Operations Research Assignment 2

Karthick Raja, Akshar Vashist, Nikhil Shinde

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1 Problem Statement

Assigning Art

It had been a dream come true for Ash Briggs, a struggling artist living in the San Francisco Bay Area. He had made a trip to the corner grocery store late one Friday afternoon to buy some milk, and on impulse, he had also purchased a California lottery ticket. One week later, he was a millionaire.

Ash did not want to squander his winnings on materialistic, trivial items. Instead he wanted to use his money to support his true passion: art. Ash knew all too well the difficulties of gaining recognition as an artist in this postindustrial, technological society where artistic appreciation is rare and financial support even rarer. He therefore decided to use the money to fund an exhibit of up-and-coming modern artists at the San Francisco Museum of Modern Art.

Ash approached the museum directors with his idea, and the directors became excited immediately after he informed them that he would fund the entire exhibit in addition to donating dollar 1 million to the museum. Celeste McKenzie, a museum director, was assigned to work with Ash in planning the exhibit. The exhibit was slated to open one year from the time Ash met with the directors, and the exhibit pieces would remain on display for two months.

Ash began the project by combing the modern art community for potential artists and pieces. He presented the following list of artists, their pieces, and the price of displaying each piece1 to Celeste.

Ash possesses certain requirements for the exhibit. He believes the majority of Americans lack adequate knowledge of art and artistic styles, and he wants the exhibit to educate Americans. Ash wants visitors to become aware of the collage as an art form, but he believes collages require little talent. He therefore decides to include only one collage. Additionally, Ash wants viewers to compare the delicate lines in a three-dimensional wire mesh sculpture to the delicate lines in a two-dimensional computer-generated drawing. He therefore wants at least one wire mesh sculpture displayed if a computer-generated drawing is displayed. Alternatively, he wants at least one computer-generated drawing displayed if a wire mesh sculpture is displayed. Furthermore, Ash wants to expose viewers to all painting styles, but he wants to limit the number of paintings displayed to achieve a balance in the exhibit between paintings and other art forms. He therefore decides to include at least one photo-realistic painting, at least one cubist painting, at least one expressionist painting, at least one watercolor painting, and at least one oil painting. At the same time, he wants the number of paintings to be no greater than twice the number of other art forms.

Ash wants all his own paintings included in the exhibit since he is sponsoring the exhibit and since his paintings celebrate the San Francisco Bay Area, the home of the exhibit.

Ash possesses personal biases for and against some artists. Ash is currently having a steamy affair with Candy Tate, and he wants both of her paintings displayed. Ash counts both David Lyman and Rick Rawls as his best friends, and he does not want to play favorites among these two artists. He therefore decides to display as many pieces from David Lyman as from Rick Rawls and to display at least one piece from each of them. Although Ziggy Lite is very popular within art circles, Ash believes Ziggy makes a mockery of art. Ash will therefore only accept one dis- play piece from Ziggy, if any at all.

Celeste also possesses her own agenda for the exhibit. As a museum director, she is interested in representing a diverse population of artists, appealing to a wide audience, and creating a politically correct exhibit. To advance feminism, she decides to include at least one piece from a female artist for every two pieces included from a male artist. To advance environmentalism, she decides to include either one or both of the pieces "Aging Earth" and "Wasted Resources." To advance Native American rights, she decides to include at least one piece by Bear Canton. To advance science, she decides to include at least one of the following pieces: "Chaos Reigns," "Who Has Control," "Beyond," and "Pioneers."

Celeste also understands that space is limited at the museum. The museum only has enough floor space for four sculptures and enough wall space for 20 paintings, collages, and drawings. Finally, Celeste decides that if "Narcissism" is displayed, "Reflection" should also be displayed since "Reflection" also suggests narcissism. Ash does not require personal compensation, and the cost for moving his pieces to the museum from his home in San Francisco is minimal. The cost of displaying his pieces therefore only includes the cost of constructing the display and insuring the pieces.

Please explore the following questions independently except where otherwise indicated.

- 1. Ash decides to allocate 4 million dollar to fund the exhibit. Given the pieces available and the specific requirements from Ash and Celeste, formulate and solve a BIP model to maximize the number of pieces displayed in the exhibit without exceeding the budget. How many pieces are displayed? Which pieces are displayed?
- 2. To ensure that the exhibit draws the attention of the public, Celeste decides that it must include at least 20 pieces. Formulate and solve a BIP model to minimize the cost of the exhibit while displaying at least 20 pieces and meeting the requirements set by Ash and Celeste. How much does the exhibit cost? Which pieces are displayed?
- 3. An influential patron of Rita Losky's work who chairs the Museum Board of Directors learns that Celeste requires at least 20 pieces in the exhibit. He offers to pay the minimum amount required on top of Ash's 4 million dollar to ensure that exactly 20 pieces are displayed in the exhibit and that all of Rita's pieces are displayed. How much does the patron have to pay? Which pieces are displayed?

