

Review: MOSFET Modeling and CMOS Circuits

Sangyoung Park

Chair for Real-Time Computer Systems

Technical University of Munich

sangyoung.park@tum.de

Why Recall MOSFET/CMOS?

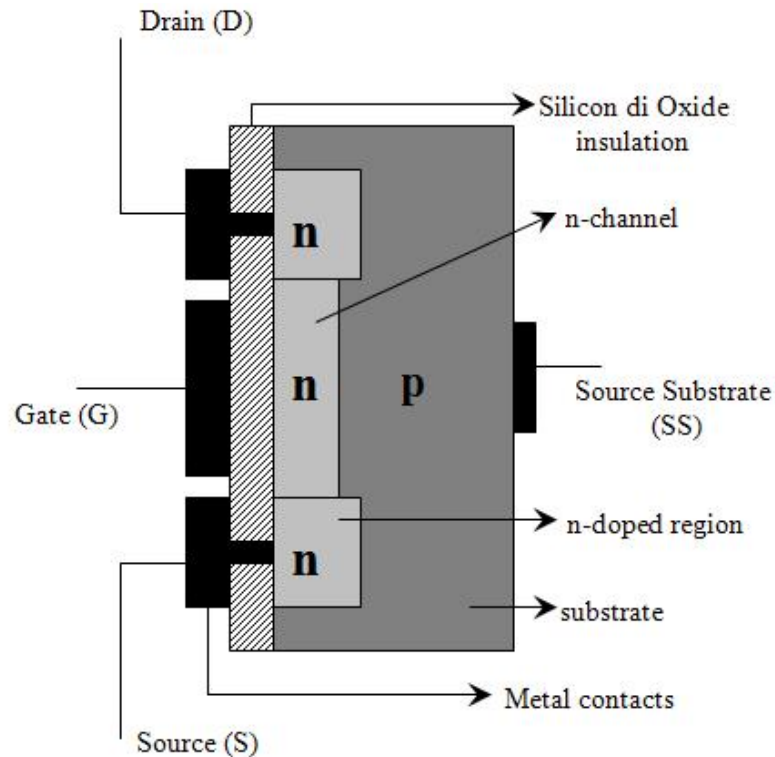
- Complementary metal–oxide–semiconductor (CMOS)
 - Nowadays electronics devices are mostly implemented with CMOS technology
 - Need not recall all the details of the equations
 - Understanding qualitative relationship between the key parameters is more important
 - Will be the basis of understanding source of power consumption and low-power techniques

Contents

- MOSFET Structure and Operation
- MOSFET Threshold Voltage
- 1st-Order Current-Voltage Characteristics
- Velocity Saturation
- Short-Channel Effect
- MOS Capacitance

