DYNAMIC PROGRAMMING

Handwritten Notes of Striver(TUF) Playlist

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DP (Dynamic programming)

1. DP1. Introduction to Dynamic programming | Memoization | Tabulation | Space optimization Techniques
2. DP2. Climbing Stairs | Learn How to write 1D Recurrance Relations
3. DP3. Frog Jump | Dynamic programming | Learn to write 1D DP
4. DP4. Frog Jump with K distance | lecture 3 follow up question
5. DP5. Maximum sum of Non-adjacent elements | House Robber | 1-D | DP on subsequences
6. DP6. House Robber 2 | 1D DP | DP on Subsequences
7. DP7. Ninja's Training | Must for 2D concepts | 2D DP
8. DP8. Grid unique path | Learn everything about DP on Grids | All Techniques
9. DP9. unique paths2 | DP on Grid with Maze obstacles
10. DP10. Minimum path sum in Grid | DP on Grids | Asked in Microsoft
11. DP11. Triangle | fixed starting point and variable ending point | DP on Grids
12. DP12. Minimum/Maximum falling path sum | Variable starting and ending point | DP on Grids
13. DP13. Cherry pickup II | 3D DP made Easy | DP on Grids
14. DP14. Subset sum Equal k to Target | Identify DP on subsequences and ways to solve them
15. partition Equal Subset sum | DP on subsequences
16. Partition A set into Two Subsets with minimum Absolute sum Difference DP on subsequences
17. Count Subsets with sum K | DP on subsequences
18. Count partitions with given difference | DP on Subsequences
19. 0/1 Knapsack | Recursion to strivers array space optimization approach | DP on subsequences
20. Minimum coins | DP on Subsequences | Infinite Supplies pattern
21. Target Sum | DP on subsequences
22. DP22. Coin Change 2 | Infinite supplies pattern | DP on subsequences
23. DP23. Unbounded Knapsack | 1-D Array space optimised Approach
24. DP24. Rod cutting problem | 1-D Array space optimised approach
25. DP25. Longest Common subsequences | Top-down | Bottom-up | Space optimised | DP on String
26. DP26. print longest Common subsequences | DP on string

27. DP27. Longest Common Substring | DP on string
28. DP28. Longest palindromic subsequences |
29. DP29. Minimum insertions to make string palindrome
30. DP30. Minimum insertions | Deletions to cover string A to string B
31. DP 31. Shortest common subsequences | DP on strings
32. DP32. Distinct subsequences | 1D Array Optimization Technique
33. DP 33. Edit distance | Recursive to 1D Array optimised solution
34. DP34. Wildcard matching | Recursive to 1D Array optimisation
35. DP35. Best time to Buy and sell stocks | DP on stocks
36. DP36. Buy and sell stocks - II | Recursion to space optimisation
37. DP37. Buy and sell stocks - III | Recursion to space optimisation
38. DP 38. Buy and sell stocks - IV | Recursion to space optimisation
39. DP39. Buy and sell stocks with cooldown | Recursion to space optimization
40. DP40. Buy and Sell stocks with Transaction Fee | Recursion to space optimisation
41. DP41. Longest increasing subsequence | Memoization
42. DP42. Printing Longest increasing subsequences | Tabulation | Algorithm
48. DP43. Longest increasing subsequence | Binary search | Intution
44. DP44 Largest divisible subset | Longest increasing subsequences
45. DP45. Longest String chain | Longest increasing subsequence | LIS
46. DP46. Longest Bfot Bitonic subsequence | LIS
47. DP47. Number of Longest increasing subsequences
48. DP48. Matrix chain Multiplication | MCM | partition DP starts
49. DP49. Matrix chain Multiplication | MCM | Bottom-up | Tabulation
50. DP50. Minimum cost to cut the stick |
51. DP51. Burst Ballons | partion DP |
52. DP52. Evaluate Boolean Expression to True | partion DP
58. DP53. palindrome partitioning - II | Front partition
54. DP54. partition array for Maximum sum | Front partition
55. DP55. Maximum Rectangle area with all 1's | DP on Rectangles
56. DP56. Count Square Submatrices with All ones | DP on Rectangles

DP (Dynamic programming)

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Those who cannot remember the past are condemned to repeat it.

i) Tabulation — Bottom up
ii) Memoization — Top-down

Memoization → Tabulation
↳ TL ↳ TC
SC SC → final soln

Pre requisite
Recursion playlist
1st 10 vdos, 6th & 7th imp

(1) fibonacci No.

0 1 1 2 3 5 8 13 21
0 1 2 3 4 5 6 7

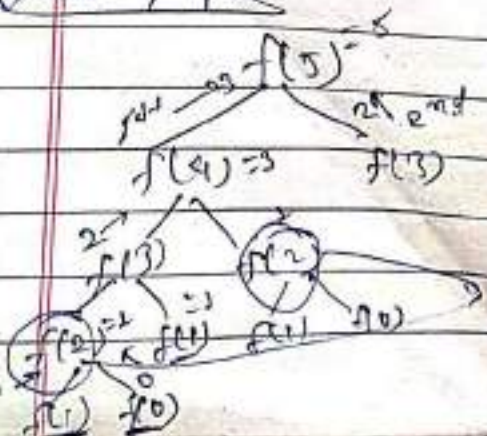
$f(n) = f(n-1) + f(n-2)$

$f(4) =$

F(n) Recursion

```
{
  if(n<=1)
    return n;
  return f(n-1) + f(n-2);
}
```

Recursion tree



f(4) →
f(3) →

overlapping sub problem
Memoization

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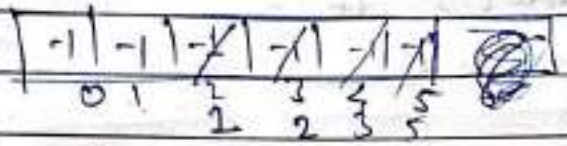
Memorization) → tend to store the value of sub problems in some map/table.



Initialy = -1

dp[n+1]

```
f(n)
{
  if (n<=1)
    return n;
  if(dp[n] != -1)
    return dp[n];
  return f(n-1) + f(n-2); (1)
}
```

① convert Recursion into Memorization.

②

dp[n]

Recursion → DP

3 Steps

declearse a dp array
size of sub problem i.e. n

Step 1 : Storing the answer which is being computed for every subproblem. will

Step 2 : Checking if the subproblem has been previously solved. if it is previously solved then the value will not be -1.

Step 3 :

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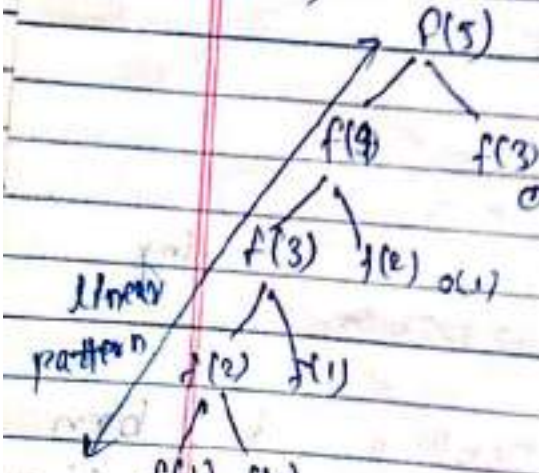
```
int f( int n , vector<int> &dp )
{
    if(n<=1 )
        return n;

    if( dp[n] != -1)
        return dp[n];

    return dp[n] = f(n-1, dp ) + f(n-2 , dp);
}
int main()
{
    int n;
    cin>>n;
    vector<int> dp(n,-1);
    cout<< f(n,dp);
    return 0;
}
```



TC = O(N)

SC = O(N) + O(N)

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Recursion → Tabulation (Bottom - Up)
(Top down)
(Base case to the required)

* Answer require to base case

dp[n+1]

dp[0] = 0 dp[1] = 1

```
for(i=2; i<=n ; i++)
    {
    dp[i] = dp[i-1] + dp[i-2];
    }
```

Best solution (space optimization)

```
int main()
{
int n;
cin>>n;
int prev2 = 0;
int prev1 = 1;
for(int i=2; i<=n; i++)
    {
    int curr = prev + prev2
    prev2 = prev;
    prev = curr;
    }
cout<< prev;
```

Intuitive solution

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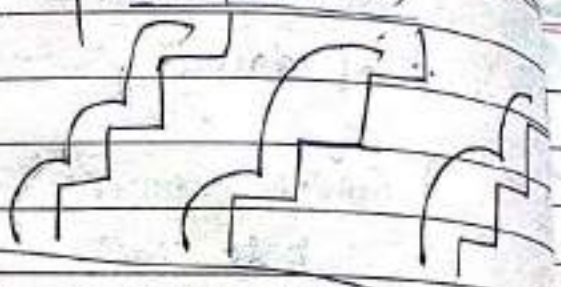
Climbing stairs problem | 1D problem



n = 3

* understand a DP problem :-

1. concept of try all possible way
   - Count
   - Best way
2. count the total no. of ways.
3. Optimal solution (min, max)

Then try to apply recursion all possible ways

Jwp.

