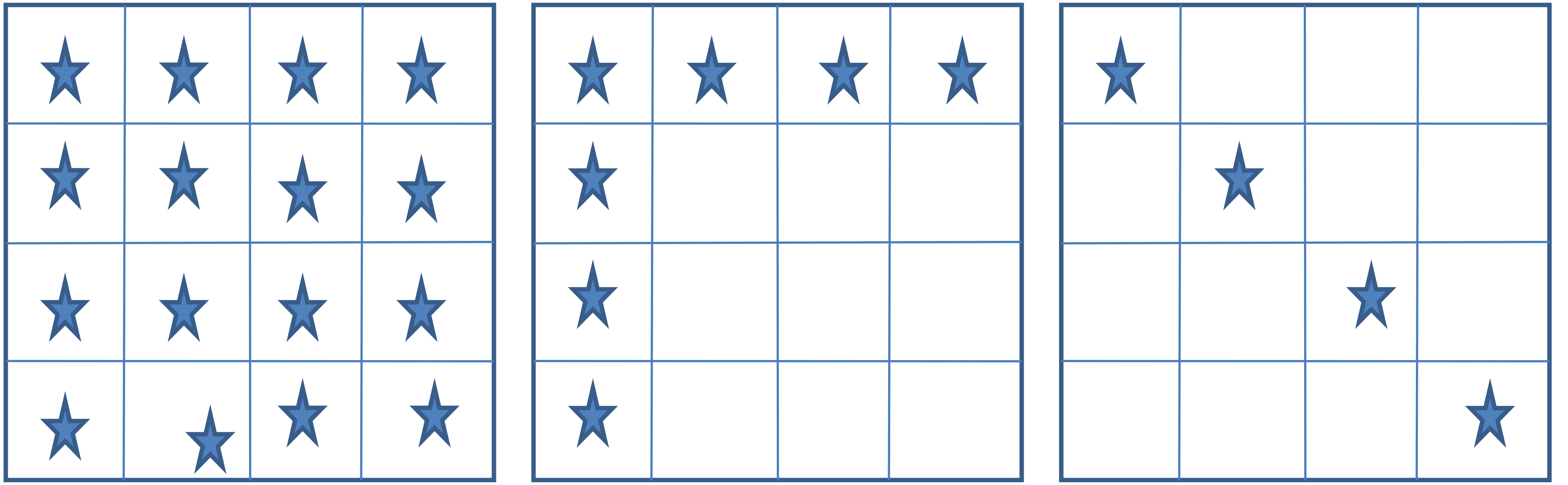
Super resolution technique 2 [Chang]

- Addresses the disadvantages in Kennedy's proposal by applying SR to the reconstructed images sourced from a single sinogram

- Low resolution images are generated by back-projecting the sinogram data onto sub-pixel shifted grids

- SR image is obtained as a union of images defined on shifted lattices (USL)

- In USL-based technique, $n*n = n^2$ shifted images are required for generating an up-sampled (by n) image
  - A reduction to (2n-1) or to n images has been proposed in [chang 2nd paper]



(a) (b) (c)

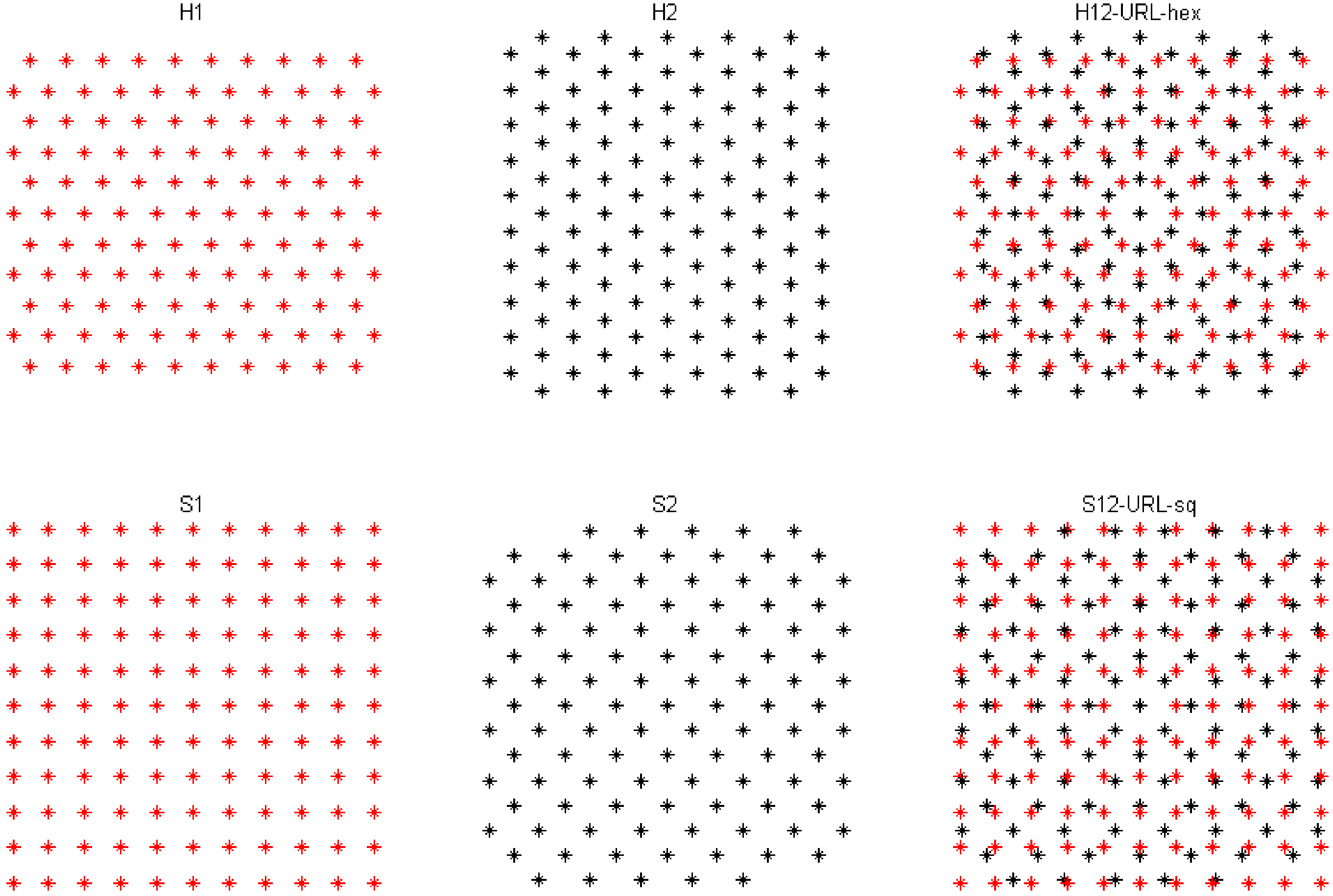
Required low resolution images (marked by stars) required for deriving an up-sampled (by 4) image with SR: (a) 16 (b) 7 with ISR-1 algorithm and (c) 4 with ISR-2 algorithm.