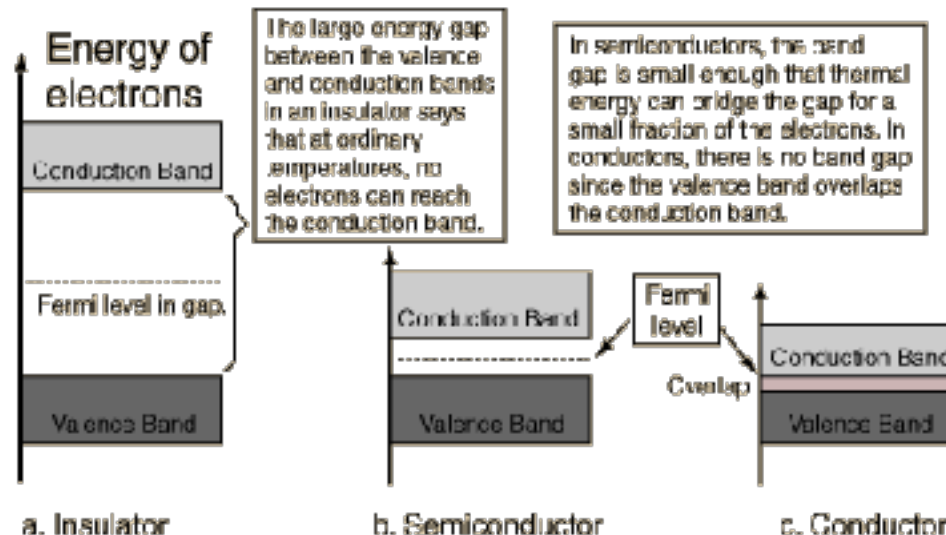
MOSFET Structure and Operation

– MOSFET structure



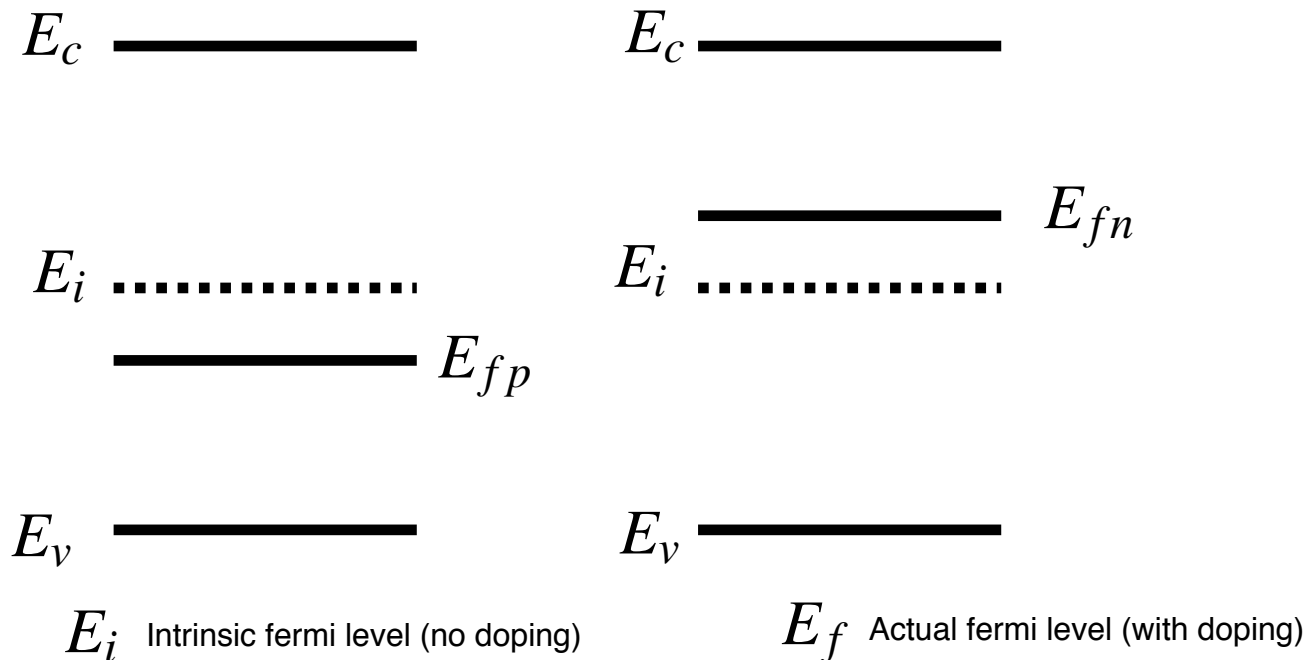
www.wikipedia.com

- Energy band diagram
 - Visualization of available energy levels for electrons in solid materials
- Fermi-level
 - Hypothetical energy level that would have 50% probability of being occupied by an electron



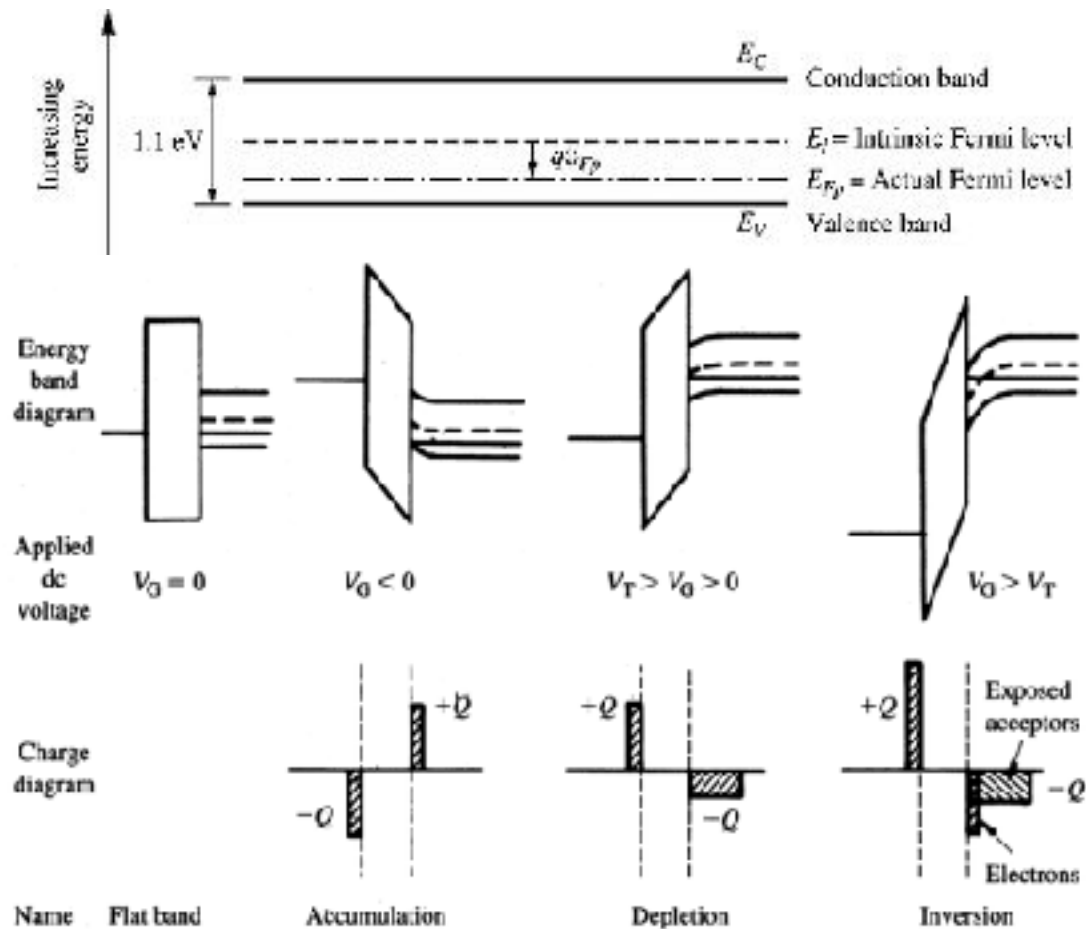
source: hyperphysics.phy-astr.gsu.edu

- Fermi level
 - Lies on the halfway in the band gap in case of semiconductors
 - Controllable by doping to p-type or n-type