Shortcut

1. Try to represent the problem in terms of index.
2. Do all possible stuff on that index according to the problem statement.
3. If question say count all ways then sum of all stuff.
4. min (of all stuff) → find min

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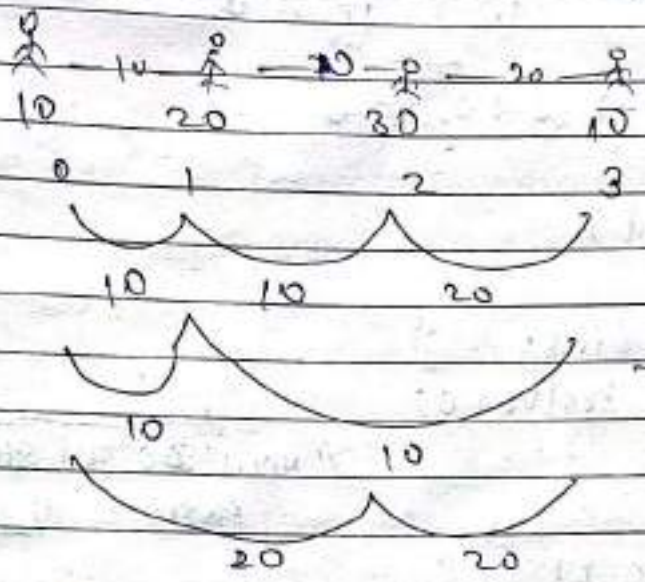
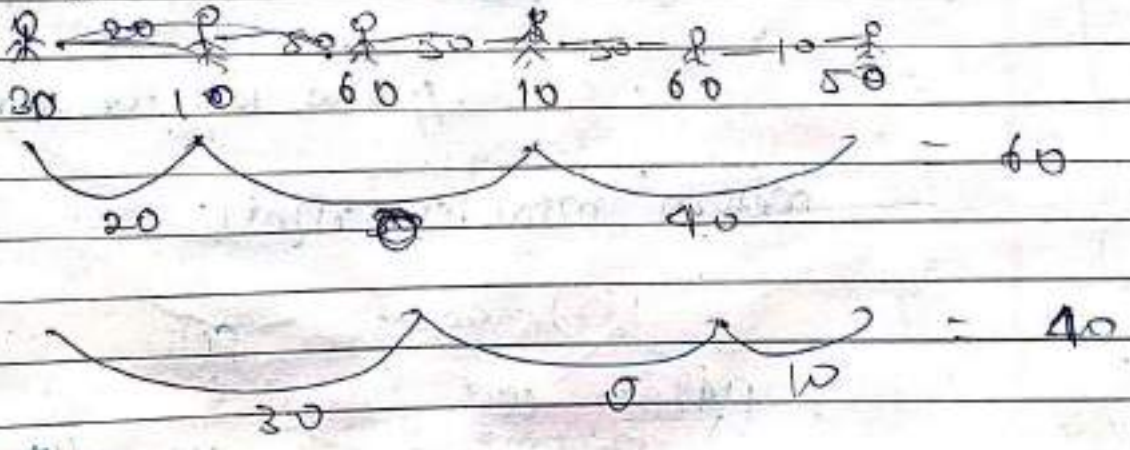
Frog JUMP problem :-



All possible ways, best ways - Means Recursion / DP

Why a greedy algo does not works ?



Greedy fails here, so we have to try Recursion.

Recursion → Try all possible ways.

↓

(min energy)

Step 1: index

Step 2: Do all steps on that index acc. to question.

Steps: Take @ the min (all steps).

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f(n-1) → min energy required to reach (n-1) from 0.

f(0) → (0 → 0) = 0

```
f(ind)
{
  if(ind == 0)
    return 0;
```

// Now DO all stuffs on the index +1 or +2 jump

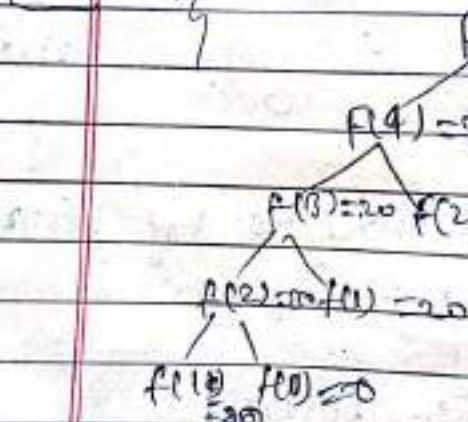
Recursive code

```
left = f(n-1)
  left = f(ind-1) + abs(a[ind] - a[ind-1]);

  if(ind > 1)
  {right = f(ind-2) + abs(a[ind] - a[ind-2]);
  }

// Now take the minimal

return min(left, right);
}
```



f(5)

left = f(4) + |10| = 40

right = f(3) + |40| = 60

min(40, 60)

= 40

and so, on

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0	1	2	3	4	5
30	10	60	10	60	50

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f(1)
{
left = f(0) + arr(20)
right = INT_MAX ——— ∵ ind ≯ 1 of arr
return (20, INT_MAX);
}
∴ f(1) = 20

f(2)
{
left = f(1) + (50) : = 70
right = f(0) + (30) : = 30
return (70, 30)
}
∴ f(2) = 30

f(3)
{ left = f(2) + (50) : = 80
right = f(1) + (0) = 20
return (80, 20) ∴ f(3) = 20
}

f(4)
{ left = f(3) + (50) = 70
right = f(2) + (0) = 30
return (70, 30)
}
f(4) = 30

f(5)
{ left = f(4) + (10) = 40
right = f(3) + (40) = 60
return (40, 60); ∴ f(5) = 40 Ans
}

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How to convert Recurence into DP

Recurence → DP

Memoization → look at the parameters are changing.

ind is changing

∵ Max size of ind is 5

∴ 5+1 = 6

dp[6]

Step1: declare an array of size n+1

```
dp[n+1] := -1
```

Step2: Before returning add it up. Store it and return.

```
return dp[ind] = min(left, right);
```

Step3: Check if it is previously computed or not.

```
if(dp[ind] != -1)
    return dp[ind];
```

TC → O(N)

SC → O(N) + O(N)

Recursion array

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...ning the subsequence

Let's code

```
int ff(int ind, vector<int> &height, vector<int> &dp):
{
    if(ind==0)
        return 0;
    if(dp[ind]!=-1) return dp[ind];
    int left = f(ind-1, height, abs(height[ind] - height[ind-1]);
    int right = INT_MAX;
    if(ind>1)
    {
        right = f(ind-2, height, abs(height[ind] - height[ind-2]);
    }
    return dp[ind] = min(left, right);
}
```

Only some test case will pass & T.L.E.

```
int frogjump(int n, vector<int> &height)
{
    vector<int> dp(n+1, -1);   — (1) Step
    return f(n-1, height, dp)
}
```

Now optimized Memoization.

TC:
TC → O(N)
SC → O(N) + O(N)
(recursion) (array dp)

Step 1: declare dp
dp[n+1]

Step 2: Store it and return (overlapping sub problem)
return dp[ind] = min(left, right);

Step 3: Check if it is previously computed or not
if(dp[ind]!=-1)
return dp[ind];

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Memoization (Top-down) — Tabulation (Bottom-up)

Tabulation Method

int dp[n] → for zero based index

```
int frogJump(int n, vector<int> &heights)
{
    vector<int> dp(n, 0);
    dp[0] = 0;
    for(int i=1; i<n; i++)
    {
        int fs = dp[i-1] + abs(height[i] - height[i-1]);
        int ss = INT_MAX;
        if(i>1)
        {
            ss = dp[i-2] + abs(height[i] - height[i-2]);
        }
        dp[i] = min(fs, ss);
    }
    return dp[n-1];
}
```

T.C → O(N)

S.C → O(N) + O(N) (Recursion reduced)

O(N) — array dp

space optimization

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```
int frogJump(int n, vector<int>& height)
{
    int prev = 0;
    int prev2 = 0;

    for(int i = 1; i < n; i++)
    {
        int fs = prev + abs(height[i] - height[i-1]);
        int ss = INT_MAX;
        if(i > 1)
            ss = prev2 + abs(height[i] - height[i-2]);

        int curi = min(fs, ss);
        prev2 = prev;
        prev = curi;
    }
    return prev;
}
```

$TC = O(N)$

$SC = O(N) + O(N)$ ~~array dp~~ ~~Recursion~~

$\therefore SC = O(1)$ both are reduced

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Recurrence & Code

```
f(ind)
{
  if(ind == 0)
    return 0;

int minSteps = INT_MAX;
  for(int j=1; j<=k ; j++)
{
    if( ind-j >=0)
    {
    Jump = f(ind-j) + abs(a[ind] - a[ind-j]
    minSteps = min( min_steps, jump);
    }
```

Now Memoize the code

