Operations Research Assignment

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1 Problem Description

The Springfield school board has made the decision to close one of its middle schools (sixth, seventh, and eighth grades) at the end of this school year and reassign all of next year's middle school students to the three remaining middle schools. The school district provides bus sing for all middle school students who must travel more than approximately a mile, so the school board wants a plan for reassigning the students that will minimize the total bus sing cost. The annual cost per student of bus sing from each of the six residential areas of the city to each of the schools is shown in the following table (along with other basic data for next year), where 0 indicates that bus sing is not needed and a dash indicates an in feasible assignment.

The school board also has imposed the restriction that each grade must constitute between 30 and 36 percent of each school's population. The above table shows the percentage of each area's middle school population for next year that falls into each of the three grades. The school attendance zone boundaries can be drawn so as to split any given area among more than one school, but assume that the percentages shown in the table will continue to hold for any partial assignment of an area to a school.

Area	No. of Students	Percentage in 6th Grade	Percentage in 7th Grade	Percentage in 8th Grade	Bussing Cost per Student		
					School 1	School 2	School 3
1	450	32	38	30	\$300	0	\$700
2	600	37	28	35	—	\$400	\$500
3	550	30	32	38	\$600	\$300	\$200
4	350	28	40	32	\$200	\$500	—
5	500	39	34	27	0	—	\$400
6	450	34	28	38	\$500	\$300	0
				School capacity:	900	1,100	1,000

You have been hired as an operations research consultant to assist the school board in determining how many students in each area should be assigned to each school.

(a) Formulate a linear programming model for this problem.

(b) Solve the model.

(c) What is your resulting recommendation to the school board?

After seeing your recommendation, the school board expresses concern about all the splitting of residential areas among multiple schools. They indicate that they "would like to keep each neighborhood together."

(d) Adjust your recommendation as well as you can to enable each area to be assigned to just one school. (Adding this restriction may force you to fudge on some other constraints.) How much does this increase the total bussing cost?

The school board is considering eliminating some bussing to reduce costs. Option 1 is to eliminate bussing only for students traveling 1 to 1.5 miles, where the cost per student is given in the table as 200. Option 2 is to also eliminate bussing for students traveling 1.5 to 2 miles, where the estimated cost per student is 300.

(e) Revise the model from part (a) to fit Option 1, and solve. Compare these results with those from part (c), including the reduction in total bussing cost.

The school board now needs to choose among the three alternative bussing plans (the current one or Option 1 or Option 2). One important factor is bussing costs. However, the school board also wants to place equal weight on a second factor: the inconvenience and safety problems caused by forcing students to travel by foot or bicycle a substantial distance (more than a mile, and especially more than 1.5 miles). Therefore, they want to choose a plan that provides the best trade-off between these two factors.

(g) Use your results from parts (c), (e), and (f) to summarize the key information related to these two factors that the school board needs to make this decision.

(h) Which decision do you think should be made? Why?

2 Cost function and Constraints

The board wants to assign the students form each area to the different schools while restricted by the following:

1. Minimize the total bussing cost

Based on different set of constraints as listed below:

2.1 Constraint Set 1

Each grade must constitute between 30 and 36 percent of each school's population

2.2 Constraint Set 2

Each grade must constitute between 28 and 38 percent of each school's population. All the students form one area should go to the same school.

2.3 Constraint Set 3

Each grade must constitute between 30 and 36 percent of each school's population. Eliminate the bussing services for students living 1 to 1.5 miles away from their schools. (The cost of bussing services are 200 dollar)

2.4 Constraint Set 4

Each grade must constitute between 28 and 38 percent of each school's population. Eliminate the bussing services for students living 1 to 2 miles away from their schools. (The cost of bussing services are 300 dollar)

3 Decision Variables

$$x(i,j)$$

where i is the indicator of the area , varies from 1 to 6 where j is the indicator of the school, varies from 1 to 3 Total of 18 variables