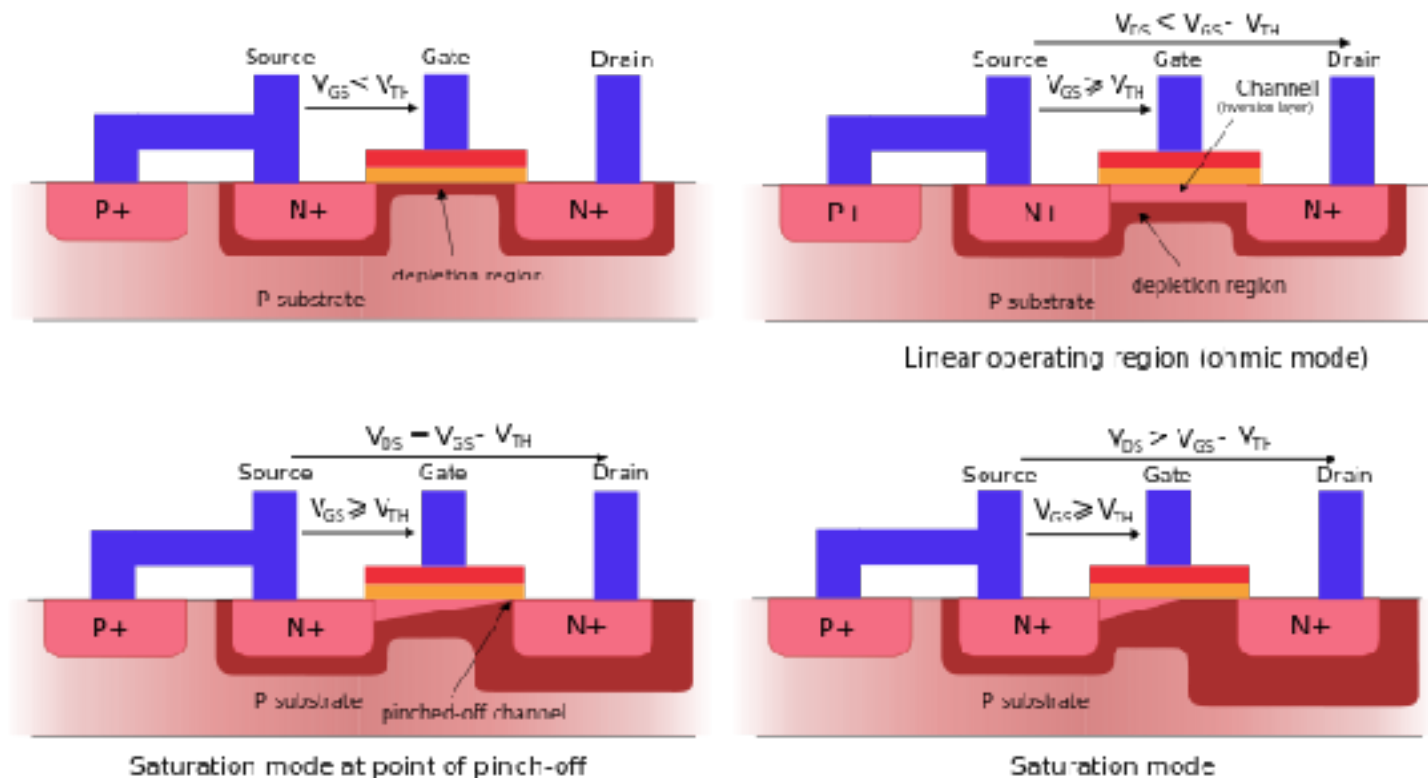
MOSFET Threshold Voltage

- Energy band diagram



MOSFET Structure and Operation

- Cut-off / sub-threshold / weak inversion mode
- Triode mode / linear region
- Saturation / active mode

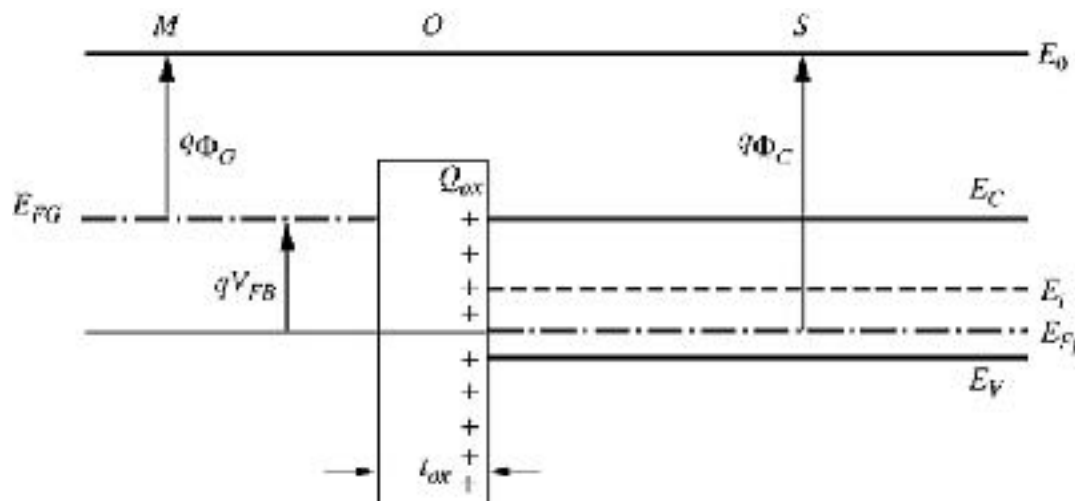


reference: http://en.wikipedia.org/wiki/File:MOSFET_functioning.svg

- Depletion
 - Mobile holes are pushed down under the gate
- Weak inversion
 - Depletion layer thickness increases and an initial layer of mobile electron appears at the surface of the silicon
- Strong inversion
 - The concentration of the mobile electrons is increased until it becomes equal to the concentration of the holes in the substrate
 - Depletion layer thickness remains constant

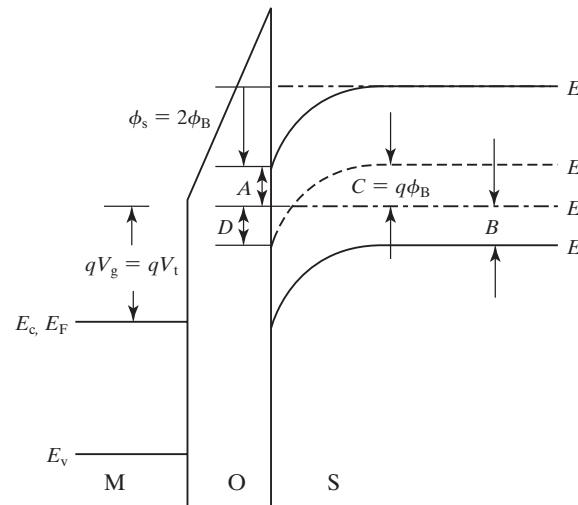
- Flat-band voltage: V_{FB}
 - The voltage that when applied to a MOS Capacitor produces zero net charge in the underlying semiconductor

$$F_{FB} = \phi_C - \phi_G$$



MOSFET Threshold Voltage

- V_{GS} required to produce strong inversion is threshold voltage V_T
 - The semiconductor surface that is normally p-type becomes n-type
 - Gate voltage where surface electron concentration equals bulk doping concentration (e.i. $A=B$)



Source: Chapter 5. C. Hu., "Modern semiconductor devices for integrated circuits", 2010

- Rearrangement of V_T

$$V_t = \phi_{gc} - 2 \cdot \phi_F - \frac{Q_{b0}}{C_{ox}} - \frac{Q_{ox}}{C_{ox}}$$