```
f(ind)
```

Tabulation

```
if{   int dp[n];
      dp[0] = 0;
  for(int i=1; i<n; i++)
    {
        for( j
    int min_steps = INT_MAX;
    for( j=1; j<=k; j++)
      { if(i-j>=0)
      }
    Jump = dp[i-1] + abs(a[i] - a[i-j]
    Min steps = Min( Min_steps, jump);
```

dp[i] = min_steps;

}

print [dp[n-1]];

T.C → O(N×K)

S.C → O(N)

Can't optimize further as S.C - O(K) + O(N)

MAXIMUM SUM OF NON-ADJACENT ELEMENTS

Let's try out all subsequences with the given constraints

Try out all means Recursion.

↓

pick the one with maximum sum.

Print all subsequences

↳ pick/Non-pick

1. Index
2. Do stuff
3. return the best you can get

Pick subsequence with no adjacent element.

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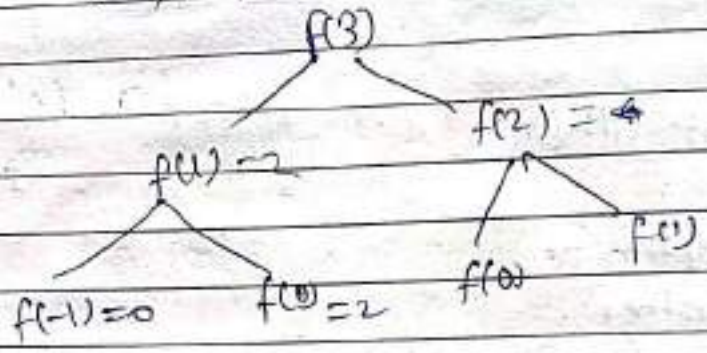
0 1 2 3

2, 1, 4, 9



f(1)

{

f(1) → sum[0...1]

pick = a[1] + f(-1) = 1 + 0 = 1

not pick = 0 + f(0) = 2

max(pick, not pick) ⇒ ∴ f(1) = 2

f(3)

{

pick = a[3] + f(1) = 9 + 2 = 11

not pick = 0 + f(2) = 6

max(pick, not pick) ∴ f(3) = 11

f(2)

{ pick = a[2] + f(0) = 4 + 2 = 6

not pick = 0 + f(1) = 2

max(pick, not pick) ∴ f(2) = 6

TC - O(N)

SC - O(N) + O(N)

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Leet code

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Recursive code

```
int f(int ind, vector<int> &nums, &dp)
{
  if(ind == 0)
      return nums[ind];
  if(ind < 0)
      return 0;
  int if(dp[ind] != -1) return dp[ind];
  int pick = nums[ind] + f(ind-2, nums, dp);

  int not_pick = 0 + f(ind-1, nums, dp);
  return max(pick, not_pick);
}
return dp[ind] = max(pick, not_pick);
int Max_Sum_Non_Adjacent(vector<int> &nums)
{
    int n = nums.size();
    Vector<int> dp(n,-1);
    return (n-1, nums, dp);
}
```

Now Memoized to save from TLE

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Tabulation

```
int dP[0] = a[0];
int neg = 0;
for(int i = 1; i<n; i++)
{
take = a[ind];
Non take = 0+
if(i>1)
take = take + dp[i-2];
Nontake = 0 + dp[i-1];
dp[i] = max(take, non take);
}
```

TC = O(N)
SC = O(N)

Space optimization :-

```
int prev = a[0]
int prev2 = 0;
for(i=1; i<n; i++)
{
take = a[ind];
if(i>1)
take = take + prev2;
Non take = 0 + prev;
```

curi = Max (take, Non_take);
prev2 = prev;
Prev = curi;

}

print (prev);

int Max_sum_Non_Adja(vector<int> &nums)

{

int n = nums.size();

int prev = nums[0];

int prev2 = 0;

for(int i=1; i<n; i++)

{

int take = nums[i];

if (i>1)

take = take + prev2;

int Not_take = 0 + prev;

int curi = max(take, Not take);

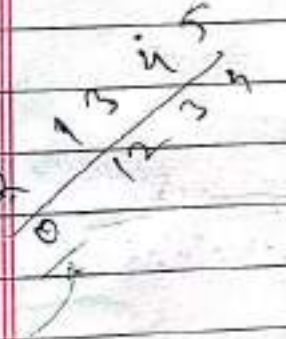
prev2 = prev

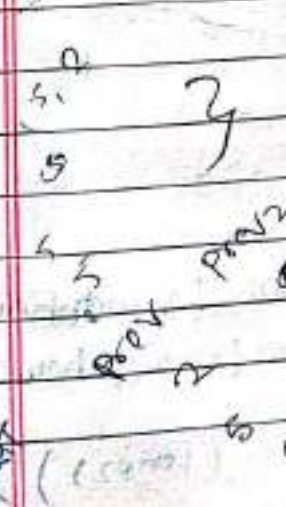
prev = curi;

}

return prev;

}





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HOUSE ROBBER

```
int max(vector<int> &nums)
{
    int n = nums.size();
    int prev = nums[0];
    int prev2 = 0;
    int
    for(int i = 1; i < n; i++)
    {   int take = nums[i];
        if(i > 1)
            take = take + prev2;

        int not_take = 0 + prev;

        int curi = max(take, not_take);
        prev2 = prev;
        prev = curi;
    }

    return prev;
}

int houseRobber(vector<int> &valueInHouse)
{
    vector<int> temp1, temp2;
    int n = valueInHouse.size();
    if(n == 1) return valueInHouse[0];
    for(int i = 0; i < n; i++)
    {   if(i != 0) temp1.push_back(valueInHouse[i]);
        if(i != n-1) temp2.push_back(valueInHouse[i]);
    }
    return max(maximumNoOfAdj(temp1), maximumNoOfAdj(temp2));
}
```

NINJA TRAINING Problem

whenever

Greedy fails

↓

Try all possible ways

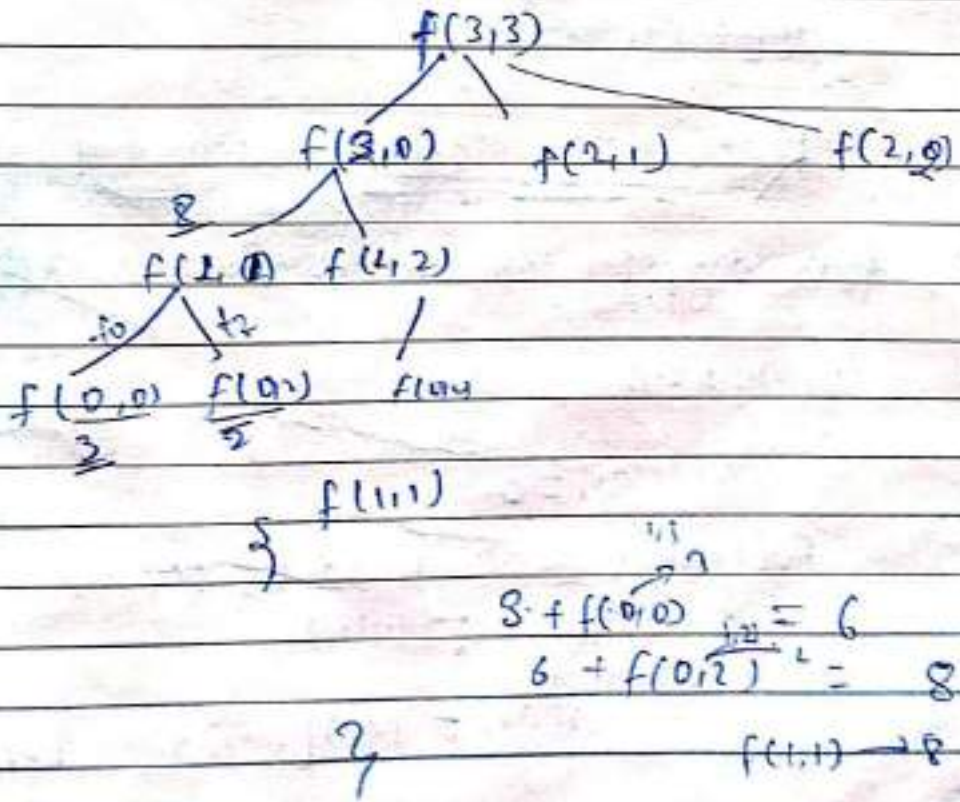
↓

Recursion

↓

1. index
2. DO Stuffs on that index
3. Find max or min

t0	t1	t2	
2	1	3	d_0
(3)	4	6	d_1
10	1	6	d_2
8	3	7	d_3



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f(1,2)
{
3+3 = 6
4+3 = 7
}

f(2,0)
{
1 + f(1,1) = 9
6 + f(1,2) = 12
}

f(2,1)
{

Overlapping sub-problem
↓
Memoization
(changing parameter
ind, last

day (N) | last (0,1,2,3) (4)

N X 4 dp size array
dp[ind][last]

Recursion code + memoized