Artist	Piece	Description of Piece	Price
Colin Zweibell	"Perfection"	A wire mesh sculpture of the human body	\$300,000
	"Burden"	A wire mesh sculpture of a mule	\$250,000
	"The Great Equalizer"	A wire mesh sculpture of a gun	\$125,000
Rita Losky	"Chaos Reigns"	A series of computer-generated drawings	\$400,000
	"Who Has Control?"	A computer-generated drawing intermeshed with lines of computer code	\$500,000
	"Domestication"	A pen-and-ink drawing of a house	\$400,000
	"Innocence"	A pen-and-ink drawing of a child	\$550,000
Norm Marson	"Aging Earth"	A sculpture of trash covering a larger globe	\$700,000
	"Wasted Resources"	A collage of various packaging materials	\$575,000
Candy Tate	"Serenity"	An all blue watercolor painting	\$200,000
	"Calm Before the Storm"	A painting with an all blue watercolor background and a black watercolor center	\$225,000

Artist	Piece	Description of Piece	Price
Robert Bayer	"Void"	An all black oil painting	\$150,000
	"Sun"	An all yellow oil painting	\$150,000
David Lyman	"Storefront Window"	A photo-realistic painting of a jewelry store display window	\$850,000
	"Harley"	A photo-realistic painting of a Harley-Davidson motorcycle	\$750,000
Angie Oldman	"Consumerism"	A collage of magazine advertisements	\$400,000
	"Reflection"	A mirror (considered a sculpture)	\$175,000
	"Trojan Victory"	A wooden sculpture of a condom	\$450,000
Rick Rawls	"Rick"	A photo-realistic self-portrait (painting)	\$500,000
	"Rick II"	A cubist self-portrait (painting)	\$500,000
	"Rick III"	An expressionist self-portrait (painting)	\$500,000
Bill Reynolds	"Beyond"	A science fiction oil painting depicting Mars colonization	\$650,000
	"Pioneers"	An oil painting of three astronauts aboard the space shuttle	\$650,000
Bear Canton	"Wisdom"	A pen-and-ink drawing of an Apache chieftain	\$250,000
	"Superior Powers"	A pen-and-ink drawing of a traditional Native American rain dance	\$350,000

Helen Row	"Study of a Violin"	A cubist painting of a violin	\$400,000
	"Study of a Fruit Bowl"	A cubist painting of a bowl of fruit	\$400,000
Ziggy Lite	"My Namesake"	A collage of Ziggy cartoons	\$300,000
	"Narcissism"	A collage of photographs of Ziggy Lite	\$300,000
Ash Briggs	"All That Glitters"	A watercolor painting of the Golden Gate Bridge	\$50,000*
	"The Rock"	A watercolor painting of Alcatraz	\$50,000
	"Winding Road"	A watercolor painting of Lombard Street	\$50,000
	"Dreams Come True"	A watercolor painting of the San Francisco Museum of Modern Art	\$50,000

2 Assumptions

On the artists' pieces of artwork, it is assumed that they only have one piece of that particular artwork and no duplicates will be used for the exhibit

3 Definition of Variables

```
Yst = Artwork done by the artist "s" for the given title "t" 
S is from the set ( a,b,c,d,e,f,g,h,i,k,l,m) t \ \text{is from the set } (1,2,3,4)
```

3.1 Model Formulation in GAMS

```
Set s 'Artist' / a,b,c,d,e,f,g,h,i,j,k,l,m / t 'Title' / 1,2,3,4 / ;

Variables y(s,t) 'Whether artwork t by artist s is selected' z 'Total Number of artwork to be displayed'; binary variables y;
```

3.2 Variables table

Artist	Piece	Variable
Zwelbell	Perfection	Yal
	Burden	Ya2
	The Great Equalizer	Ya3
Losky	Chaos Reigns	Yb1
	Who has control	Yb2
	Domestication	Yb3
	Innoncence	Yb4
Marson	Aging Earth	Yel
	Wasted Resources	Ye2
Tate	Serenity	Yd1
	Calm before the storm	Yd2
Bayer	Void	Ye1
	Sun	Ye2
Lyman	Storefront Window	Yſl
	Harley	Yt2
Oldman	Consumerism	Yg1
	Reflection	Yg2

	Trojan victory	Yg3
Rawls	Rick	Yh1
	Rick II	Yh2
	Rick III	Yh3
Reynolds	Beyond	Yil
	Pioneers	Yi2
Canton	Wisom	Yj1
	Superior power	Yj2
	Living Land	Yj3
Row	Study of a violin	Yk1
	Study of a fruit bowl	Yk2
Lite	My Namesake	Yl1
	Narcissism	Y12
Briggs	All that glitters	Yml
	The Rock	Ym2
	Winding Road	Ym3
	Dreams come true	Ym4

4 Objective Function

Three problems must be addressed to provide solution to the questions. For every problem there is a corresponding objective function and Additional constraint.