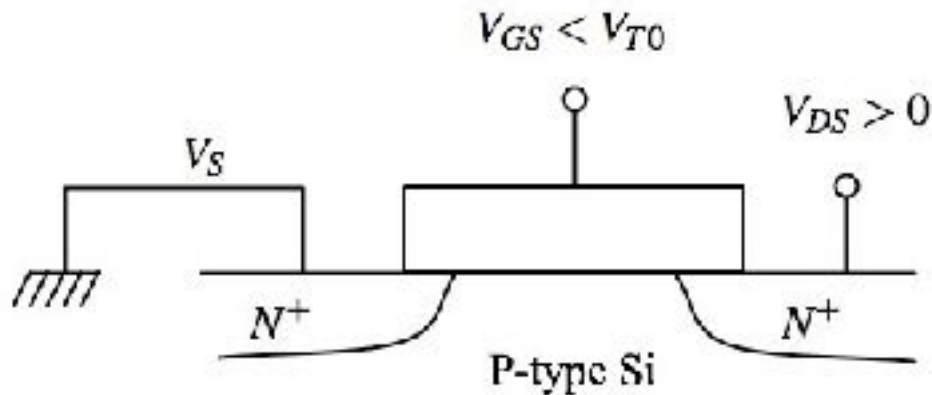
$$V_t = V_{t0} + \gamma \cdot (\sqrt{|-2 \cdot \phi_f + V_{sb}|} - \sqrt{|2 \cdot \phi_f|})$$

- Source to bulk voltage has an effect on threshold voltage
- Body effect coefficient

$$\gamma = \frac{2 \cdot N_a \cdot \epsilon_{si}}{C_{ox}}$$

1st-Order Current-Voltage Characteristics

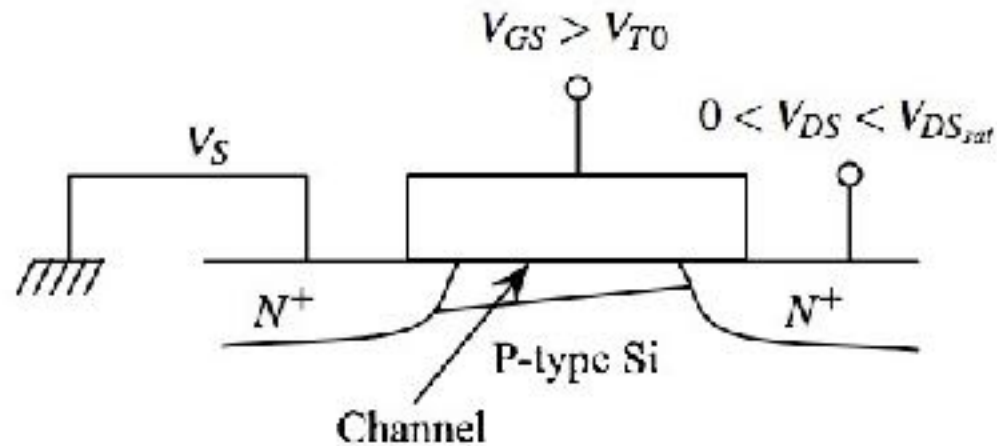
- Cut off
 - No inversion layer exists
 - Drain current is approximately zero



1st-Order Current-Voltage Characteristics

- Linear when

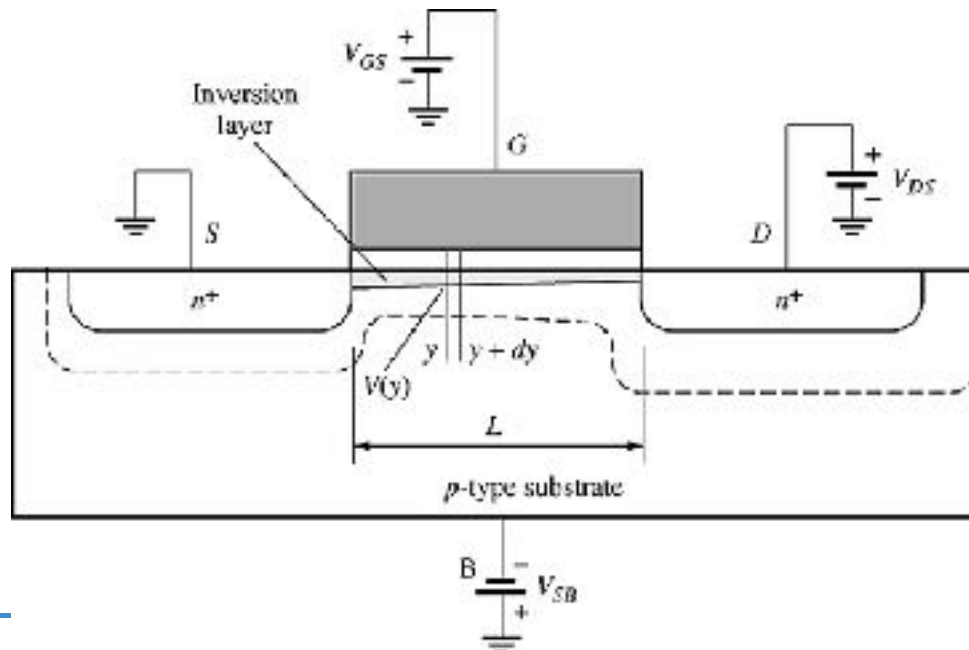
$$V_{GD} = V_{GS} - V_{DS} > V_T$$



1st-Order Current-Voltage Characteristics

- Drain current: $I_{DS} = Q_n \times v \times W$
- In the 1st order model, the velocity is linearly proportional to the E field