Proposed solution

- **Problem**: To obtain higher resolution tomographic images of good quality
- **Proposed solution**: we derive the high resolution image from union of rotated low resolution images defined on hexagonal or square lattices.
- **Notation** :

  URL-sq: union of 2 rotated images defined on square lattices

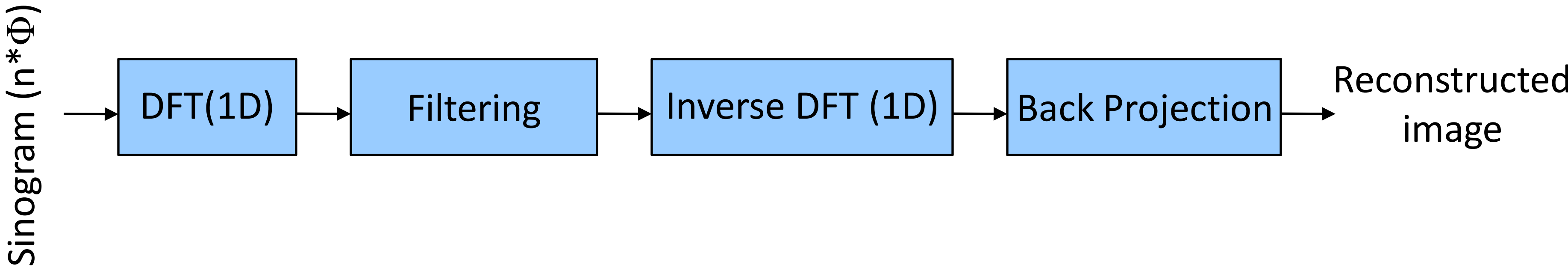
  URL-hex: union of 2 rotated images defined on hexagonal lattices.



H1: shows samples on hex lattice with basis vector (1/2,1),(0,√3/2). H2: shows the samples on rotated hex lattice generated by rotating H1 basis vectors by 30 . H12: shows quasi-periodic sampled samples generated by combining H1 and H2.

S1: shows samples on square lattice with basis vector (1,0),(0,1). S2: shows the samples on rotated square lattice generated by rotating S1 basis vectors by 45 . S12: shows quasi-periodic sampled samples generated by combining S1 and S2.

Implementation: Filtered back projection

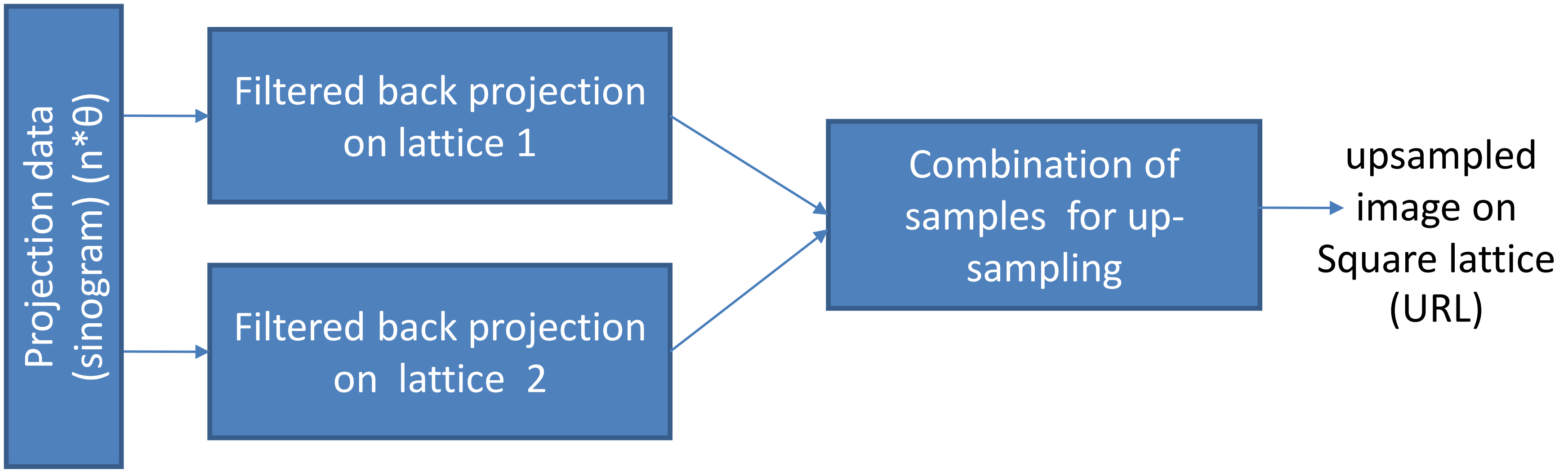


Processing stage in filtered back projection

- Sinogram of size (n*Φ) where n is number of detectors and Φ is number if angular samples, is filtered and back projected to generate the desired image.

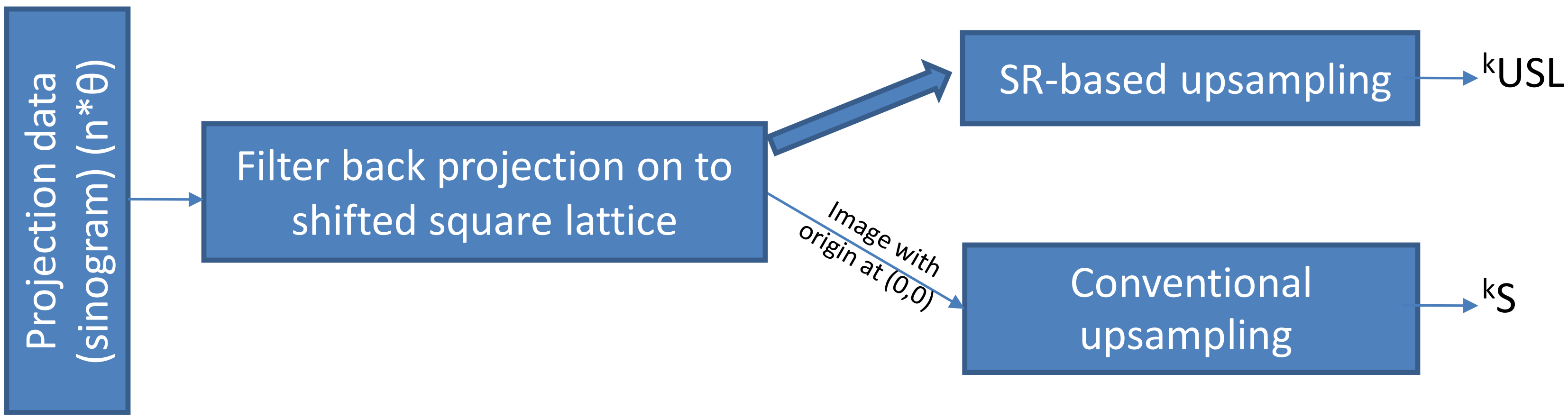
- It is possible to generate the samples on a lattice at arbitrary orientation (θ) using this algorithm

Generating URL images :



- Samples are interpolated using non-uniform thin plate splines.
- ^{k}URL-sq : When lattice 1 and 2 are square and rotated square.
- ^{k}URL-hex : when lattice 1 and 2 are hex and rotated hex.
- K: up-sampling factor

Generating USL and direct up-sampled image for comparison



Generating direct up-sampled image: ^{k}S

^{k}S is derived using a bicubic interpolation kernel

Generating ^{k}USL image

- ^{k}USL: Image was generated by implementing ISR-2 algorithm, i.e. combining only diagonal shifted images.
-
- Low resolution sub pixel shifted images are generated using Fessler's toolbox.