4.1 Question 1:

Maximising the number of pieces displayed in the exhibit without exceeding the budget.

```
\begin{array}{l} \text{Max} = \text{Ya1} + \text{Ya2} + \text{Ya3} + \text{Yb1} + \text{Yb2} + \text{Yb3} + \text{Yb4} + \text{Yc1} + \text{Yc2} + \text{Yd1} + \text{Yd2} + \text{Ye1} + \text{Ye2} + \text{Yf1} + \text{Yf2} + \text{Yg1} + \text{Yg2} + \text{Yg3} + \text{Yh1} + \text{Yh2} + \text{Yh3} + \text{Yi1} + \text{Yi2} + \text{Yj1} + \text{Yj2} + \text{Yj3} + \text{Yk1} + \text{Yk2} + \text{Yl1} + \text{Yl2} + \text{Ym1} + \text{Ym2} + \text{Ym3} + \text{Ym4} \end{array}
```

GAMS code:

```
 \begin{array}{l} \operatorname{artWorks...} \ z = = \ y(\ 'a',\ '1') + y(\ 'a',\ '2') + y(\ 'a',\ '3') + y(\ 'b',\ '1') + y(\ 'b',\ '2') + y(\ 'b',\ '3') + y(\ 'b',\ '4') + y(\ 'c',\ '1') \\ + \ y(\ 'c',\ '2') + \ y(\ 'd',\ '1') + y(\ 'g',\ '1') + y(\ 'g',\ '1') + y(\ 'g',\ '2') + y(\ 'g',\ '1') + y(\ 'g',\ '2') + y(\ 'g',\ '1') + y
```

4.2 Question 2:

This is used to minimize the cost of the exhibit while displaying at least 20 pieces and meeting the requirements set by Ash and Celeste.

```
. Min= 300^{*}Ya1 + 250^{*}Ya2 + 125^{*}Ya3 + 400^{*}Yb1 + 500^{*}Yb2 + 400^{*}Yb3 + 550^{*}Yb4 + 700^{*}Yc1 + 575^{*}Yc2 + 200^{*}Yd1 + 225^{*}Yd2 + 150^{*}Ye1 + 150^{*}Ye2 + 850^{*}Yf1 + 750^{*}Yf2 + 400^{*}Yg1 + 175^{*}Yg2 + 450^{*}Yg3 + 500^{*}Yh1 + 500^{*}Yh2 + 500^{*}Yh3 + 650^{*}Yi1 + 650^{*}Yi2 + 250^{*}Yj1 + 350^{*}Yj2 + 450^{*}Yj3 + 400^{*}Yk1 + 400^{*}Yk2 + 300^{*}Yl1 + 300^{*}Yl2 + 50^{*}Ym1 + 50^{*}Ym2 + 50^{*}Ym3 + 50^{*}Ym4
```

GAMS Code:

```
 \begin{array}{l} \operatorname{artwork}...\ z = & = 300^*\mathrm{y}(\mathrm{`a'},\mathrm{`1'}) + 250^*\mathrm{y}(\mathrm{`a'},\mathrm{`2'}) + 125^*\mathrm{y}(\mathrm{`a'},\mathrm{`3'}) + 400^*\mathrm{y}(\mathrm{`b'},\mathrm{`1'}) + 500^*\mathrm{y}(\mathrm{`b'},\mathrm{`2'}) + 400^*\mathrm{y}(\mathrm{`b'},\mathrm{`3'}) \\ + 550^*\mathrm{y}(\mathrm{`b'},\mathrm{`4'}) + 700^*\mathrm{y}(\mathrm{`c'},\mathrm{`1'}) + 575^*\mathrm{y}(\mathrm{`c'},\mathrm{`2'}) + 200^*\mathrm{y}(\mathrm{`d'},\mathrm{`1'}) + 225^*\mathrm{y}(\mathrm{`d'},\mathrm{`2'}) + 150^*\mathrm{y}(\mathrm{`e'},\mathrm{`1'}) + 150^*\mathrm{y}(\mathrm{`e'},\mathrm{`1'}) \\ + 850^*\mathrm{y}(\mathrm{`f'},\mathrm{`1'}) + 750^*\mathrm{y}(\mathrm{`f'},\mathrm{`2'}) + 400^*\mathrm{y}(\mathrm{`g'},\mathrm{`1'}) + 175^*\mathrm{y}(\mathrm{`g'},\mathrm{`2'}) + 450^*\mathrm{y}(\mathrm{`g'},\mathrm{`3'}) + 500^*\mathrm{y}(\mathrm{`h'},\mathrm{`1'}) + 500^*\mathrm{y}(\mathrm{`h'},\mathrm{`2'}) \\ + 500^*\mathrm{y}(\mathrm{`h'},\mathrm{`3'}) + 650^*\mathrm{y}(\mathrm{`i'},\mathrm{`1'}) + 650^*\mathrm{y}(\mathrm{`i'},\mathrm{`2'}) + 250^*\mathrm{y}(\mathrm{`j'},\mathrm{`1'}) + 350^*\mathrm{y}(\mathrm{`j'},\mathrm{`2'}) + 450^*\mathrm{y}(\mathrm{`j'},\mathrm{`3'}) + 400^*\mathrm{y}(\mathrm{`k'},\mathrm{`1'}) \\ + 400^*\mathrm{y}(\mathrm{`k'},\mathrm{`2'}) + 300^*\mathrm{y}(\mathrm{`1'},\mathrm{`1'}) + 300^*\mathrm{y}(\mathrm{`1'},\mathrm{`2'}) + 50^*\mathrm{y}(\mathrm{`m'},\mathrm{`1'}) + 50^*\mathrm{y}(\mathrm{`m'},\mathrm{`2'}) + 50^*\mathrm{y}(\mathrm{`m'},\mathrm{`3'}) + 50^*\mathrm{y}(\mathrm{`m'},\mathrm{`4'}); \end{array}
```