```
int f(int day, int last, vector<vector<int>> &points,dp)
{
  if(day == 0)
  {
    int maxi = 0;
    for(int task = 0; task < 3; task++)
    { if(task != last)
      {
        maxi = max(maxi, points[0][task]);
      }
    }
    return maxi;
```

```
if(dp[day][last] != -1)
    return dp[day][last];
int maxi=0;
for(int task=0; task<3; task++)
{
    if(task != last)
    {
        int point = points[day][task] + f(day-1, task, points, dp);
        maxi = max(maxi, point);
    }
}
return maxi;
}
return dp[day][last]= maxi;

int ninjaTraining(int n, vector<vector<int>> &points)
{
    vector<vector<int>> dp(n, vector<int>(4,-1));
    return f(n-1, 3, points, dp);
}
```

TL - O(N*4)*3
SC - O(N) + O(N*4)

Tabulation :-

```
dp[0][0] = max(arr[0][1], arr[0][2])
dp[0][1] = max(arr[0][0], arr[0][2])
dp[0][2] = max(arr[0][0], arr[0][1])
dp[0][3] = max(arr[0][0], arr[0][1], arr[0][2])
```

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Tab

Tabulation

```
int ninjatraining(int n, vector<vector<int>> &points)
{
    vector<vector<int>> dp(n, vector<int>(4, 0));

    dp[0][0] = max(points[0][1], points[0][2]);
    dp[0][1] = max(points[0][0], points[0][2]);
    dp[0][2] = max(points[0][0], points[0][1]);
    dp[0][3] = max(points[0][0], points[0][1], points[0][2]);

    for(int day = 1; day < n; day++)
    {
        for(int last = 0; last < 4; last++)
        {
            dp[day][last] = 0;

            for(int task = 0; task < 3; task++)
            {
                if(task != last)
                {
                    int activity = points[day][task]
                                 + dp[day-1][task];
                    dp[day][last] = max(dp[day][last], activity);
                }
            }
        }
    }
    return dp[n-1][3];
}
```

TC = O(N*4*3)

SC = O(N*4)

Space optimization

```
int ninjatraining(int n, vector<vector<int>> &points)
{
    vector<int> prev(4,0);

    prev[0] = max(points[0][1], points[0][2]);
    prev[1] = max(points[0][0], points[0][2]);
    prev[2] = max(points[0][0], points[0][1]);
    prev[3] = max(points[0][0], points[0][1], points[0][2]);

    for(int day=1; day<n; day++)
    {
        vector<int> temp(4,0);
        for(int last = 0; last < 4; last++)
        {
            temp[last] = 0;

            for(int task = 0; task < 3; task++)
            {
                if(task != last)
                {
                    temp[last] = max(temp[last],
                    points[day][last] + prev[task]);
                }
            }
        }
        prev = temp;
    }
    return prev[3];
}
```

TC = O(N*4*3)
SC = O(4)
constant

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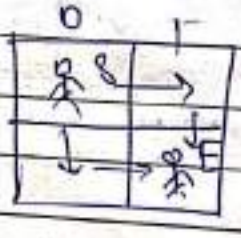
DP on Grids / 2D Matrix

→ Count paths
→ Count paths with obstacles
→ Min path sum
→ Max path sum
→ Triangle
→ 2 start points

6 problems

Total unique path

Unique path



(0,0) → (1,1)
2 ways

(0,0) → (m-1, n-1)

Try all possible ways
Means apply Recursion

How to write Recursion
1. Express in terms of index — (i,j) row col
2. Explore all steps
3. Sum up all ways / min / max

f(i,j) → no of unique ways
(0,0) → (i,j)


```
f(i,j)
{ Base case
    return 1 if reach destination
    return 0 if can't reach destination
}
```

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```
ip f(i,j)
{
  if(i==0 && j==0)
      return 1;

  IF(i<0 || j<0)
      return 0;
  if(dp[i][j] != -1) return dp[i][j];
  up = f(i-1, j);
  left = f(i, j-1);

  return up+left;
}
```

Base case

Explore

Explore

Return up

dp[i][j] =

Recursion → DP

Memoization

overlapping sub problem.

	0	1	2
0	•		
1			
2		•	

Let m = 3, n = 3

f(2,2)

up / left

f(1,2) f(2,1)

f(1,2) =

f(0,2) f(1,1)

f(-1,2) f(0,1)

f(-1,1) f(0,0)

TC = O(N×m)

SC = O(n-1 + m-1) + O(N×m)

DP

f(0,1)

{

up = f(-1,1) → 0

left = (0,0) → 1

return 0+1 → 1

upper

left

2 path

TC = (2^(m×n))

SC → O(path length)

(m-1)+(n-1)

(0,0) (0,1) (0,2) (0,3)

(1,3)

(2,3)

(3,3)

ed

=

D

deletion

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Tabulation :-

```
dp[m][m];
if( dp[0][0] = 1

for( i=0 → m-1)
{   for(j=0 → n-1)
    {
    }
}
```

Memoization ⟷ Tabulation

1. Declare base case
2. Express various states in for loop
3. copy the recurance & write.

```
if( i==0 && j==0 )
    dp[0][0] = 1
else
{
}
if(i>0)  up = dp[i-1][j];
if(j>0)  left = dp[i][j-1];

dp[i][j] = up + left;
}
}

print(dp[i][j]);
```

TC = O(N×M)
SC = O(N×M) + recursion

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→ If there is a previous row & previous column, we can space optimised it.

```
dp[i][j] = dp[i-1][j] + dp[i][j-1];
```

best way to optimised

```
dp[n] ⟶ 0
for(i=0 ⟶ m-1)
{  for(j=0 ⟶ n-1)
   {  if(i==0 && j==0)
         dp[i][j] = 1;
      else {
         dp[i][j] = dp[i-1][j] + dp[i][j-1]
      }
   }
}
```

Code

```
int f(int i, int j, vector<vector<int>> &dp)
{
   if(i==0 && j==0)
      return 1;
   if(i<0 || j<0) return 0;
   if(dp[i][j] != -1)
      return dp[i][j];
   int up = f(i-1, j, dp);
   int left = f(i, j-1, dp);
   return dp[i][j] = up + left;
}
```

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```
int uniquepath (int m, int n)
{
 int dp[m][n];

 for(int i=0; i<m; i++)
 {
  for(int j=0; j<n; j++)
  {
   if(i==0 && j==0)
    dp[i][j] = 1;
   else
   {
    int up=0;
    int left=0;
    if(i>0) up
    up = dp[i-1][j];
    left = dp[i][j-1];

    dp[i][j] = up+left;
   }
  }
 }
 return dp[n-1][j-1];
}
```

TLE

```
int uniquePaths(int m, int n)
{
    vector<int> prev(n,0);
    for(int i=0; i<m; i++)
    {
        vector<int> cur(n,0);
        for(int j=0; j<n; j++)
        {
            if(i==0 && j==0)
                cur[j]=1;
            else
            {
                int up=0;
                int left=0;
                if(i>0)
                    up=prev[j];
                if(j>0)
                    left=cur[j-1];
                dp[i][j]=up+left;
            }
        }
        prev=cur;
    }
    return prev[n-1];
}
```

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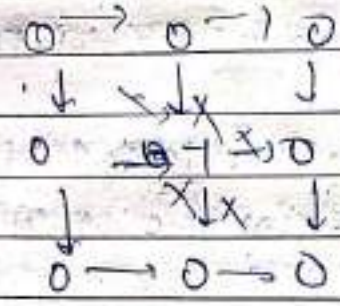
Unique paths 2 DP on Grid with Maze obstacle




```
int f(i,j)
{
if(i>=0 && j>=0 a[i][j]==-1)
    return 0;

if(i==0 && j==0)
    return 1;

if(i<0 || j<0)
    return 0;

if(dp[i][j]!=-1) return dp[i][j];
up= f(i-1,j);
left = f(i, j-1);

return up+left;
return dp[i][j] = up+left;
}
```

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Tabulation :-

```
for(i=0 → m-1)
{
    for(j=0 → n-1)
    {
        if( i==0 && j==0)
            dp[i][j] = 1;
        else
    if(i>0)   up = dp[i-1][j];
    if(j>0)   left = dp[i][j-1]

            return up+left;