- Two types SR images ^{k}USL were generated using Vanderwalle's toolbox

  a) ^{k}USL-inter : samples from low resolution images are placed on high resolution grid and interpolated using bi-cubic interpolation kernel.

  b) ^{k}USL-struct : High resolution image was generated from low resolution images by implementing structure adaptive convolution algorithm.

Phantoms

- Three analytic phantoms were designed to generate the projection data and used in our experiments:

a)*Dots* to compare the contrast and spatial resolution of the up-sampled image.

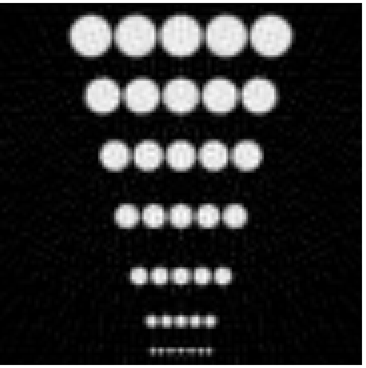
b)*Lines* to study the effect on lines at different angular orientation .

c)*Concentric rings* (with spacing between the circles being inversely proportional to the distance from the centre) to study preservation of on curved structure.

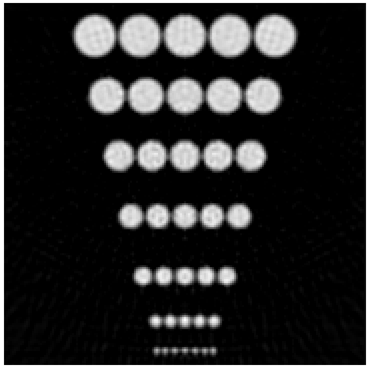
- Sinograms were generated assuming ideal parallel beam projections.
  - i.e. scanner characteristics are not taken under consideration

Results

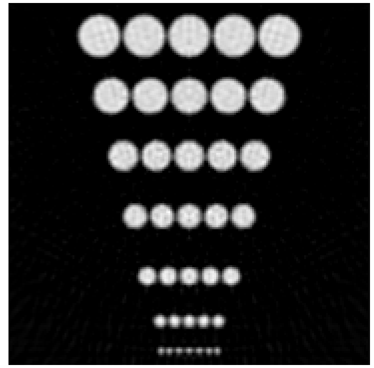
Dots up-sampled by factor 2



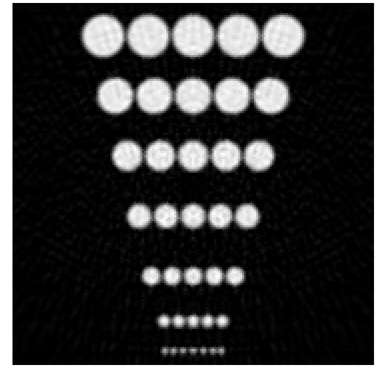
^{2}S



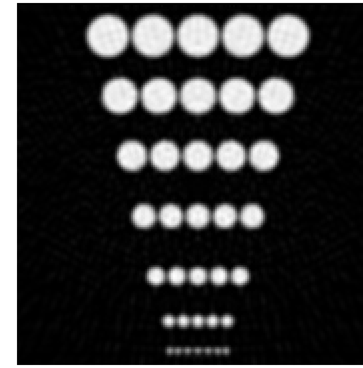
^{2}URL_sq



^{2}URL_hex

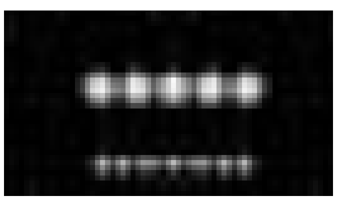


^{2}USL_inter

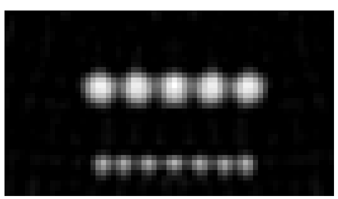


^{2}USL_struct

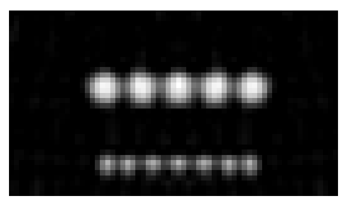
Zoomed ROI :