4.3 Question 3:

This is used to minimize the cost of the exhibit while displaying exactly 20 pieces and all of Rita's pieces. The chair of the Museum board offered to pay the minimum amount required on top of Ash's 4 Million

```
\begin{array}{l} \mathrm{Min} = 300^{*}\mathrm{Ya1} + 250^{*}\mathrm{Ya2} + 125^{*}\mathrm{Ya3} + 400^{*}\mathrm{Yb1} + 500^{*}\mathrm{Yb2} + 400^{*}\mathrm{Yb3} + 550^{*}\mathrm{Yb4} + 700^{*}\mathrm{Yc1} + 575^{*}\mathrm{Yc2} \\ + 200^{*}\mathrm{Yd1} + 225^{*}\mathrm{Yd2} + 150^{*}\mathrm{Ye1} + 150^{*}\mathrm{Ye2} + 850^{*}\mathrm{Yf1} + 750^{*}\mathrm{Yf2} + 400^{*}\mathrm{Yg1} + 175^{*}\mathrm{Yg2} + 450^{*}\mathrm{Yg3} \\ + 500^{*}\mathrm{Yh1} + 500^{*}\mathrm{Yh2} + 500^{*}\mathrm{Yh3} + 650^{*}\mathrm{Yi1} + 650^{*}\mathrm{Yi2} + 250^{*}\mathrm{Yj1} + 350^{*}\mathrm{Yj2} + 450^{*}\mathrm{Yj3} + 400^{*}\mathrm{Yk1} + 400^{*}\mathrm{Yk2} + 300^{*}\mathrm{Yl1} + 300^{*}\mathrm{Yl2} + 50^{*}\mathrm{Ym1} + 50^{*}\mathrm{Ym2} + 50^{*}\mathrm{Ym3} + 50^{*}\mathrm{Ym4} \end{array}
```

GAMS code:

```
\begin{split} z = & e = 300^*y(\text{`a'},\text{`1'}) + 250^*y(\text{`a'},\text{`2'}) + 125^*y(\text{`a'},\text{`3'}) + 400^*y(\text{`b'},\text{`1'}) + 500^*y(\text{`b'},\text{`2'}) + 400^*y(\text{`b'},\text{`3'}) + \\ & 550^*y(\text{`b'},\text{`4'}) + 700^*y(\text{`c'},\text{`1'}) + 575^*y(\text{`c'},\text{`2'}) + 200^*y(\text{`d'},\text{`1'}) + 225^*y(\text{`d'},\text{`2'}) + 150^*y(\text{`e'},\text{`1'}) + 150^*y(\text{`e'},\text{`2'}) \\ & + 850^*y(\text{`f'},\text{`1'}) + 750^*y(\text{`f'},\text{`2'}) + 400^*y(\text{`g'},\text{`1'}) + 175^*y(\text{`g'},\text{`2'}) + 450^*y(\text{`g'},\text{`3'}) + 500^*y(\text{`h'},\text{`1'}) + 500^*y(\text{`h'},\text{`2'}) \\ & + 500^*y(\text{`h'},\text{`3'}) + 650^*y(\text{`i'},\text{`1'}) + 650^*y(\text{`i'},\text{`2'}) + 250^*y(\text{`j'},\text{`1'}) + 350^*y(\text{'j'},\text{`2'}) + 450^*y(\text{'j'},\text{`3'}) + 400^*y(\text{`k'},\text{`1'}) \\ & + 400^*y(\text{`k'},\text{`2'}) + 300^*y(\text{'l'},\text{`1'}) + 300^*y(\text{'l'},\text{`2'}) + 50^*y(\text{`m'},\text{`1'}) + 50^*y(\text{`m'},\text{`2'}) + 50^*y(\text{`m'},\text{`3'}) + 50^*y(\text{`m'},\text{`4'}); \end{split}
```