    }
```

Code recursion + Memoization

```
int mod = (int)(1e9+7);
int f(int i, int j, vector<vector<int>> &mat, vector<vector<int>> &dp)
{
    if(i>=0 && j>0 && mat[i][j]==-1) return 0;
    if(i==0 && j==0) return 1;
    if(i<0 || j<0) return 0;
    if(dp[i][j]!=-1) return dp[i][j];
    int up = f(i-1, j, mat, dp);
    int left = f(i, j-1, mat, dp);

    return (up + left)%mod;
return dp[i][j] = (up+left)%mod;
}
int mazeObstacles(int n, int m, vector<vector<int>> &mat)
{
    vector<vector<int>> dp(n, vector<int>(m,-1));
    return f(n-1, mat, dp);
}
```

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Tabulation :

```
int mod = (int)(1e9 +7);
int dp[n][m];
for(int i=0; i<n; i++)
{ for(int j=0; j<m; j++)
  { if(mat[i][i] == -1) dp[i][j] = 0;
  else if(i==0 && j==0) dp[i][j] = 1;
  else { int up=0, left=0;
         if(i>0) up = up[i-1][j];
         if(j>0) left = dp[i][j-1];
         dp[i][j] = (up+left) % mod;

  }
}
return dp[n-1][m-1];
}
```

Space optimization

```
int mod =(int)(1e9 +7);

int mazeObstacle(int n, int m, vector<vector<int>> &mat)
{
   Vector<int> prev (m,0);
   for(int i=0; i<n; i++)
   { vector<int> cur(m,0);
     for(int j=0; j<m; j++)
     { if(mat[i][j] == -1)
           curr[j] = 0;
       else if(i==0 && j==0) curr[j] = 1;
       else { int up=0, left=0;
              if(i>0) up = prev[j];
              if(j>0) left = cur[j-1];
```

```
cur[j] = (up + left) % mod;
}
}
prev = cur;
}
return prev[n-1];
}
```

MINIMUM PATH SUM IN GRID

	0	1	2
0	5	9	6
1	11	5	2

2 x 3 ↓ →

(0,0) → (0,2)

5 + 11 + 5 + 2 = 23 ↓↓

5 + 9 + 5 + 2 = 21 → min

Why not follow greedy

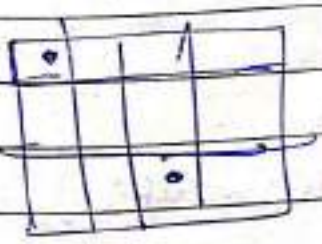
10	8	2
10	5	100
1	1	2

greedy = 10 + 8 + 2 + 100 + 2

Min = 10 + 10 + 1 + 1 + 2 ✓

Try out all paths

↳ (Recursion) ↳ Tell me the path which gives min cost



1.) index (i,j)

2.) Explore all Paths

3.) Take the min/~~max~~ path

dp[i,j]

(0,0) → (i,j)

f(n-1, m-1) → (0,0) → (n-1, m-1)

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f(i,j)
{
if(i==0 && j==0)
return a[0][0];

if(i<0 || j<0)
return INT_MAX;

(2) if(dp[i][j] != -1) return dp[i][j];

up = a[i][j] + f(i-1, j);
left = a[i][j] + f(i, j-1);

(1) dp[i][j] =
return min(up, left);

}

(3) Recursion → Memorization
overlapping
sub problem

(4) dp[n][m] → -1

Code

int f(int i, int j, vector<vector<int>> &grid,
vector<vector<int>> &dp)
{
if(i==0 && j==0)
return grid[i][j];
if(i<0 || j<0) return 1e9;
if(dp[i][j] != -1) return dp[i][j];
int up = grid[i][j] + f(i-1, j, grid, dp);
int left = grid[i][j] + f(i, j-1, grid, dp);

```
return dp[i][j] = min(left, up);
}
int minSumPath(vector<vector<int>> &grid)
{
    int n = grid.size();
    int m = grid[0].size();

vector<vector<int>> dp(n, vector<int>(m,-1));

    return f(n-1, m-1, grid, dp);
}
```

Tabulation :-

```
int minSumPath(vector<vector<int>> &grid)
{
    int n=grid.size();
    int m = grid[0].size();

vector<vector<int>> dp(n, vector<int>(m,0));
    for(int i=0; i<n; i++)
    {
        for(int j=0; j<m; j++)
        {   if(i==0 && j==0)
            dp[i][j] = grid[i][j];
        else {  int up=grid[i][j];
            if(i>0) up += dp[i-1][j];
            else up += 1e9;
```

```
int left = grid[i][j];
    if(j>0) left += dp[i][j-1];
    else left += 1e9;

    dp[i][j] = min(left, up);
  }
 }
}
```

Space optimisation

```
vector<int> cur(m,0);
for(int j=0; j<m; j++)
{
    if(i==0 && j==0)
     cur[j] = grid[i][j];
    else{
        int up = grid[i][j];
        if(i>0)  up += prev[j];
        else    up += 1e9;
        int left = grid[i][j];
        if(j>0) left += cur[j-1];
        else left += 1e9;
        cur[j] = min(left,up);
    }
}
prev=cur;
}
return prev[n-1];
}
```

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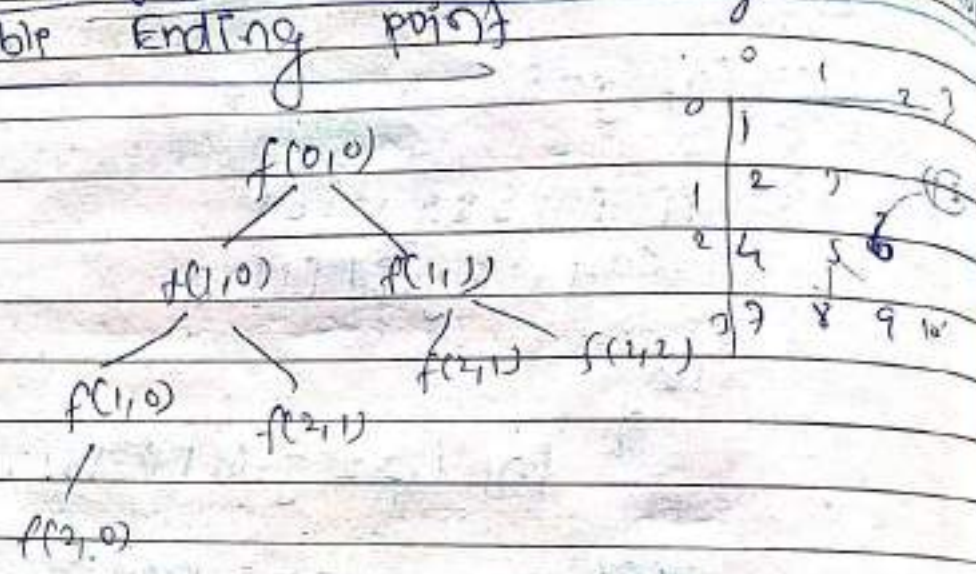
Triangle fixed starting point and Variable Ending point



int f(int i, int j, vector<vector<int>> & triangle, int n,
vector<vector<int>> dp)

TLE

{ if(i == n-1)
return triangle[n-1][j];
}

if dp[i][j] != -1)
{
return dp[i][j];

int d = triangle[i][j] + f(i+1, j, triangle, n, dp);
int dg = triangle[i][j] + f(i+1, j+1, triangle, n, dp);

return dp[i][j] = min(d, dg);
}

int minimumTriangle(vector<vector<int>> &triangle, int n){
vector<vector<int>> dp(n, vector<int>(n, -1));
return f(0, 0, triangle, n);
}

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To avoid TLE → Tabulation

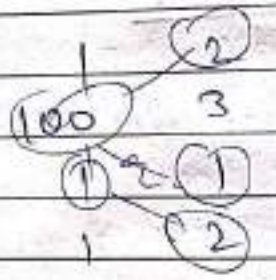
```
int minsumtriangle( vector<vector<int>> &triangle, int n)
{ vector<vector<int>> dp(n, vector<int>(n,0));
  for(int j=0; j<n; j++)
  {
    dp[n-1][j] = triangle[n-1][j];
  }
  for(int i=n-2; i>=0; i--)
  {
    for(int j=i; j>=0; j++)
    {
      int d = tri[i][j] + dp[i+1][j];
      int dg = triangle[i][j] + dp[i+1][j+1];
      dp[i][j] = min(d,dg);
    }
  }
  return dp[0][0];
}
```

→ Space optimization :

```
int Minsumpath(vector<vector<int>> &triangle, int n)
{ vector<int> front(n,0), cur(n,0);
  for(int j=0; j<n; j++)
  { front[j] = triangle[n-1][j]; }
  for(int i=n-2; i>=0; i--)
  { for(int j=i; j>=0; j--)
    { int d = triangle[i][j] + front[j];
      int dg = triangle[i][j] + front[j+1];
      cur[j] = min(d,dg); }
  } both loop ends
  front = cur;
  return front[0];
}
```

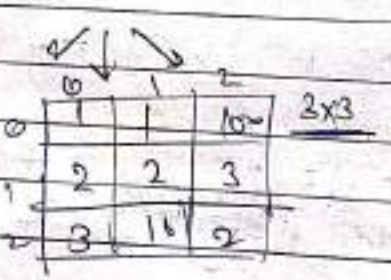
D-12

MINIMUM/MAXIMUM FALLING PATH SUM :-



Greedy → fails (because of uniformity)
Greedy will pass only when elements are uniform

We can not apply greedy ~~where~~ if uniformity is not there.



f(2,0)

f(1,-1) f(1,0) f(1,1)

f(0,-1) f(0,0) f(0,1)

f(1,0)

{

ld = 2 + f(0,-1)

u = 2 + f(0,0)

rd = 2 + f(0,1)

-ve

}

Recursion code