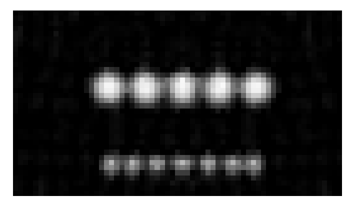
^{2}S



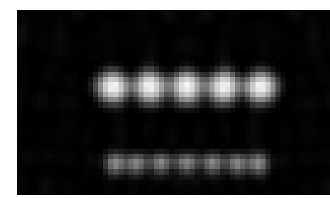
^{2}URL_sq



^{2}URL_hex



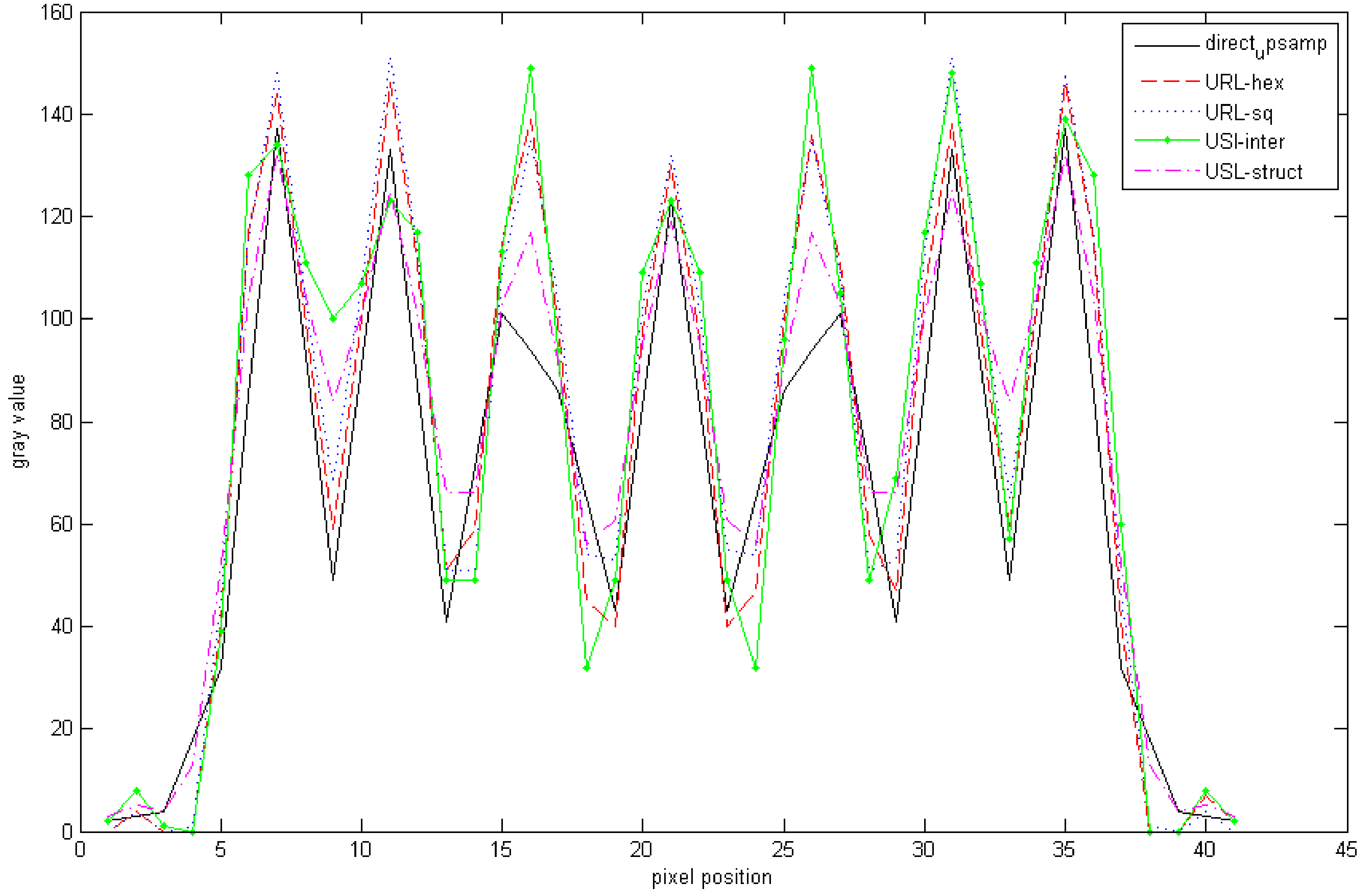
^{2}USL_inter



^{2}USL_struct

Size of Low resolution image: 101*101

Size of SR image (upsampled by 2) : 202*202

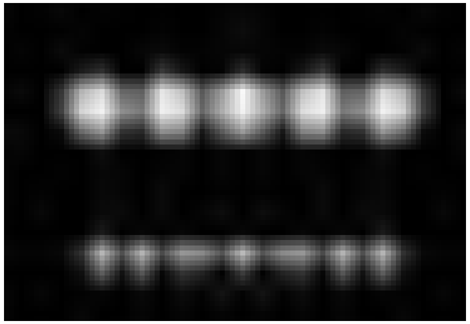


Line profile of the last line of dots

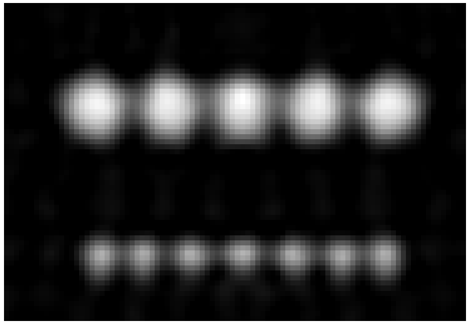
- From the zoomed images its evident that direct up-sampling method deforms the shape of the dots and USL-struct introducing smoothing which blurs the image.

- From the radial line profile we can conclude that contrast and resolution URL and USL-inter is better than USL-struct and direct upsampled image.

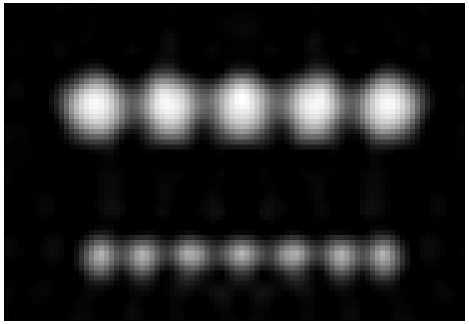
Zoomed roi of dots up-sampled by factor 4