5 Constraints

The following constraints were formulated due to the conditions established by Ash Briggs and Celeste McKenzie to accommodate many painters as much as possible while optimizing the given space and budget. These constraints provide information and limitations on what pieces to include in the exhibit.

```
Yc2 + Yg1 + Yl1 + Yl2 = 1;
```

This constraint is to include only one collage.

```
Yb1 + Yb2 \le Ya1 + Ya2 + Ya3;
```

This is to include at least one wire mesh sculpture displayed if a computer-generated drawing is displayed and vice versa.

```
Yf1 + Yf2 + Yh1 >= 1;
```

Include at least one photo-realistic painting.

```
Yh2 + Yk1 + Yk2 >= 1;
```

Include at least one cubist painting.

$$Yh3 >= 1;$$

Include at least one expressionist painting.

$$Yd1 + Yd2 + Ym1 + Ym2 + Ym3 + Ym4 >= 1;$$

Include at least one watercolor painting.

$$Ye1 + Ye2 + Yi1 + Yi2 + Yj3 >= 1;$$

Include at least one oil painting.

$$Yd1 + Yd2 + Ye1 + Ye2 + Yf1 + Yf2 + Yh1 + Yh2 + Yh3 + Yi1 + Yi2 + Yj3 + Yk1 + Yk2 + Ym1 + Ym2 + Ym3 + Ym4 <= 2*(Ya1 + Ya2 + Ya3 + Yb1 + Yb2 + Yb3 + Yb4 + Yc1 + Yc2 + Yg1 + Yg2 + Yg3 + Yj1 + Yj2 + Yl1 + Yl2);$$

The number of paintings to be no greater than twice the number of other art forms.

$$Yd1 = 1;$$

Yd2 = 1;

Ash wants both of her paintings of Candy Tate displayed whom he has a steamy affair.

$$Yf1 + Yf2 >= 1;$$

$$Yh1 + Yh2 + Yh3 >= 1;$$

Display as many pieces from David Lyman as from Rick Rawls and to display at least one piece from each of them

$$Yf1 + Yf2 = Yh1 + Yh2 + Yh3;$$

Artworks by Lyman must be equal to that of Rawls since Ash doesn't want to play favorites among them.

$$Yl1 + Yl2 \le 1$$
;

Ash will only accept one display from Ziggy or nothing at all.

$$\begin{array}{l} (\mathrm{Yc1} + \mathrm{Yc2} + \mathrm{Ye1} + \mathrm{Ye2} + \mathrm{Yf1} + \mathrm{Yf2} + \mathrm{Yg1} + \mathrm{Yg2} + \mathrm{Yg3} + \mathrm{Yh1} + \mathrm{Yh2} + \mathrm{Yh3} + \mathrm{Yi1} + \mathrm{Yi2} + \mathrm{Yj1} + \mathrm{Yj2} + \mathrm{Yj3} + \mathrm{Yl1} + \mathrm{Yl2} + \mathrm{Ym1} + \mathrm{Ym2} + \mathrm{Ym3} + \mathrm{Ym4})/2 <= \mathrm{Ya1} + \mathrm{Ya2} + \mathrm{Ya3} + \mathrm{Yb1} + \mathrm{Yb2} + \mathrm{Yb3} + \mathrm{Yb4} + \mathrm{Yd1} + \mathrm{Yd2} + \mathrm{Yk1} + \mathrm{Yk2}; \\ \end{array}$$

Due to Celeste's want to advance criticism, this constraint means to include at least one piece from a female artist for every two pieces included from a male artist.

$$Yc1 + Yc2 >= 1;$$

Include either one or both of the pieces "Aging Earth" and "Wasted Resources".

$$Yj1 + Yj2 + Yj3 >= 1;$$

To advance Native American Rights, include at least one by Bear Canton.

$$Yb1 + Yb2 + Yi1 + Yi2 >= 1;$$

To advance science, at least one of "Chaos Reigns", "Who Has Control", "Beyond", and Pioneers."

```
Yg2 >= Yl2;
```

If "Narcissism" is displayed, "Reflection" must also be displayed because "Reflection" also suggests narcissism.

Due to the limited space in the museum, the following constraints were formulated: Ya1 + Ya2 + Ya3 + Yc1 + Yg2 + Yg3 = 4;

Wall space is also limited and only 20 paintings, collages and drawings can be used for the exhibit. Yb1 + Yb2 + Yb3 + Yb4 + Yc2 + Yd1 + Yd2 + Ye1 + Ye2 + Yf1 + Yf2 + Yg1 + Yh1 + Yh2 + Yh3 + Yi1 + Yi2 + Yj1 + Yj2 + Yj3 + Yk1 + Yk2 + Yl1 + Yl2 + Ym1 + Ym2 + Ym3 + Ym4 = 20;

Other constraints were formulated due to the additional questions given in the problem. For the first question, the cost constraint where each binary variable is multiplied to their corresponding cost. The summation of cost must be at most 4 million dollar.