$$v = \mu E \text{ where } E = \frac{dV(y)}{dy}$$



- Drain current

$$I_{DS} dy = W \mu_n C_{ox} (V_{GS} - V(y) - V_T) dV$$

$$I_{DS} \int_0^L dy = W \mu_n C_{ox} \int_0^{V_{DS}} (V_{GS} - V - V_T) dV$$

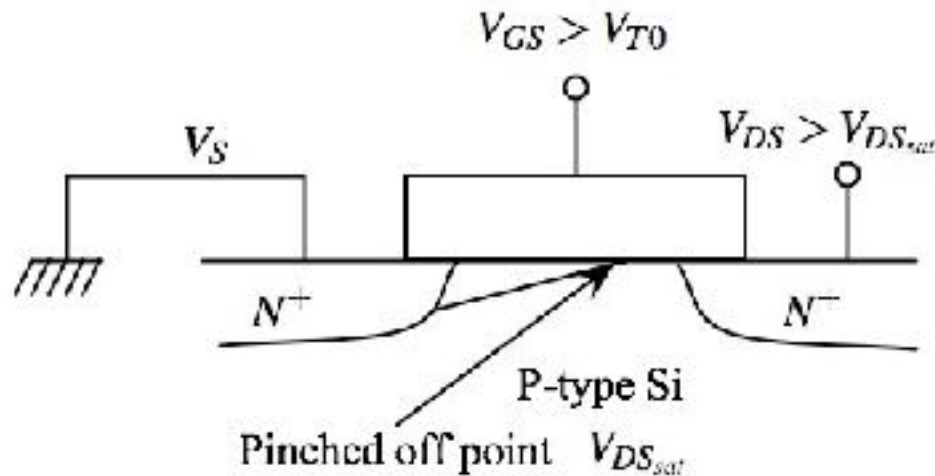
$$I_{DS} = \frac{k}{2} (2(V_{GS} - V_T)V_{DS} - V_{DS}^2)$$

$$\text{where } k = \frac{W}{L}, \text{ and } k' = \mu_n C_{ox}$$

1st-Order Current-Voltage Characteristics

- Saturation (pinched off)

$$V_{GS} > V_T, V_{DS} > V_{GS} > V_T$$



1st-Order Current-Voltage Characteristics

- Saturation voltage

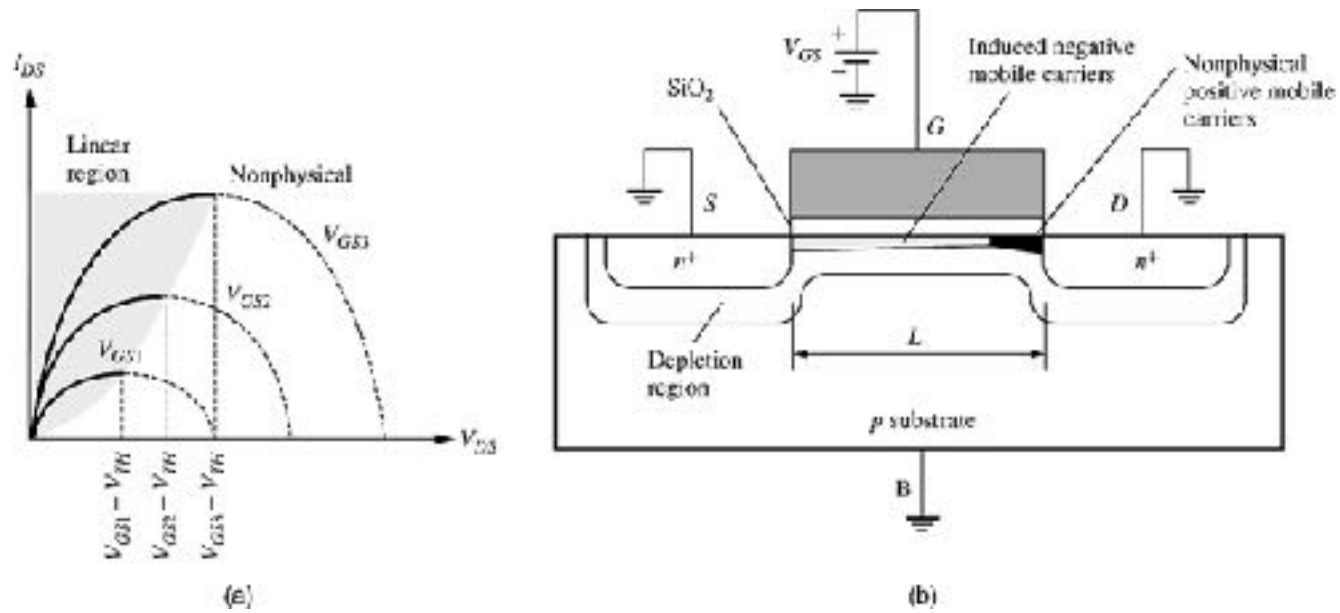
$$V_{Dsat} = V_{GS} - V_T$$

- Beyond the saturation voltage, I_{DS} is obtained by

$$V_{DS} = V_{GS} - V_T$$
$$I_{DS} = \frac{k}{2}(V_{GS} - V_T)^2$$

1st-Order Current-Voltage Characteristics

- Channel-length modulation



- Channel-length modulation
 - Measured V-I characteristics show a weak function of V_{DS}