```
int f(int i, int j, vector<vector<int>> & matrix, vector<vector
        <int>> &dp)
{   if( j<0 || j>=matrix[0].size() )
        return -1e9;
    if(i==0)
        return matrix[0][j];

    if(dp[i][j] != -1) return dp[i][j];

    int u = matrix[i][j] + f(i-1, j, matrix, dp);
    int ld = matrix[i][j] + f(i-1, j-1, matrix, dp);
    int rd = matrix[i][j] + f(i-1, j+1), matrix, dp);

    return dp[i][j] = max(u, max(ld, rd));
}

int getmax( vector<vector<int>> &matrix )
{   int n = matrix.size();
    int m = matrix[0].size();

    vector<vector<int>> dp(n, vector<int>(m, -1));
    int maxi = -1e9;
    for(int j=0; j<m; j++)
    {
        maxi = max( maxi, f(n-1, j, matrix, dp));
    }
    return maxi;
}
```

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Tabulation :-

```
int getmax(vector<vector<int>> &matrix)
{
    int n = matrix.size();
    int m = matrix[0].size();

    vector<vector<int>> dp(n, vector<int>(m, 0));

    for(int j=0; j<m; j++)
    {
        dp[0][j] = matrix[0][j];
    }

    for(int i=1; i<n; i++)
    {
        for(int j=0; j<m; j++)
        {
            int u = matrix[i][j] + dp[i-1][j];
            int ld = matrix[i][j];
            if(j-1>=0)
                ld += dp[i-1][j-1];
            else
                ld += -1e8;
            int rd = matrix[i][j];
            if(j+1<m)
                rd += dp[i-1][j+1];
            else rd += -1e8;
            dp[i][j] = max(u, max(ld, rd));
        }
    }
```

```
int maxi = -1e8;
for(int j=0; j<m;j++)
{
    maxi = max( maxi, dp[n-1][j]);
}
return maxi;
}
```

prev[j]

Space optimization code =

int curr[m], prev[m];

Just replace dp[i][j] → prev[j]

& make → prev = curr;

and → return prev.prev

in this case

~~return max= max(, max);~~

return max(maxi, prev);

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Cherry pickup

Rules

1. Express everything in terms (i_1, j_1) & (i_2, j_2)
2. Explore all the paths ↙↓↘
3. Max sum.

Fixed starting point

Variable ending point

↓

$(0,0)$ $(0, m-1)$ (At any index in the last row)

(All paths by Alice) + (All paths by Bob)

Recursion Recursion

take Together

$f(0,0, 0, m-1)$

Alice Bob

Base case

i) Out of Boundary

ii) last row case (Destination)

Boundary can always write first.

```
f(i1, j1, i2, j2)
{
if( j1<0 || j1>=m || j2<0 || j2>=m)
    return -1e8;
```

↙↓↘ means they reach at same time.

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```
if (i == n-1)
{
    if (j1 == j2)
        return a[i][j1];
    else
        return a[i][j1] + a[i][j2];
}
```

Base condition

Explore all the paths

(i+1, j-1) (i+1, j) (i+1, j+1)

In one step move Alice & Bob both

```
maxi = 0;
for(dj1 -> -1 -> +1)
{
    for dj2 -> -1 -> +1
    {
        maxi = max( maxi, f(i+1, j1+dj1, j2+dj2) );
    }
}
if(j1 == j2)
    return a[i][j1] + maxi;
else
    return a[i][j1] + a[i][j2] + maxi;
```

TC → $3^n \times 3^n$ = exponential

SC = O(N) ASC

Auxiliary stack space

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Recursion code

```
int f( int i, int j1, int j2, int r, int c,
 vector<vector<int>> &grid, vector<vector<vector<int>>> &dp)

{
   if(j1<0 || j2<0 || j1>=c || j2>=c)
   {
     return -1e8;
   }
   if(i==r-1)
   {
     if(j1==j2)
        return grid[i][j1];
     else return grid[i][j1] + grid[i][j2];
   }
  if(dp[i][j1][j2]!=-1)
   { return dp[i][j1][j2];
   }
int maxi=-1e8;
 for(int dj1=-1; dj2<=+1; dj1++)
   {
    for( int dj2=-1; dj<=+1 ; dj2++)
       {
         int value=0;
    if( j1==j2 ) value=grid[i][j1];
    else
        value = grid[i][j1] + grid[j][j2];
   value += f(i+1, j1+dj1, j2+dj2, r, c, grid, dp);
        maxi = max(maxi, value);
   }
  }
 return dp[i][j1][j2] = maxi;
}
```

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```
int maximumchocolate( int r, int c, vector<vector<int>>
                      & grid)
{
  Vector<vector<vector<int>>> dp[3, vector<vector<int>>
        (c, vector<int>(c,-1)));

  return f(0,0, c-1, r, c, grid, dp);
}
```

Tabulation

```
int MAXchocolate( int n, int m, vector<vector<int>> & grid)
{
vector<vector<vector<int>>> dp(n, vector<vector<int>>(m, vector<int>
(m,0)));
   for(int j1 = 0; j1<m; j1++)
    }
      for(int j2 =0; j2<m; j2++)
      {
        if(j1==j2)
        dp[n-1][j1][j1] = grid[n-1][j1];
      else
      dp[n-1][j1][j2] = grid[n-1][j1] + grid[n-1][j2];
    }
  }
for( int i= n-2; i>=0; i--)
  { for(int j1=0; j1<m; j1++)
    { for( int j2=0;j2<m; j2++)
      { int maxi = -1e8;
        for(int dj1=-1; dj1<=+1; dj1++)
        {
          for(int dj2=-1; dj2<=+1; dj2++)
```

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```
{ int value = 0;
  if(j1==j2) value = grid[i][j1];
  else
    value = grid[i][j1] + grid[i][j2];

  if(j1+dj1>=0 && j1+dj1<m
  && j2+dj2>=0 && j2+dj2<m)
      value += -1e8;
  maxi = max(maxi, value);
  }
 }
 dp[i][j1][j2] = maxi;
 }
 }
 }
 return dp[0][0][m-1];
}
```

```
int dp[r][c][c];
// 1d → two variable
2D → 1D DP
3D → 2D DP
```

Space Optimization :-

```
int maximumChocolate(int n, int m, vector<vector
    <int>> &grid)
{
  vector<vector<int>> front(m, vector<int>(m,0));
  vector<vector<int>> cur(m, vector<int>(m,0));

  for(int j1=0; j1<m; j1++)
  { for (int j2=0; j2<m; j2++)
    { if(j1==j2) front[j1][j2]=grid[n-1][j1];
```

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```
else front[j1][j2] = grid[n-1][j1] + grid[n-1][j2];
    }
  }
  for(int i = n-2 ; i>=0 ; i--)
  {
    for(int j1=0; j1<m; j1++)
    {
      for(int j2=0; j2<m; j2++)
      {
        int maxi = -1e8;
        for(int dj1 = -1 ; dj1 <= +1 ; dj1++)
        {
          for(int dj2 = -1; dj2 <= +1 ; dj2++)
          {
            int value = 0;
            if(j1 == j2) value = grid[i][j1];
            else value = grid[i][j1] + grid[i][j2];
            if(j1+dj1 >= 0 && j1+dj1 < m &&
               j2+dj2 >= 0 && j2+dj2 < m)
              value += front[j1+dj1][j2+dj2];
            else
              value += -1e8;
            maxi = max(maxi, value);
          }
        }
        curr[j1][j2] = maxi;
      }
    }
    front = curr;
  }
  return front[0][m-1];
}
```

DP-14

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SUBSET SUM EQUA TO K

DP on subsequence / subsets

(contigous) Non contigous

[1, 3, 2] → [1, 2], [3, 2]

→ Generate all subsequence & select if any of them gives a sum of K

Power set ← Bit manipulation

Recursion

① Express (ind, target)

② Explore possibilities of the index.

f(n-1, target)

Tabulation

① Base case

② Form the Nested loops ⇒

ind → [1 → n-1]

target → [1 → target]

arr → [2, 3, 1, 1] Target = 5

f(3, 4)
index target

f(2, 5) f(2, 4)

f(1, 3)

f(1, 2)

f(0)

f(0, 2)

f(-1, 0) f(-1, 2)

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Recursion