The second question modifies the objective function where the cost is multiplied to each binary variable. The additional constraint is that the summation of all pieces must be at least 20. All the variables are added and must be greater than or equal to 20.

For the last question, all the pieces when added must be equal to 20 pieces. Also, another constraint says that all of Rita Lowky's pieces must be displayed so the variables representing her pieces are summed and equated to 4.

5.1 GAMS code for Question 1 Constraints

```
 \begin{split} & \text{artWorks.. } z = = y(\text{'a','1'}) + y(\text{'a','2'}) + y(\text{'a','3'}) + y(\text{'b','1'}) + y(\text{'b','2'}) + y(\text{'b','3'}) + y(\text{'b','4'}) + y(\text{'c','1'}) \\ & + y(\text{'c','2'}) + y(\text{'d','1'}) + y(\text{'d','2'}) + y(\text{'e','1'}) + y(\text{'e','2'}) + y(\text{'f','1'}) + y(\text{'f','2'}) + y(\text{'g','1'}) + y(\text{'g','2'}) \\ & + y(\text{'g','3'}) + y(\text{'h','1'}) + y(\text{'h','2'}) + y(\text{'h','3'}) + y(\text{'h','3'}) + y(\text{'i','1'}) + y(\text{'j','2'}) + y(\text{'j','1'}) + y(\text{'j','2'}) + y(\text{'j','2'}) \\ & + y(\text{'k','1'}) + y(\text{'h','2'}) + y(\text{'l','1'}) + y(\text{'l','2'}) + y(\text{'m','2'}) + y(\text{'m','3'}) + y(\text{'m','4'}); \\ & \text{collageConstraints.. } y(\text{'c','2'}) + y(\text{'g','1'}) + y(\text{'l','1'}) + y(\text{'l','2'}) + y(\text{'a','2'}) + y(\text{'a','3'}) ; \\ & \text{meshReqConstraints.. } y(\text{'b','1'}) + y(\text{'b','2'}) + y(\text{'h','1'}) + y(\text{'a','2'}) + y(\text{'a','3'}) ; \\ & \text{realisticConstraint.. } y(\text{'f','1'}) + y(\text{'f','2'}) + y(\text{'h','1'}) + g = 1 ; \\ & \text{cubist.. } y(\text{'h','2'}) + y(\text{'k','1'}) + y(\text{'k','2'}) + g = 1 ; \\ & \text{expressionist.. } y(\text{'h','3'}) = G = 1 ; \\ & \text{water.. } y(\text{'d','1'}) + y(\text{'d','2'}) + y(\text{'m','1'}) + y(\text{'m','2'}) + y(\text{'m','3'}) + y(\text{'m','4'}) = G = 1 ; \\ & \text{water.. } y(\text{'d','1'}) + y(\text{'d','2'}) + y(\text{'m','1'}) + y(\text{'m','2'}) + y(\text{'m','3'}) + y(\text{'m','4'}) = G = 1 ; \\ \end{aligned}
```

```
oil. y('e','1')+y('e','2')+y('i','1')+y('i','2')+y('j','3')=G=1;
paint Sculpture...y('d','1') + y('d','2') + y('e','1') + y('e','2') + y('f','1') + y('f','2') + y('h','1') + y('h','2') + y('h','3') + y('h','1') 
, '1') + y('k', '2') + y('m', '1') + y('m', '2') + y('m', '3') + y('m', '4') = L = 2*(y('a', '1') + y('a', '2') + y('a', '3') + y('b', '1') + y('b', '1') + y('b', '1') + y('b', '1') + y('a', '1') 
 (2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')
(1') + v(1', 2');
abct1.. y('d','1') = e = 1;
abct2.. y('d', '2') = e = 1;
abf1.. y('f','1')+y('f','2')=G=1;
abf2.. y('h','1')+y('h','2')+y('h','3') =G=1;
abf3.. y('f','1')+y('f','2') = e = y('h','1')+y('h','2')+y('h','3');
ziggy.. y('l','1') + y('l','2') = L = 1;
 \text{criticism..} \ (y(\text{`c'},\text{`1'}) + y(\text{`c'},\text{`2'}) + y(\text{`e'},\text{`1'}) + y(\text{`e'},\text{`2'}) + y(\text{`f'},\text{`1'}) + y(\text{`f'},\text{`2'}) + y(\text{`g'},\text{`1'}) + y(\text{`g'},\text{`2'}) + y(\text{`g'},\text{`3'}) + y(\text{`h'},\text{`1'}) + y(\text{`h'},\text{`1'}) + y(\text{`h'},\text{`1'}) + y(\text{`h'},\text{`1'}) + y(\text{`h'},\text{`1'}) + y(\text{`h'},\text{`1'}) + y(\text{`h'},\text{`h'},\text{`1'}) + y(\text{`h'},\text{`h'},\text{`h'},\text{`h'}) + y(\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'},\text{`h'}
 (2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')
(2') + y(m', 3') + y(m', 4')/2 = L = y(a', 1') + y(a', 2') + y(a', 3') + y(b', 1') + y(b', 2') + y(b', 3') + y(b
(3') + y('d','1')+y('d','2')+y('k','1')+y('k','2');
env.. y('c','1') + y('c','2') = G = 1;
native.. y('j','1')+y('j','2')+y('j','3') = G= 1;
science.. y('b','1') + y('b','2') + y('i','1') + y('i','2') = G = 1;
narcissim.. y('g', '2') = G = y('l', '2');
sculpt.. y('a','1')+y('a','2')+y('a','3')+y('c','1')+y('g','2')+y('g','3')=L=4;
space...y('b','1')+y('b','2')+y('b','3')+y('b','4')+y('c','2')+y('d','1')+y('d','2')+y('e','1')+y('e','2')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y('f','1')+y
 (2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')
 (1)' + y(k', 2)' + y(1', 1)' + y(1', 2)' + y(m', 2)' + y(m', 2)' + y(m', 3)' + y(m', 3)' + y(m', 4)' = L = 20;
cost.. \ 300*y('a','1') + 250*y('a','2') + 125*y('a','3') + 400*y('b','1') + 500*y('b','2') + 400*y('b','3') + 400*y('b','3
550^*y('b','4') + 700^*y('c','1') + 575^*y('c','2') + 200^*y('d','1') + 225^*y('d','2') + 150^*y('e','1') + 150^*y('e','2')
+850*y('f','1') + 750*y('f','2') + 400*y('g','1') + 175*y('g','2') + 450*y('g','3') + 500*y('h','1') + 500*y('h','2')
+500*y('h','3') +650*y('i','1') +650*y('i','2') +250*y('j','1') +350*y('j','2') +450*y('j','3') +400*y('k','1') \\
 +400*y('k','2') +300*y('l','1') +300*y('l','2') +50*y('m','1') +50*y('m','2') +50*y('m','3') +50*y('m','4')
```