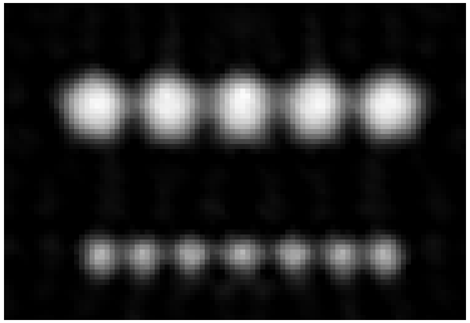
^{4}S



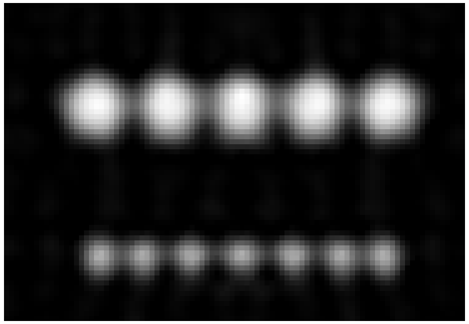
^{4}URL_sq



^{4}URL_hex

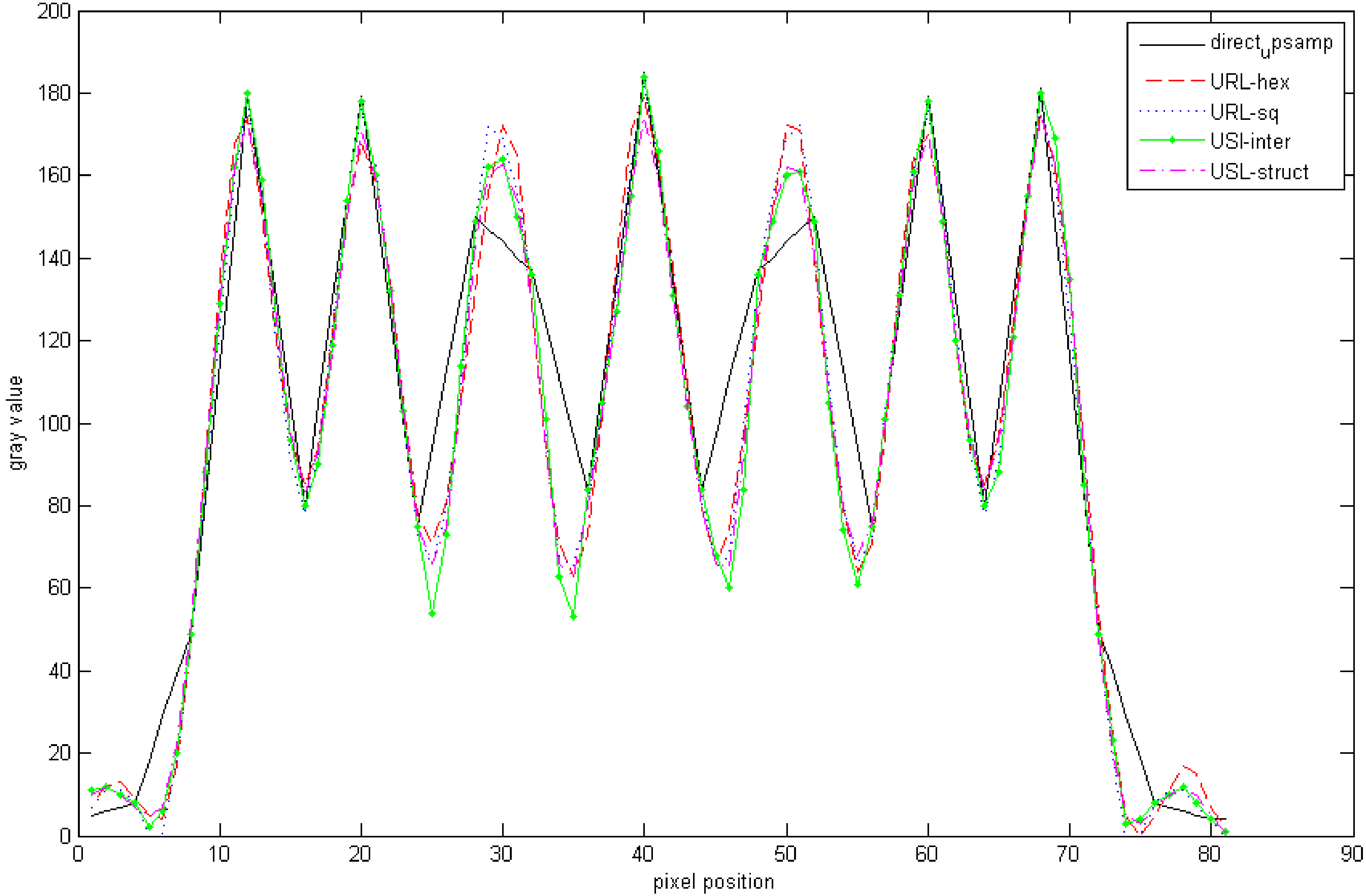


^{4}USL_inter



^{4}USL_struct

Size of Low resolution image: 101*101
Size of SR image (upsampled by 4) : 404*404

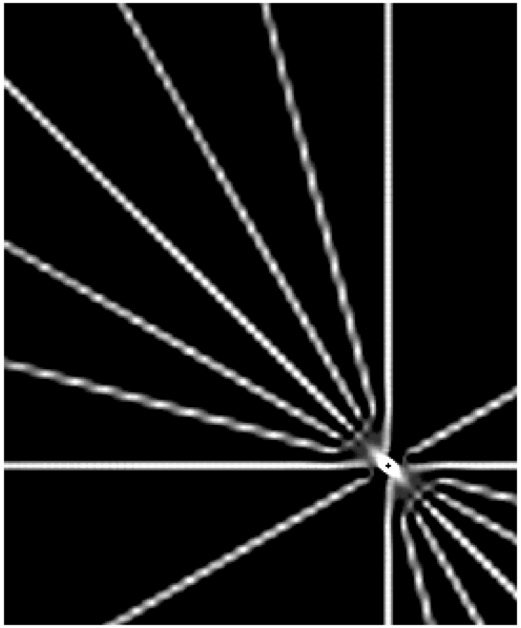


Line profile of the last line

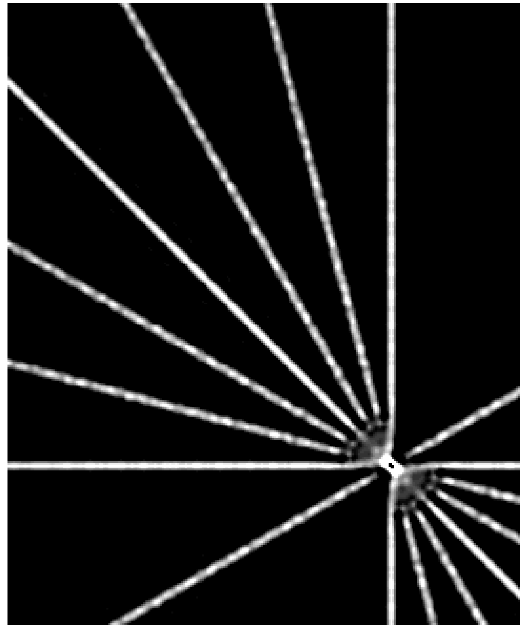
- From the zoomed images its evident that direct upsampling and USL-inter deforms the shape of dots
- Percentage of distortion is more for direct up-sampling

- Radial line profiles shows that the contrast and resolution of URL and USL-inter is nearly same and is better than USL-struct and direct upsampled image

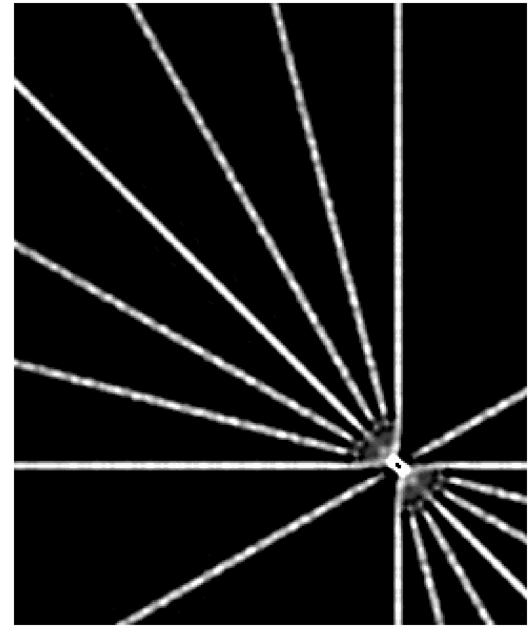
Lines at different orientation



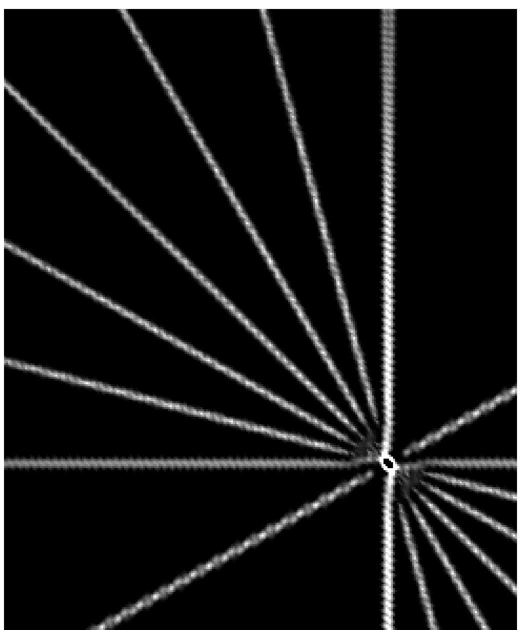
^{4}S



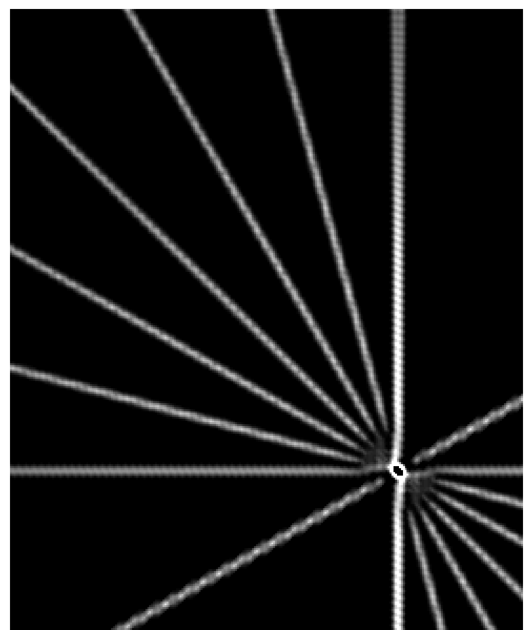
^{4}URL_sq



^{4}URL_hex



^{4}USL_inter



^{4}USL_struct

Size of Low resolution image: 121*121
Size of SR image (upsampled by 4) :484*484

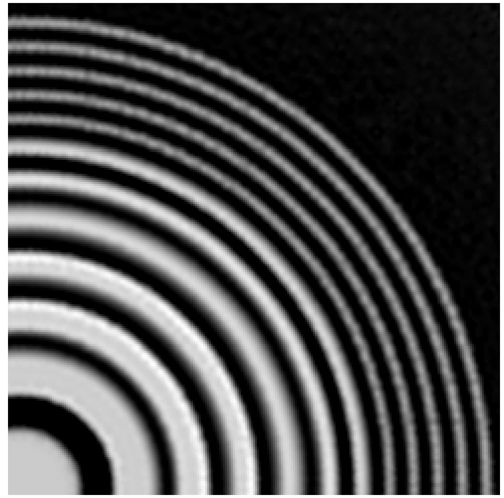
- URL images are able to preserve the definition of lines better than USL and direct upsampled image

- Direct up-sampling introduces maximum distortion in image.