```
bool f(int ind, int target, vector<int>& arr,
       vector<vector<int>>& dp)
{
  if(target == 0) return true; if(ind == 0) return (arr[0] == target);
  if(dp[ind][target] != -1)
      return dp[ind][target];

  bool notTake = f(ind-1, target, arr, dp);

  bool take = false;

  if(arr[ind] <= target)
      take = f(ind-1, target-arr[ind], arr, dp);

  return dp[ind][target] = take | notTake;
}

bool subsetSumK(int n, int k, vector<int>& arr)
{
  vector<vector<int>> dp(n, vector<int>(k+1, -1));

  return f(n-1, k, arr, dp);
}
```

$TC = O(2^n)$

$SC = O(N)$

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Tabulation:-

```
bool subsetSumToK( int n, int k, vector<int> &arr){

vector<vector<int>> dp(n, vector<int> (k+1, 0));

for(int i=0; i<n; i++)
    dp[i][0] = true;

dp[0][arr[0]] = true;

for(int ind = 1; ind<n; ind++)
{
  for(int target = 1; target<=k; target++)
    {
      bool notTake = dp[ind-1][target];
      bool take = false;

      if(arr[ind] <= target)
        take = dp[ind-1][target-arr[ind]];

      dp[ind][target] = take | notTake;
    }
}
  return dp[n-1][k];
}
```

TC - O(N*K)
SC = O(N*K)

Whenever you see dp[ind-1] means we can optimise space.

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```
bool subsetSumToK( int n, int k, vector<int>&arr)
{
    vector<bool> prev(k+1,0), cur(k+1,0);
    prev[0] = cur[0] = true;
    prev[arr[0]] = true;
    for(int ind=1; ind<n; ind++)
    {
        for(int target=1; target<=k; target++)
        {
            bool notTake = prev[target];
            bool take = false;

            if(arr[ind] <= target)
                take = prev[target-arr[ind]];

            cur[target] = take | notTake;
        }
        prev = cur;
    }
    return prev[k];
}
```

O(1)

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DP-15

Partion Equal Subset Sum

{2, 3, 3, 3, 4, 5}

{2, 3, 5} Subset 1

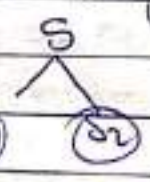
{3, 3, 4} Subset 2

Subset with n sum

Exactly 2 subset

$S_1 = S_2$

Entire array sum = S



$S_1 = S_2 = S/2$

S = total sum

if S = odd → (X) not possible

if S = even

then look for target sum $\frac{S}{2}$

If one subset you got ($\frac{S}{2}$) then you already got 2nd subset.

NOW this question converts into

arr → [ ]

subset sum = $S/2$

where

S = total sum of given array.

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Space optimization code :-

```
bool subsetSumTOK( int n, int K, vector<int> &arr)
{
   vector<bool> prev(K+1, 0), cur(K+1, 0);
    prev[0] = cur[0] = true;
    prev[arr[0]] = true;

    for( int ind = 1; ind < n; ind++)
       {  for( int target = 1; target <= K; target++)
          {
             bool notTake = prev[target];
             bool take = false;

             if(arr[ind] <= target)
               take = prev[target - arr[ind]];
                cur[target] = take | notTake;

          } prev = cur;
      }
      return prev[K];
   }

bool Canpartition( vector<int> &arr, int n)
{  int totsum = 0;
   for(int i=0; i<n; i++)
       { totsum += arr[i];
       }
   if(totsum % 2 != 0 ) return false;
   int target = totsum/2;
   return subsetTOK( n, target, arr);
}
```

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DP-16

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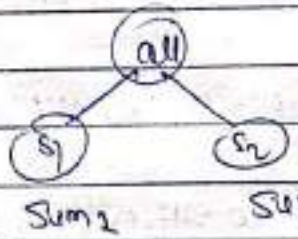
Partition A set into Two Subset with Minimum Absolute sum

[1, 2, 3, 9]

[1, 2, 3, 4]

{1,2} {3,4} = |7-3| = 4
{1,3} {2,4} = |6-4| = 2
{1,4} {2,3} = |5-5| = 0



abs|Sum1 - Sum2| is minimum

Subset sum → Tabulation ✓

If we check for a target = K
We can derive if every possible target between (1 & K) is ✓/✗

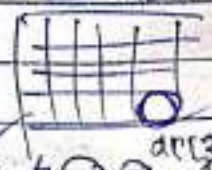
[3, 2, 7]

S_1 min 0
S_1 max 12

all possible S_1

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12
✓ ✗ ✓ ✓ ✗ ✓ ✗ ✓ ✗ ✓ ✓ ✗ ✓

dp[3][12+1] Ans



int dp[n][totsum+1]

Subset sum code

```
int minSubsetSumDifference( vector<int> &arr, int n)
{
    int totSum = 0;
    for(int i=0; i<n; i++)
        totSum += arr[i];

    int k = totSum;
vector<vector<bool>> dp(n, vector<bool>(k+1, 0));
    for(int i=0; i<n; i++) dp[i][0] = true;
    if(arr[0] <= k) dp[0][arr[0]] = true;
    for(int ind = 1; ind<n; ind++)
        { for(int target = 1; target<=k; target++)
            { bool notTake = dp[ind-1][target];
              bool take = false;
            if(arr[ind] <= target)
              take = dp[ind-1][target - arr[ind]];
              dp[ind][target] = take | notTake;
            }
        }

    int mini = 1e9
    for(int s1 = 0; s1 <= totSum/2; s1++)
        { if (dp[n-1][s1] == true)
            { mini = min(mini, abs((totSum
                -s1) - s1));
            }
    return mini;
}
```

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DP-17

Number of subsets.

{1, 2, 2, 3}

{1, 2} target = 3

{1, 2}

{3}

no. of subset = 3

if cnt of count

in Base case Always
return 1 if true
return 0 if false

Recurrance

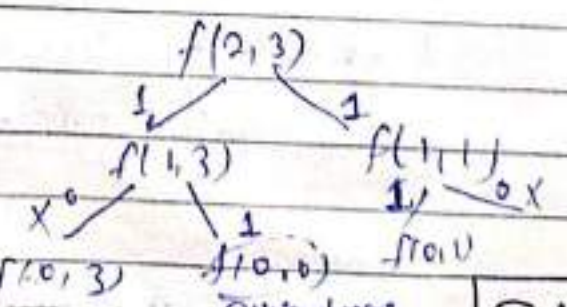
1.) Express in terms of (ind, target)
2.) Explore all the possibilities
3.) Sum up all the possibilities & return.

0	1	2	3
1	2	2	3

F(n-1, target) (ind+1)

```
not_pick = f(ind-1, target);
pick = 0;

if (a[ind] <= s) {
    pick = f(ind-1, target-a[ind]);
}
return pick + notpick;
}
```



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no. of subset = 2 {1,2}, {3}

Tabulation

1. Base case
2. Look at the changing parameter & write nested loop
3. Copy the recursion

```
int dp[N][sum+1];
for(i=0 → n-1) dp[i][0] = 1
if(a[0]<=S)  dp[0][a[0]] = 1

for(i=1 → n-1)
    for(s=0 → sum)
    {
        copy the recursion;
    }
```

TC - O(N x sum)
SP - O(N x sum)

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Recursion code :-

```
int f(int ind, int sum, vector<int> &num, vector<vector<int>> &dp)
{
    if(num==0) return 1;
    if(dp[ind][num] != -1) return dp[ind][num];

    int notTake = f(ind-1, sum, num, dp);
    int take = 0;

    if(num[ind] <= sum)
        take = f(ind-1, sum-num[ind], num, dp);

    return dp[ind][num] = notTake + take;
}

int findWays(vector<int> &num, int tar)
{
    int n = num.size();

    vector<vector<int>> dp(n, vector<int>(tar+1, -1));

    return f(n-1, tar, num, dp);
}
```

Tabulation :

```
int findways(vector<int> &num, int tar)
{
    int n = num.size();
    vector<vector<int>> dp(n, vector<int>(tar+1, 0));
    for(int i=0; i<n; i++)
        { dp[i][0] = 1;
    if (num[0] <= tar)
        dp[0][num[0]] = 1;
    for(int ind = 1; ind < n; ind++)
        {
        for(int sum = 0; sum <= tar; sum++)
            {
            int nottake = dp[ind-1][sum];
            int take = 0;
            if(num[ind] <= sum)
            {
                take = dp[ind-1][sum - num[ind]];
            }
            dp[ind][sum] = nottake + take;
        }
    }
    return dp[n-1][tar];
}
```