5.2 GAMS code for Question 2 Constraints

```
y('c', '2') + y('g', '1') + y('l', '1') + y('l', '2') = e = 1;
y('b','1')+y('b','2') = L = y('a','1')+y('a','2')+y('a','3');
y('f','1')+y('f','2')+y('h','1') =G=1;
y('h','2')+y('k','1')+y('k','2') =G=1;
y('h','3') = G = 1;
v('d','1')+v('d','2')+v('m','1')+v('m','2')+v('m','3')+v('m','4')=G=1;
y('e','1')+y('e','2')+y('i','1')+y('i','2')+y('j','3') =G=1;
y('d','1')+y('d','2')+y('e','1')+y('e','2')+y('f','1')+y('f','2')+y('h','1')+y('h','2')+y('h','3')+y('i','1')+y('i','2')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3')+y('j','3'
  (2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')
(1') + y(1', 2');
y('d','1') = e = 1;
y('d', '2') = e = 1;
y('f','1')+y('f','2') =G=1;
y('h','1')+y('h','2')+y('h','3') =G=1;
y('f','1')+y('f','2') = e = y('h','1')+y('h','2')+y('h','3');
y('l','1') + y('l','2') = L = 1;
 (y(c',1')+y(c',2')+y(e',1')+y(e',2')+y(f',1')+y(f',2')+y(f',2')+y(g',1')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+y(g',2')+
 (2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')+y(2')
(2') + y(m', 3') + y(m', 4'))/2 = L = y(a', 1') + y(a', 2') + y(a', 3') + y(b', 1') + y(b', 3') + y(
 ,'4') + y('d','1')+y('d','2')+y('k','1')+y('k','2');
y('c','1') + y('c','2') = G = 1;
```

```
\begin{split} y(\text{`j'},\text{`1'}) + y(\text{`j'},\text{`2'}) + y(\text{`j'},\text{`3'}) &= G = 1; \\ y(\text{`b'},\text{`1'}) + y(\text{`b'},\text{`2'}) + y(\text{`i'},\text{`1'}) + y(\text{`i'},\text{`2'}) &= G = 1; \\ y(\text{`g'},\text{`2'}) &= G = y(\text{`l'},\text{`2'}); \\ y(\text{`a'},\text{`1'}) + y(\text{`a'},\text{`2'}) + y(\text{`a'},\text{`3'}) + y(\text{`c'},\text{`1'}) + y(\text{`g'},\text{`2'}) + y(\text{`g'},\text{`3'}) &= L = 4; \\ y(\text{`b'},\text{`1'}) + y(\text{`b'},\text{`2'}) + y(\text{`b'},\text{`3'}) + y(\text{`b'},\text{`4'}) + y(\text{`c'},\text{`2'}) + y(\text{`d'},\text{`1'}) + y(\text{`d'},\text{`2'}) + y(\text{`e'},\text{`1'}) + y(\text{`e'},\text{`2'}) + y(\text{`f'},\text{`1'}) + y(\text{`f'},\text{`2'}) + y(\text{`g'},\text{`1'}) + y(\text{`b'},\text{`2'}) + y(\text{`b'},\text{`3'}) + y(\text{`b'},\text{`3'}) + y(\text{`b'},\text{`3'}) + y(\text{`b'},\text{`1'}) + y(\text{`i'},\text{`2'}) + y(\text{`j'},\text{`1'}) + y(\text{`j'},\text{`2'}) + y(\text{`j'},\text{`3'}) + y(\text{`b'},\text{`3'}) + y(\text{`b'},\text{`2'}) + y(\text{`b'},\text{`3'}) + y(\text{`b'},\text{`3'}) + y(\text{`b'},\text{`3'}) + y(\text{`b'},\text{`3'}) + y(\text{`b'},\text{`1'}) + y(\text{`b'},\text{`2'}) + y(\text{`b'},\text{`3'}) + y(\text{`b'},\text{`1'}) + y(\text{`b'},\text{`2'}) + y(\text{`b'},\text{`3'}) + y(\text{`b'},\text{
```