$$L' = L - \Delta L$$



Width of the depletion layer between the pinch-off point and the drain

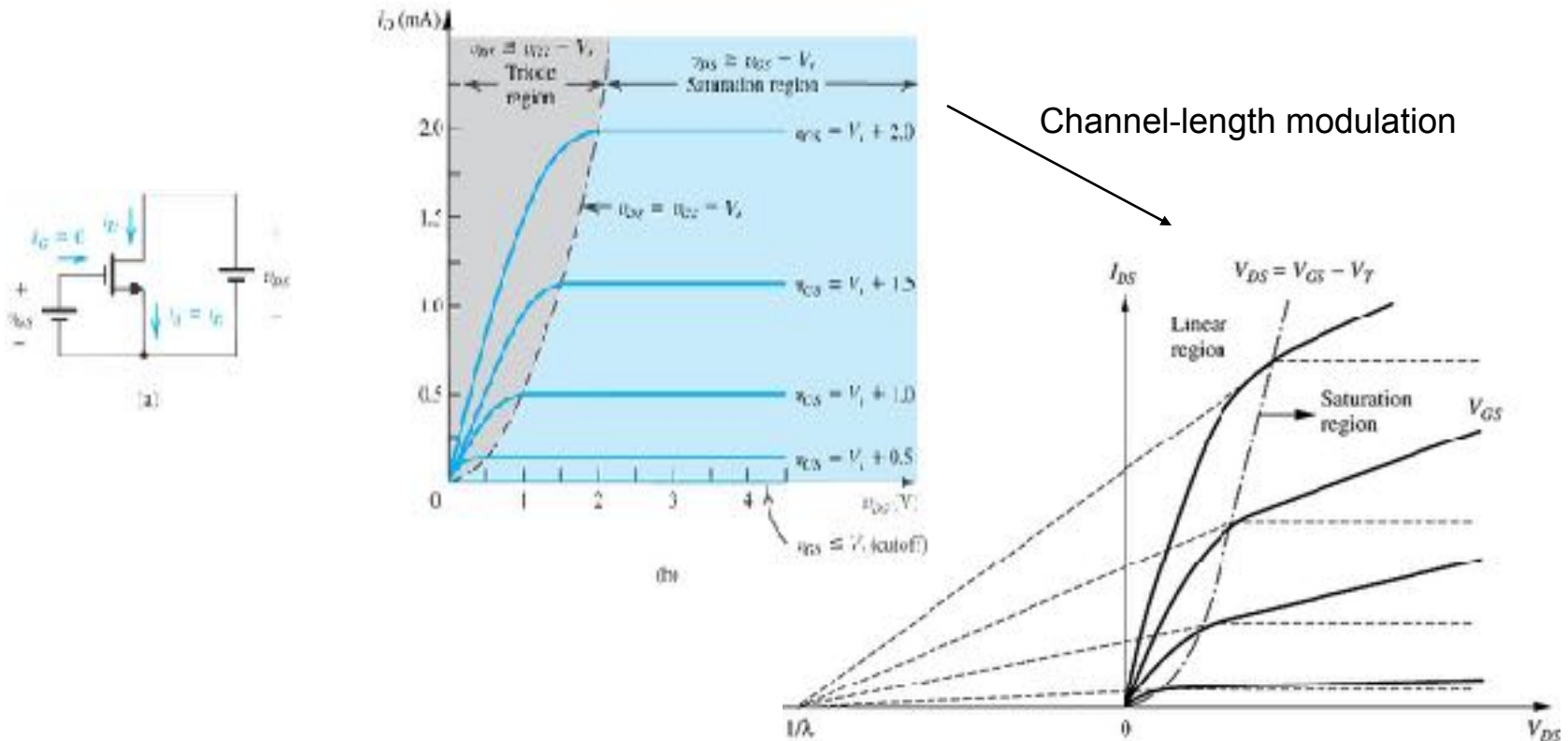
$$\Delta L = \sqrt{2 \frac{\epsilon_{si}}{qN_A} (V_{DS} - V_{Dsat})^2}$$

- Corrected saturation current

$$I_{DS} = \frac{kW}{2L'} (V_{GS} - V_T)^2 = \frac{kW}{2L} (V_{GS} - V_T)^2 \frac{1}{1 - \frac{\Delta L}{L}}$$
$$I_{DS} = \frac{k'}{2} (V_{GS} - V_T)^2 (1 + \lambda V_{DS})$$

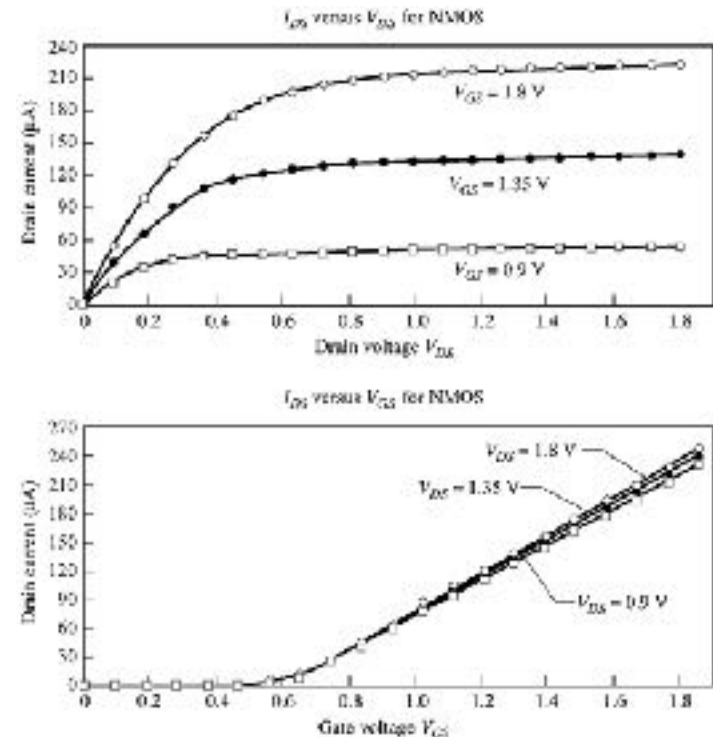
1st-Order Current-Voltage Characteristics

- The i_D - v_{DS} characteristics for a device with $k'_n (W/L) = 1.0 \text{ mA/V}^2$



Velocity Saturation

- Quadratic model is valid for long-channel devices
- Modern DSM devices
 - Channel length has been scaled to the point where the vertical and horizontal electric fields may interact
 - Saturation occurs at the pinch-off point is no longer valid
 - Saturation occurs due to velocity saturation
 - 0.18 μ m device is rather linear than quadratic



Velocity Saturation

- Horizontal field acts to push the carriers to their velocity limit and cause
 - Early saturation
 - Mobility degradation
 - Due to electron scattering caused by dangling bonds at the Si-SiO₂ interface

$$\mu_e = \frac{\mu_o}{1 + \left(\frac{V_{GS} - V_T}{\theta_{tox}} \right)^\eta}$$