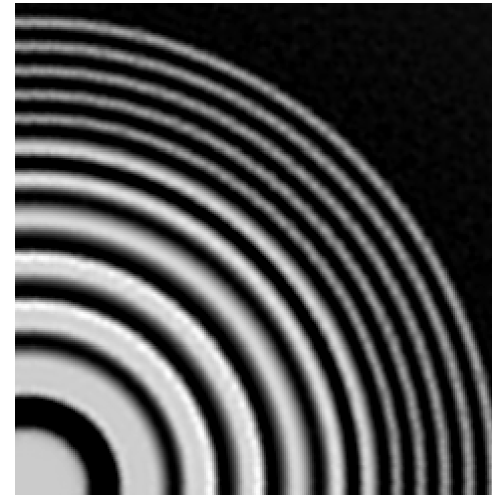
- In USL images distortion is prominent in vertical line. Width is narrower near the centre but as it reaches the periphery it increases.

Concentric rings

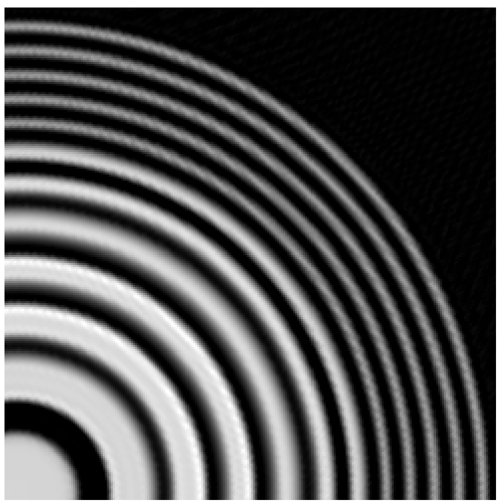
^{4}S



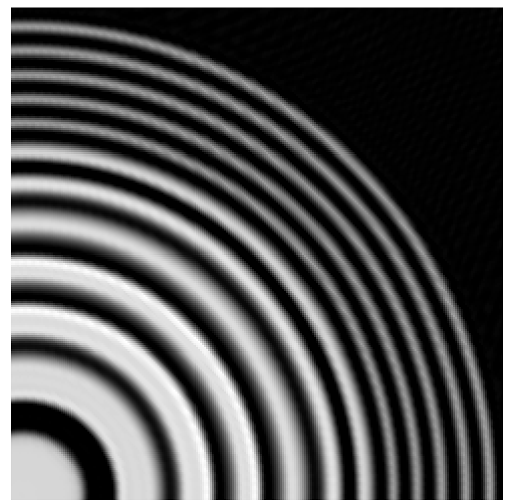
^{4}URL_sq



^{4}URL_hex

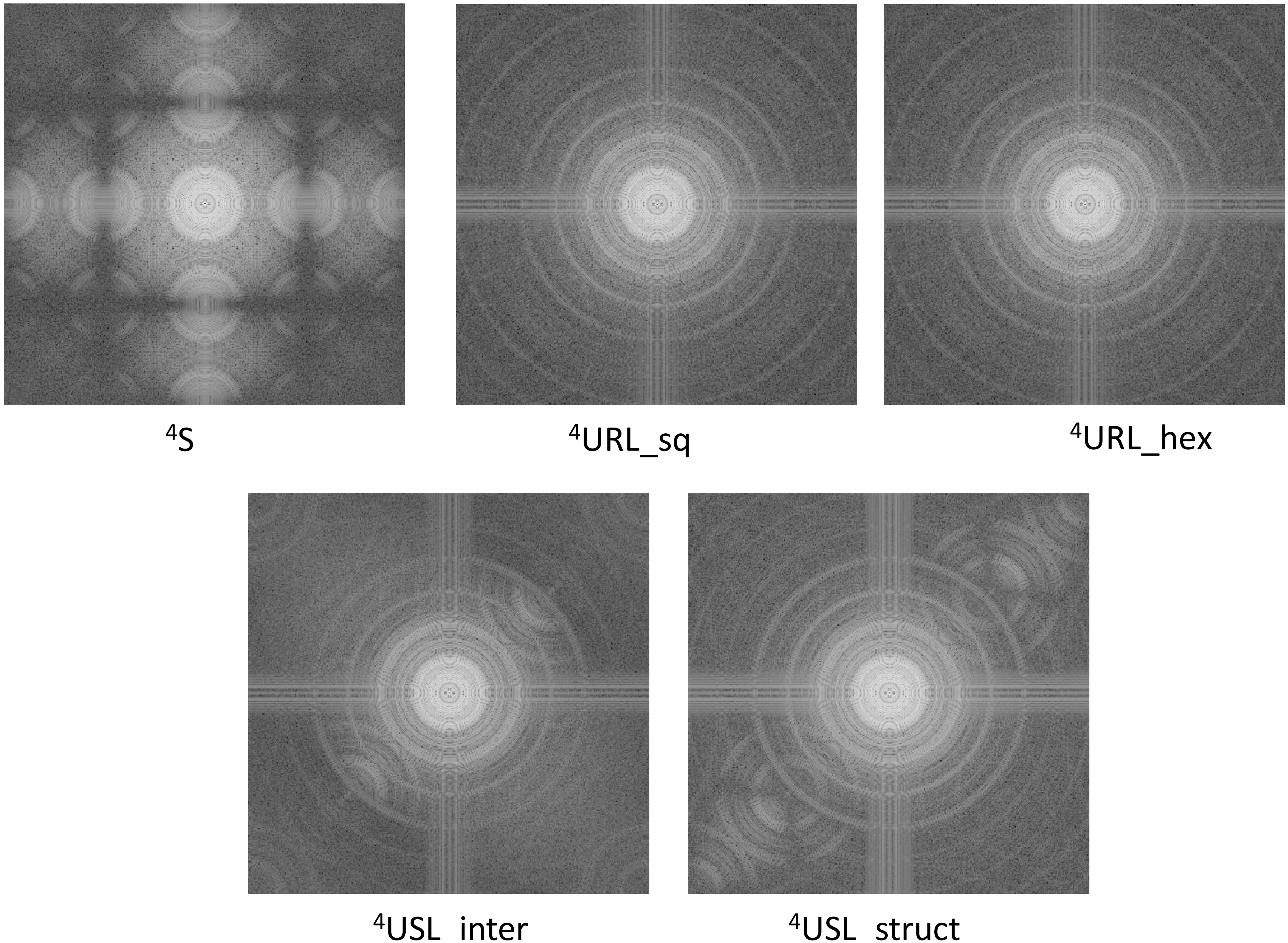


^{4}USL_inter

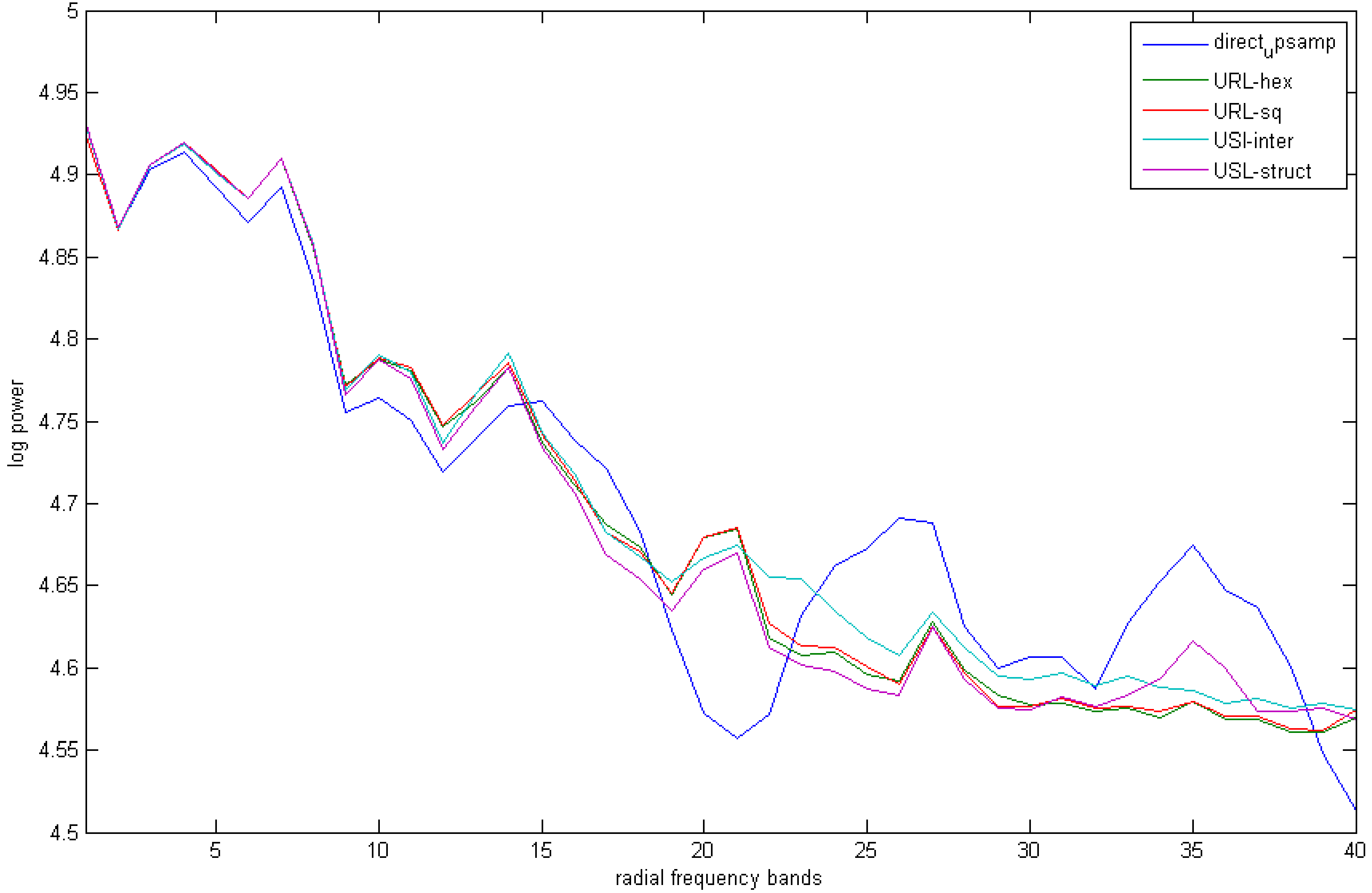


^{4}USL_struct

Roi of concentric rings upsampled by 4



Spectra of rings upsampled by factor 4



1D Power Spectral Plot for Concentric Ring images
Up-sampled by a factor 4