5.3 GAMS code for Question 3 Constraints

All the conditions in the Question 2 with an added constraint for rita

$$y('b','1')+y('b','2')+y('b','3')+y('b','4') =G=4$$

6 Solutions and Discussion

The problem of assigning art is solved using Binary Integer Programming wherein it is determined whether a specific artwork is to be placed on the museum or not. The model generated different results depending on the current objective and the constraints needed to be satisfied.

6.1 Question 1:

For the first part, it is indicated that 4 million dollars is available to fund the exhibit. Given this, we want to maximize the number of pieces that can be displayed without exceeding the budget.

The total cost of the chosen pieces were found to be 3950000 dollars

8			
Serial.No	Artist	Art	Price
1	Angie Oldman	Reflection	175000
2	Ash Briggs	All that glitters	50000
3	Ash Briggs	Dreams come true	50000
4	Ash Briggs	The Rock	50000
5	Ash Briggs	Winding Road	50000
6	Bear Canton	Wisdom	250000
7	Candy Tate	Calm before the strom	225000
8	Candy Tate	Serenity	200000
9	Colin Zweibell	The Great Equlizer	125000
10	David Lyman	Harley	750000
11	Helen Row	Study of the Violin	400000
12	Norm Marson	Wasted Resource	575000
13	Rick Rawls	RICK III	500000
14	Rita Losky	Chaos Reign	400000
15	Robert Bayer	Void	150000

6.2 Question 2:

For the second situation, the goal is to minimize the cost of placing a total of at least 20 artworks on the exhibit. The chosen artworks are stated in Table 3 with their corresponding costs. The total cost of displaying the said artworks is 5,400,000.

Perfection	300,000	Reflection	175,000
Burden	250,000	Rick III	500,000
The Great Equalizer	125,000	Wisdom	250,000
Chaos Reigns	400,000	Superior Power	350,000
Wasted Resources	575,000	Study of a Violin	400,000
Serenity	200,000	Study of a Fruit Bowl	400,000
Calm Before the Storm	225,000	All that Glitters	50,000
Void	150,000	The Rock	50,000
Sun	150,000	Winding Road	50,000
Harley	750,000	Dreams Come True	50,000

6.3 Question 3:

Lastly, in the third situation, it is required to choose 20 artworks with an available fund of 4,000,000. Moreover, a patron of Rita Losky's work is willing to pay the minimum additional amount that will exceed the budget as long as all Losky's work will be displayed. Chosen artworks and their individual costs are summarized in Table below. The total cost is 5,800,000, therefore, the patron should pay 1,800,000.

Burden	250,000
The Great Equalizer	125,000
Chaos Reigns	400,000
Who Has Control?	500,000
Domestication	400,000
Innoncence	550,000
Wasted Resources	575,000
Serenity	200,000
Calm Before the Storm	225,000
Void	150,000
Sun	150,000
Harley	750,000
Reflection	175,000
Rick III	500,000
Wisdom	250,000
Study of a Fruit Bowl	400,000
All that Glitters	50,000
The Rock	50,000
Winding Road	50,000
Dreams Come True	50,000
TOTAL	5,800,000

7 Appendix

- 1. GAMS code and Solver Results for Question 1 $\,$
- 2. GAMS code and Solver Results for Question $2\,$
- 3. GAMS code and Solver Results for Question 3 $\,$