↑ ↑
Empirical values

Velocity Saturation

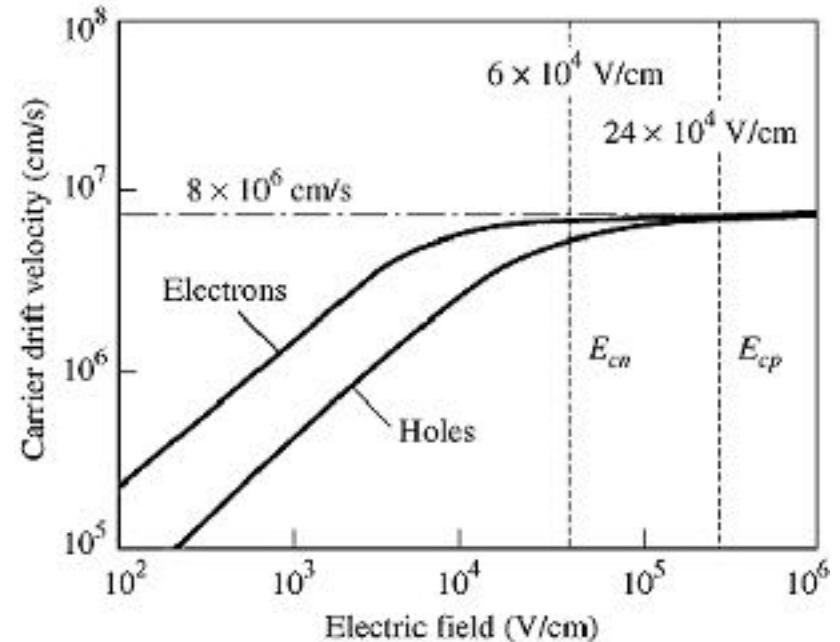
- Piece-wise continuous model

$$v = \mu_e \frac{E_y}{1 + \frac{E_y}{E_c}} \quad E_y < E_c$$

$$v = v_{sat} \quad E_y \geq E_c$$

- Drain current

$$I_{DS} = W v_{sat} C_{ox} \frac{(V_{GS} - V_T)^2}{V_{GS} - V_T + E_c L}$$



- Short-channel effect
 - V_T roll-off: charge partitioning model

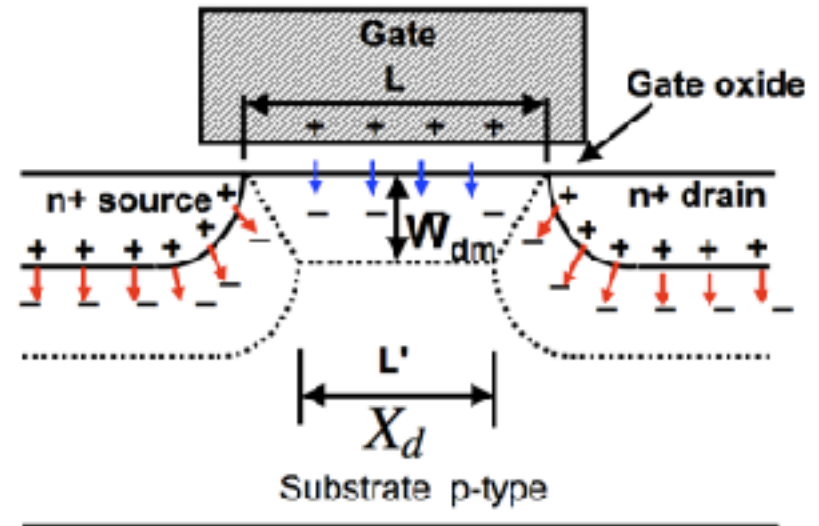
$$V_T = V_{FB} - 2\phi_F - \frac{Q_B}{C_{ox}}$$

- Long channel

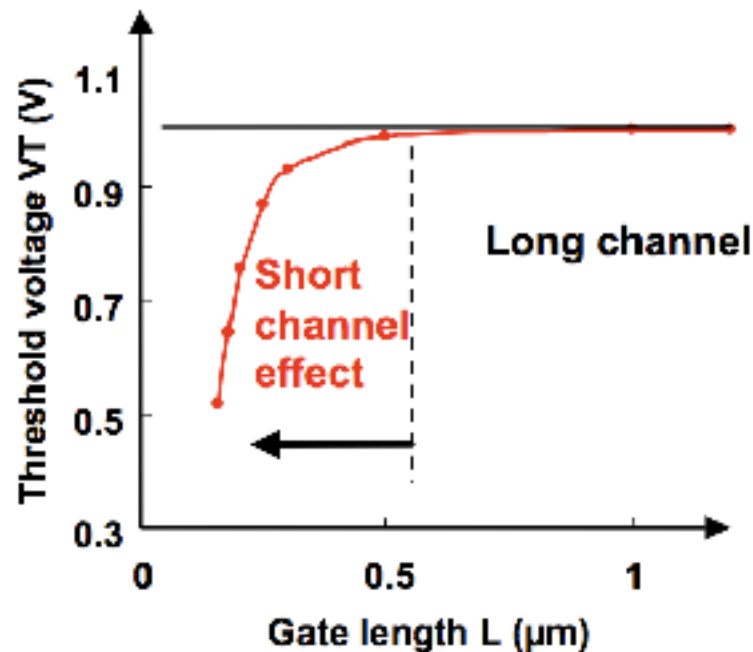
$$Q_B \propto X_d \times L$$

- Short channel

$$Q'_B \propto X_d \times \frac{L+L'}{2} < Q_B \rightarrow V_T \text{ decreases}$$

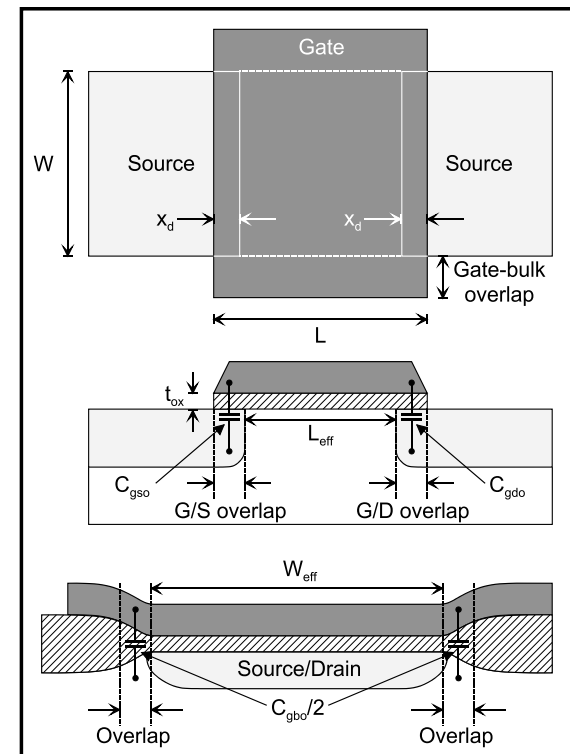
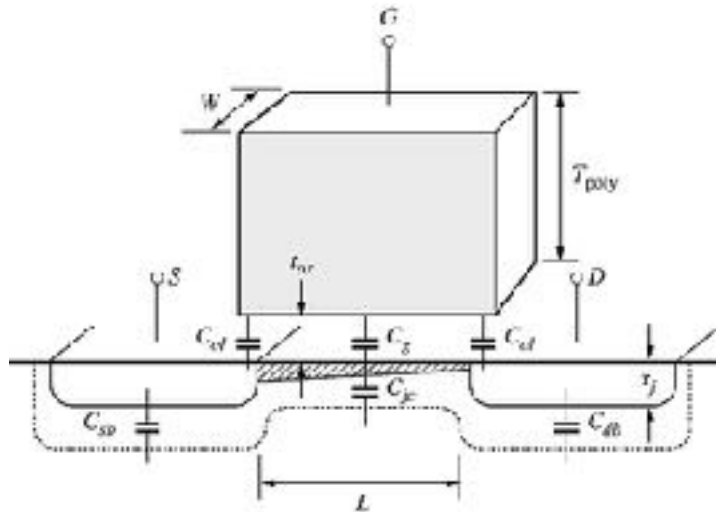