The below table shows the distribution of the students from each area to be put in three different schools

	Number of Student in		
Area	School 1	School 2	School 3
1	X_{11}	X_{12}	X_{13}
2	X_{21}	X_{22}	X_{23}
3	X_{31}	X_{32}	X_{33}
4	X_{41}	X_{42}	X_{43}
5	X_{51}	X_{52}	X_{53}
6	X_{61}	X_{62}	X_{63}

$$c$$

c is the cost function to be minimized
c = sum(cost(i,j)*x(i,j))

The corresponding GAMS code is

Table cost(i,j) 'Total cost incurred by the school on bussing per student from area [1-6] to school [1-3]'

1 2 3

1 300 0 700

2 999999 400 500

3 600 300 200

4 200 500 999999

5 0 999999 400

6 500 300 0 ;

obj.. c =e= sum((i,j), cost(i,j)*x(i,j));

4 Construction of Linear Program in GAMS

4.1 Assigning the variables

The corresponding code for the variable declaration in GAMS is

Set

i 'Area identifier' / 1*6 /

j 'School identifier' / 1*3 /

k 'grade of the students' / 6*8 /;

4.2 Linear Constraints

1. The students in the area should be divided among the three schools. The condition could be written as:

X11 + X12+ X13 = 450
X21+ X22 + X23 = 600
X31+ X32+ X33 = 550
X41+ X42+ X43 = 350
X51+ X52+ X53 = 500
X61+ X62+ X63 = 450

The corresponding GAMS code is

Parameter

n(i) 'Students from area i'

/ 1 450

2 600

3 550

4 350

5 500

6 450 /

Equation

assign(i).. sum(j, x(i,j)) =e= n(i) ;assign(i).. sum(j, x(i,j)) =e= n(i) ;

2. The number of students assigned to each school should not exceed the limits as provided.

X11+ X21+ X31+ X41+ X51+ X61 ¡= 900

X12+ X22+ X32+ X42+ X52+ X62 ¡= 1,100

X13+ X23+ X33+ X43+ X53+ X63 ¡= 1,000

GAMS code

Parameter

max(j) 'maximum capacity of the school'

/ 1 900

2 1100

3 1000 /;

Equation

maxcap(j).. sum(i, x(i,j)) =L= max(j) ;

3.Total number of students in each grade for each school should be with in the range of 30 percent to 36 percent of the total number of students in each school.

The school board also has imposed the restriction that each grade must constitute between 30 and 36percent of each school's population.

30 percent total number of student in school I lesser than 6th graders and 36 percent total number of student in school I greater than 6th graders

30 percent total number of student in school I lesser than 7th graders and 36 percent total number of student in school I greater than 7th graders

30 percent total number of student in school I lesser than 8th graders and 36 percent total number of student in school I greater than 8th graders

GAMS code

maxpercent(k,j).. sum(i, x(i,j)*percent(i,k)) =L= 0.36*sum(i, x(i,j)) ;

minpercent(k,j).. sum(i, x(i,j)*percent(i,k)) =G= 0.30*sum(i, x(i,j));

4.3 Linear Constraints - Scenario 2

Adjusting the recommendation as to enable each area to be assigned to just one school.

Choosing the schools with least cost

Mod(x11, 450) = 0 Mod(x12, 450) = 0 Mod(x13, 450) = 0

Mod(x21, 600) = 0 Mod(x22, 600) = 0 Mod(x23, 600) = 0

Mod(x31, 550) = 0 Mod(x32, 550) = 0 Mod(x33, 550) = 0

Mod(x41, 350) = 0 Mod(x42, 350) = 0 Mod(x43, 350) = 0

Mod(x51, 500) = 0 Mod(x52, 500) = 0 Mod(x53, 500) = 0

Mod(x61, 450) = 0 Mod(x62, 450) = 0 Mod(x63, 450) = 0

Also an adjustment to be made at the percentage limit of students in each school per grade to reach an feasible solution.