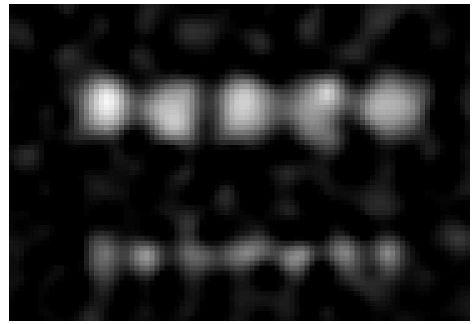
- ROI images: URL and USL method preserves the definition of curve
- Curve in case of URL image is more crisp where as for USL image, curves are smooth and wider

- From the spectra and 1-D power spectral plot it’s evident that URL and USL suppress imaging and improves the quality of up-sampled image
- In 1-D power spectral plot, the curve of direct up-sampled image shows prominent variation which signifies presence of imaging.

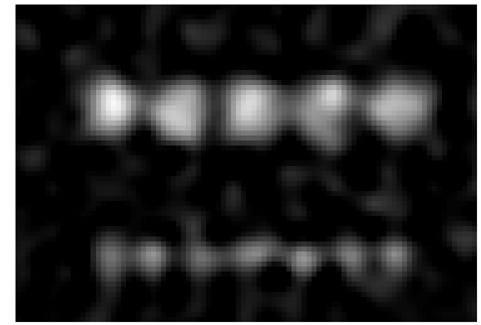
Noise Study

Zoomed roi of Dots up-sampled by factor 4, Gaussian noise added with mean 0, variance 0.001