- V_T roll-off impact
 - Threshold voltage reduces as L reduces
 - I_{off} increases
 - Reduced control of the gate on the channel



MOS Capacitance

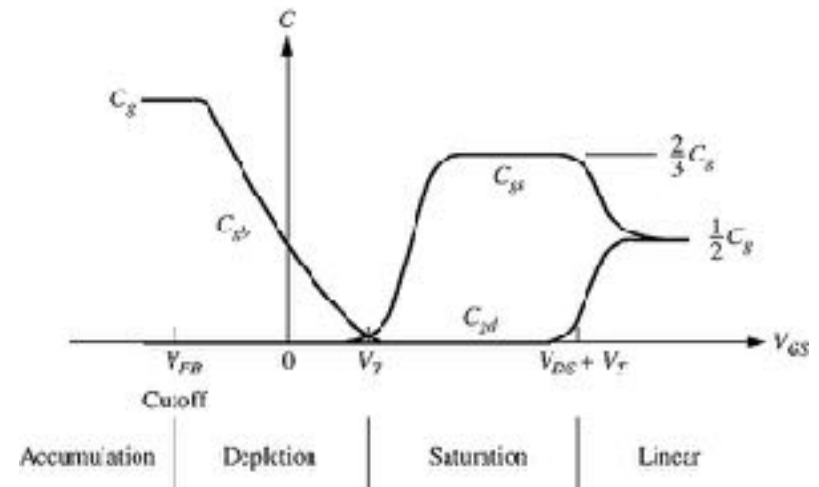
- Very important for transient simulation
 - Thin oxide capacitance: C_{gs} , C_{gd} and C_{gb}) represented by C_g
 - Junction capacitance (C_{sb} and C_{db})
 - Depletion layer capacitance (C_{jc}) associated with C_{gb}



MOS Capacitance

- Thin oxide capacitance
 - The most important MOS capacitance
 - Two plates of the capacitance are defined as the gate and the channel
 - C_g remains constant for over 25 years because both L and t_{ox} are scaled at the same rate

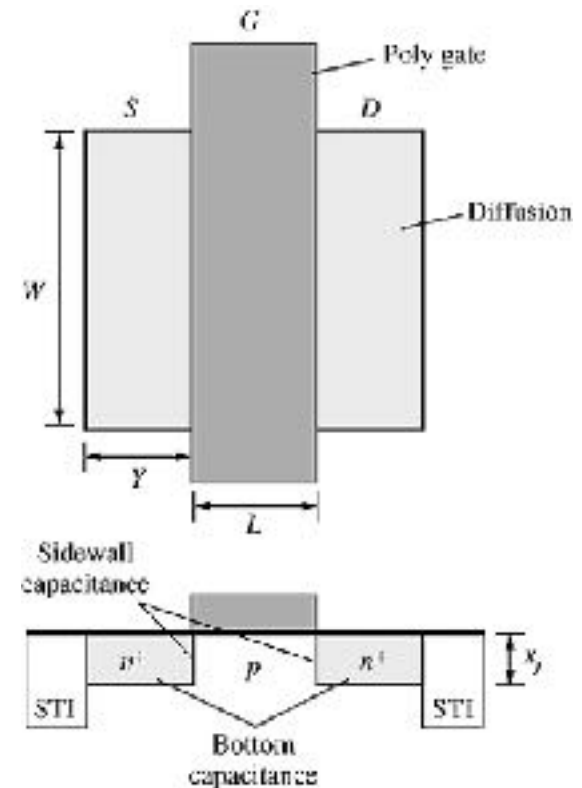
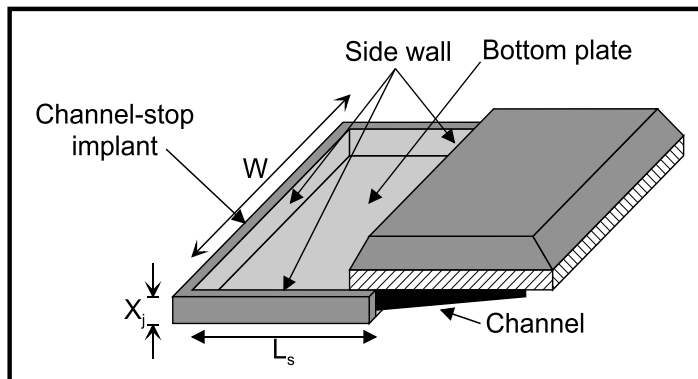
$$C_G = WLC_{ox} = WL \frac{\epsilon_{ox}}{t_{ox}} = WC_g$$



MOS Capacitance

- Area of source and drain
 - Bottom area: $A_b = WY$
 - Side wall area: $A_{sw} = Wx_j$
- Junction capacitance

$$C_J = \frac{C_{jb}A_b}{\left(1 - \frac{V_J}{\phi_B}\right)^{m_j}} + \frac{C_{jsw}A_{sw}}{\left(1 - \frac{V_J}{\phi_B}\right)^{m_{jsw}}}$$



- Slides are modified from lecture notes of “Advanced Computer System Design” from Seoul National University (Lecturer: Prof. Naehyuck Chang)