GAMS code

Equation

maxpercent(k,j).. sum(i, x(i,j)*percent(i,k)) =L= 0.38*sum(i, x(i,j)) ;

minpercent(k,j).. sum(i, x(i,j)*percent(i,k)) =G= 0.29*sum(i, x(i,j));

modular(i,j).. x(i,j)-n(i)*(x(i,j)/n(i)) =e= 0;

4.4 Linear Constraints - Scenario 3

To delete the bussing cost for students that are 1 to 1.5 miles away from their schools, the following constraints should be added to the basic groups:

Forcing the constraint to zero, where the bussing cost is 200 dollars.

cost of x(3,3) =0 and cost of x(4,1) = 0

GAMS code

Table cost(i,j) 'Total cost incurred by the school on bussing per student from area [1-6] to school [1-3]'

1 2 3

1 300 0 700

2 999999 400 500

3 600 300 0

4 0 500 999999

5 0 999999 400

6 500 300 0 ;

4.5 Linear Constraints - Scenario 4

To delete the bussing cost for students that are 1 to 2 miles away from their schools.

Forcing the constraint to zero, where the bussing cost is 300 and 200 dollars.

cost of x(3,3) = cost of x(4,1) = cost of x(1,1) = cost of x(3,2) = cost of x(6,2) = 0

GAMS code

Table cost(i,j) 'Total cost incurred by the school on bussing per student from area [1-6] to school [1-3]'

1 2 3

1 0 0 700

2 999999 400 500

3 600 0 0

4 0 500 999999

5 0 999999 400

6 500 0 0 ;

5 Results

Results are tabulated as below with model number, results for x(i,j) and Minimized cost.

Constraint Set 1: Base Model			
Area Number	School 1	School 2	School 3
1	-	450	-
2	-	422.22	177.778
3	-	227.778	322.22
4	350	-	-
5	366.67	-	133.33
6	83.33	-	366.67
	Minimized cost: $555,555.56		

Constraint Set 2: Model Modified 1			
Area Number	School 1	School 2	School 3
1	-	450	-
2	-	600	-
3	-	-	550
4	350	-	-
5	500	-	-
6	-	-	450
	Minimized cost: $420,000		

Constraint Set 3: Model Modified 2			
Area Number	School 1	School 2	School 3
1	-	450	-
2	-	600	-
3	-	50	500
4	350	-	-
5	318.87	-	181.818
6	131.818	-	318.182
	Minimized cost: $393,640		

Constraint Set 4: Model Modified 3			
Area Number	School 1	School 2	School 3
1	38.710	411.290	-
2	-	236.559	363.441
3	-	77.957	472.043
4	350	-	-
5	435.484	-	64.516
6	75.806	374.194	-
	Minimized cost: $340,050		

Models	Optimal cost of Bussing	Decrease (after constraints)
Base Model	$555,555.56	-
Model Modified 1	$420,000	***$135555.56***
Model Modified 2	$393,640	***$161915.56***
Model Modified 3	$340,050	***$215505.56***

5.1 safety concern

To decide on a plan lets look into the cost saving and the number of people made to walk for 1-1.5, 1.5-2 Miles.

Option	Less than 1 Mile	Walking 1 to 1.5 Mile	Walking 1.5 to 2 Mile	More than 2 Miles Bussing	Gain on Adopting
Option 0	1400	900	-	600	$135,555.56
Option 1	***1087.75***	***850***	***50***	***913.636***	***$161915.56***
Option 2	846.775	822	491	740.332	$215505.56

If Cost and Safety are to be taken with equal weight., then we would recommend option 1. since it saves 215505.56 dollars and only 850 students are made to walk more than 1 to 1.5 .

6 Appendix

The List of documents are attached.

1. GAMS Code and Solver results for Model 0
2. GAMS Code and Solver results for Model 1
3. GAMS Code and Solver results for Model 2
4. GAMS Code and Solver results for Model 3