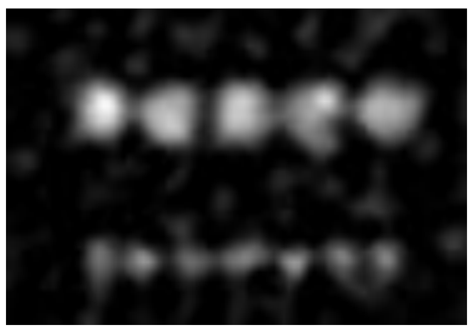
^{4}S



^{4}URL_sq



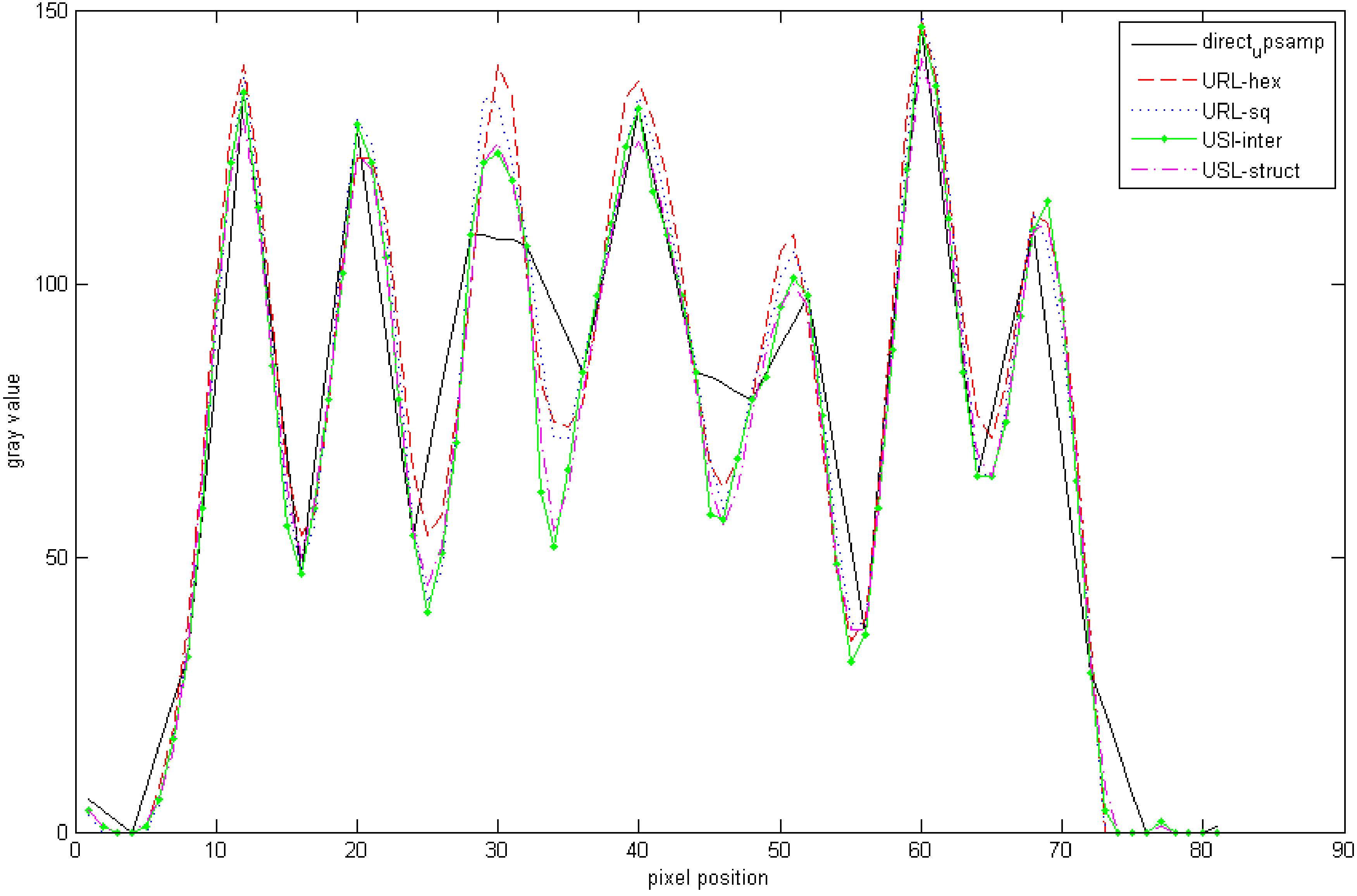
^{4}URL_hex



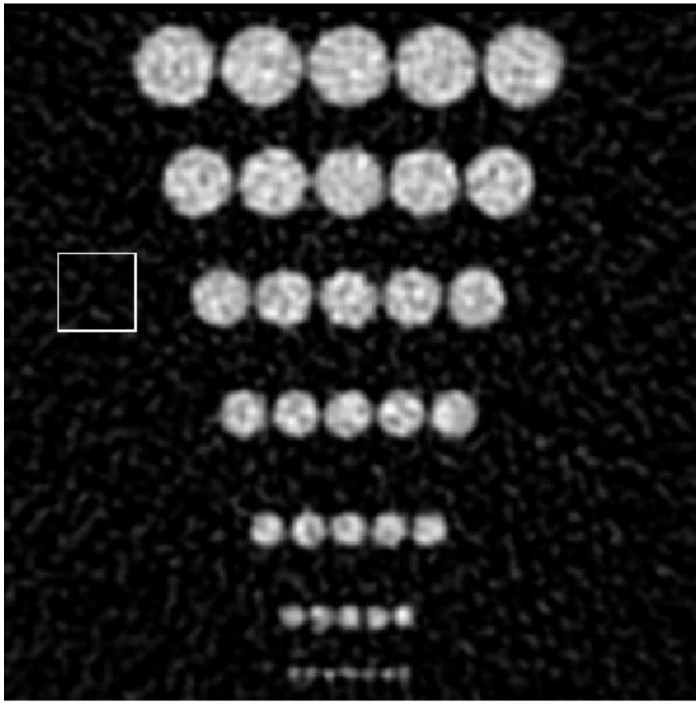
^{4}USL_inter

^{4}USL_struct

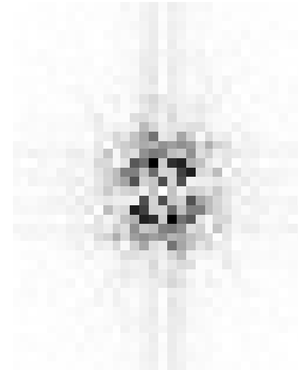
Size of Low resolution image: 101*101
Size of SR image (upsampled by 4) : 404*404



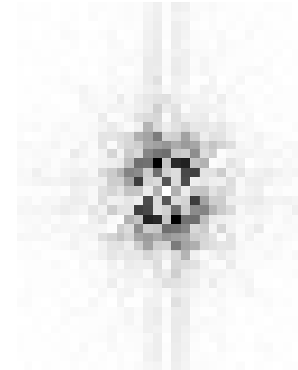
Line profile of the last line



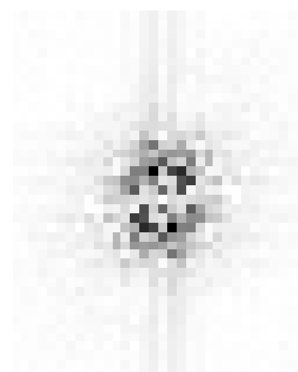
(a)



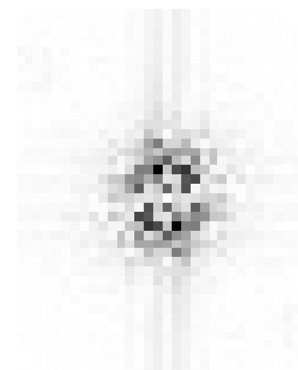
(b)



(c)



(d)



(e)

(a) ROI used for computing noise spectra, (b),(c),(d),(e) are noise spectra of URL-sq, URL-hex , USL-inter , USL- struct

- Line profile shows that the contrast and resolution of URL and USL images is nearly equal.
- Direct upsampling deforms the shape of dots and depreciate the resolution.
- Noise spectra of both URL and USL images is band limited, but URL images are more noisy than USL images.
- URL images are generated by combining 2 low resolution images where as USL images are generated by combining 4 low resolution images

Conclusion

- The results of the proposed method shows considerable improvement over direct up-sampled image in terms of contrast and preserves definition of image.
- But when compared with ISR-2 generated image, the difference in image quality is not significant.
- A key advantage of the proposed method is that only two images are required for generating a high resolution image whereas ISR-2requires k low resolution images for an up-sampling factor of k.

Thank you