Lesson 11

11.6

$$F = k_f \cdot \omega^2$$
$$M = k_m \cdot \omega^2$$

Comments:

$$k_m - \text{constant of proportionality for moment/angular torque coefficient}$$
$$k_f - \text{constant of proportionality for force/thrust coefficient}$$
$$\omega - \text{rotation rate of a rotor } (rad/sec)$$

11.8

$$F_z = m_z \cdot \ddot{z}$$
$$F = m \cdot g$$
$$F_{net} = m \cdot a$$
$$F_{thrust} = F - F_{net}$$
$$F_{thrust} = m \cdot g - m \cdot a$$
$$F_{thrust} = m \cdot (g - a)$$
$$F_{net} = F_{thrust} - F$$
$$F_{net} = F_{thrust} - m \cdot g$$

Comments:

$$m_z - \text{mass of vehicle}$$
$$F_{thrust} - \text{thrust force}$$
$$F_{net} - \text{net force}$$
$$g - \text{gravitational constant - 9.81 } (m/sec^2)$$

11.9

Two rotors:

Strange equation related to F_{net} - it should be $F_{net} = F_{thrust} - F$ but we have $F_{net} = F - F_{thrust}$. Maybe because Z is positive down.

$$\ddot{z} = \frac{F_{net}}{m}$$
$$\ddot{z} = \frac{m \cdot g - F_{thrust}}{m}$$
$$\ddot{z} = \frac{m \cdot g - k_f \cdot (\omega_1^2 + \omega_2^2)}{m}$$
$$\ddot{z} = g - \frac{k_f}{m} \cdot (\omega_1^2 + \omega_2^2)$$
$$M_z = I_z \cdot \ddot{\psi}$$
$$\ddot{\psi} = \frac{M_z}{I_z}$$
$$\ddot{\psi} = \frac{k_m}{I_z} \cdot (\omega_2^2 - \omega_1^2)$$

Comments:

$g -$ gravitational constant - 9.81 (m/sec^2)
$k_m -$ constant of proportionality for moment/angular torque coefficient
$k_f -$ constant of proportionality for force/thrust coefficient
$\ddot{\psi} -$ angular/rotational acceleration (rad/sec^2)
$\omega_1, \omega_2 -$ angular velocities
$I_z -$ moment of inertia around the z-axis
$\ddot{z} -$ vertical/linear acceleration (m/sec^2)

11.18

$$F_y = F_{thrust} \cdot \sin\phi$$
$$\ddot{y} = \frac{F_y}{m}$$
$$\ddot{y} = \frac{F_{strust} \cdot \sin\phi}{m}$$

Comments:

$\ddot{y} -$ resulting acceleration (m/sec^2)
$F_y -$ resulting force in y direction
$m -$ mass of vehicle

11.19

$$M_x = F \cdot d_{perp,x}$$
$$d_{perp,x} = L \cdot \cos\theta$$
$$M_x = F \cdot L \cdot \cos\theta$$

Comments:

$$M_x - \text{moment about the x axis/roll}$$
$$L - \text{length of arm/distance between the center of mass and the propeller axis}$$
$$d_{perp,x} - \text{perpendicular distance to the x axis}$$
$$d_{perp,x} =< L$$

11.23

$$c = F_1 + F_2$$
$$c = k_f \cdot \omega_1^2 + k_f \cdot \omega_2^2$$
$$M_x = (F_1 - F_2) \cdot L$$
$$M_x = (k_f \cdot \omega_1^2 - k_f \cdot \omega_2^2) \cdot L$$
$$\ddot{z} = g - \frac{c \cdot \cos(\phi)}{m}$$
$$\ddot{y} = \frac{c \cdot \sin(\phi)}{m} \ddot{\phi} = \frac{M_x}{I_x}$$

Comments:

$$c - \text{collective thrust}$$
$$L - \text{length of arm/distance between the center of mass and the propeller axis}$$
$$\ddot{z} - \text{vertical (z) acceleration}$$
$$\ddot{y} - \text{lateral (y) acceleration}$$