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space optimization code :-

```
int findways( vector<int> &num, int tar)
{
    int n = num.size();
    vector<int> prev(tar+1,0), cur(tar+1);
    prev[0] = curr[0] = 1;
    if(num[0] <= tar)
        prev[num[0]] = 1;

for(int ind = 1; ind < n; ind++)
    {
        for(int sum=0; sum<=tar; sum++)
        {
            int notTake = prev[sum];
            int take = 0;
            if(num[ind] <= sum)
                take = prev[sum - num[ind]];

            cur[sum] = notTake + take;
        }
        prev = cur;
    }
    return prev[tar];
}
```

Count partitions with given sum difference:

arr → {0, 0, 1} Sum = 1

Ans = 4

{0, 1}

{0, 1}

{0, 0, 1}

{1}

no. of zeros → (2)

pow (2, n) x arr 1

power set $2^2 \times 1$

$4 \times 1 = 4$ subset

$S_1 >= S_2$

& $S_1 - S_2 = D$

$S_1 = totsum - S_2$

$D = totsum - S_2 - S_2$

$D = totsum - 2S_2$

$S_2 = \frac{totsum - D}{2}$

Count of Subset = $\left(\frac{Totsum - D}{2}\right)$

DP-17 → modified target

Totsum − D >= 0

totsum − D has to be even

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Recursion + Memoization code

```
int mod = (int)(1e9+7);
int f(int ind, int sum, vector<int> &num, vector<vector
    <int>> &dp)
{
 if(ind==0)
  { if(sum==0 && num[0])
     return 2;
    if(sum==0 || sum==num[0])
     return 1;
    return 0;
  }
  if(dp[ind][sum]!=-1)
     return dp[ind][sum];

  int notTake = f(ind-1, sum, num, dp);
  int take = 0;
  if(num[ind] <= sum)
  take = f(ind-1, sum-num[ind], num, dp);

  return dp[ind][sum] = (notTake + take)%mod;
 }

 return dp[n-1][tar];
}
int countPartitions(int n, int d, vector<int> &arr)
{
 int totSum = 0;
 for(auto &it: arr) totSum += it;
 if(totSum-d<0 || (totSum-d)%2) return false;
 return findWays(arr, (totSum-d)/2);
}
```

```
int findways(vector<int> &num, int tar)
{
    int n = num.size();
    vector<vector<int>> dp(n, vector<int>(tar+1, -1));
    return f(n-1, tar, num, dp);
}

int countpartitions(int n, int d, vector<int> &arr)
{
    int totsum = 0;
    for(auto &it: arr)
        totsum += it;
    if((totsum - d < 0) || (totsum - d) % 2)
        return false;
    return findways(arr, (totsum - d)/2);
}
```

Tabulation :-

```
int findWays(vector<int> &num, int tar){
    int n = num.size();
    vector<vector<int>> dp[n, vector<int>(tar+1, 0)];
    if(num[0] == 0)
        dp[0][0] = 2;
    else dp
        dp[0][0] = 1;
    if
```

Space Optimization Code

```
int findways(vector<int> &num, int
{
    int n= num.size();
    vector<int> prev(tar+1,0),cur(tar+1,0);
    if(num[0] ==0)
        prev[0]=2;
    else prev[0]=1;

    if(num[0] != 0 && num[0] <= tar)
        prev[num[0]] =1;
    for(int ind =1; ind<n; ind++)
    {
        for(int sum=0; sum<= tar; sum++)
        {
            int notTake = prev[sum];
            int take=0;
            if(num[ind]<=sum)
            take = prev[sum-num[ind]];
            cur[sum] = (notTake + take)%mod;
        }
        prev = curr;
    }
    return prev[tar];
}
int
```

DP-19

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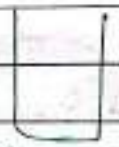
0-1 Knapsack

$n = 3$

wt. →	3	4	5
val →	30	50	60



3 + 5 = 8 weight
val = 60 + 30
= 90 ✓

4 + 3 = 7
val = 50 + 30 = 80 ✗

① Choose 1st costly products.
Greedy Not works here

wt ⇒	3	2	5
val ⇒	30	40	60

wt. 2 + 3 = 5
val. 40 + 30 = 70

Law of uniformity was not there that why case Greedy fails.

Try out all combination
↓
Take the best by value combination

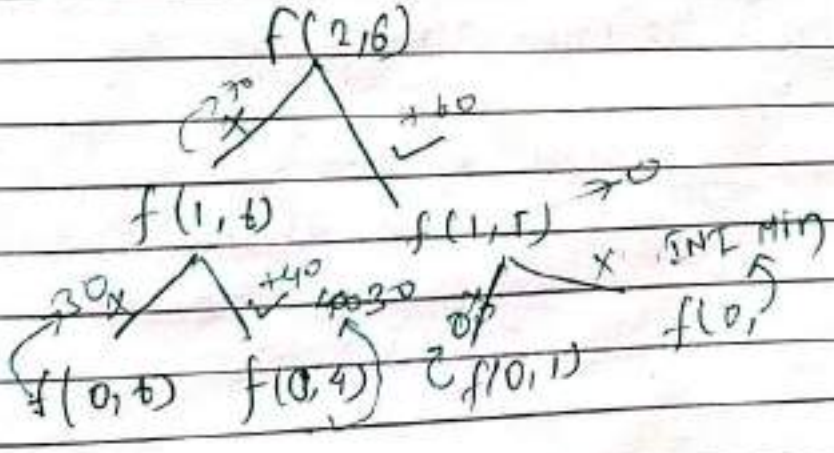
Recursion → (i) Express in terms of (index, wt)
(ii) Explore all possibilities
(iii) looking for max/min

3	4	5
30	50	60

f(ind, w)
{ if(ind == 0)
{ if(wt[0] <= w) return val[0]; }
else {return 0;}
Not take = 0 + f(ind-1, w)
take = INT_MIN;
if(wt[ind] <= w)
take = val[ind] + f(n-1, w-wt[ind]);
return max(not take, take);
}

wt → 6
val → 100
0

wt → 3 2 5
val 30 40 60
wt = 6



Memoization.

ind, w
dp[ind+1][w+1];

TC = O(n x w)
SP = O(N x W) + O(N)

Tabulation

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Tabulation :-

1. Base case.
2. Write the changing parameters ind, w
3. Loops
   no. of loops = no. of changing parameter.

int dp[ind][w];

Base case :-

for(int dp[ind][w] = 0;

for(i = val[0] → w) dp[0][i] = val[0];

Recursion + Memoization

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```
int f( int ind, int w, vector<int> &wt, vector<int> &val,
       vector<vector<int>> &dp){
{
  if(ind == 0)
  {
    if( wt[0]<= w)
      { return val[0];
      }
    return 0;
  }

  if(dp [ind][w] != -1)
    {
      return dp [ind][w];
    }
  int Not_take = 0 + f(ind-1, w, wt, val, dp);

  int take =INT_MIN;
  if( wt[ind] <= w)
    {
    take = val[ind] + f(ind-1, w-wt[ind], wt,
                        val, dp);
    }
  return dp[ind][w] = max( take, Not_take);

int knapSack( vector<int> weight, vector<int> value,
  int n, int w)
  {
    vector< vector<int>> dp(n, vector<int>(w+1, -1));
    return f(n-1, w, weight, value, dp);
  }
```

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Tabulation:

```
int knapsack(vector<int> wt, vector<int> val,
int n, int weight)
{

  vector<vector<int>> dp(n, vector<int>(weight+1, 0));

  for(int w=wt[0]; w<=weight; w++)
  {
    dp[0][w] = val[0];
  }

  for(int ind=1; ind<n; ind++)
  {
    for(int w=0; w<=weight; w++)
    {
      int not_take = 0 + dp[ind-1][w];
      int take = INT_MIN;
      if(wt[ind] <= w)
      {
        take = val[ind] +
          dp[ind-1][w-wt[ind]];
      }

      dp[ind][w] = max(take, not_take);
    }
  }

  return dp[n-1][weight];
}
```

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Space optimization

```
int knapsack( vector<int> wt, vector<int> val, int n, int
weight)
{
    vector<int> prev(weigh+1,0), cur(weight+1,0);
    vector<vector<int>> dp(n, vector<int> (maxweight+1,0));

    for(int w = wt[0]; w<=
for(int ind=1; ind<n; ind++)
    {
    for(int w=0; w<=Max weight ; w++)
        {
        int not_take = 0 + prev[w];

        int take = INT_MIN;
        if( wt[ind] <= w)
        {
        take = val[ind] + prev[w-wt[ind]];
        }

        cur[w] = max(take, not_take);
        }
        prev = cur;
    }
    return prev[weight];
}
```

Single array space optimization

```
int knapsack(vector<int> wt, vector<int> val,
  int n, int weight)
{
   vector<int> prev(weight+1, 0);

  for (int w=wt[0]; w<=weight; w++)
      {
        prev[w] = val[0];
      }
  for(int ind=1; ind<n; ind++)
      {
        for(int w = weight; w>=0; w--)
        {
          int not_take = 0 + prev[w];
          int take = INT_MIN;

          if(wt[ind] <= w)
          {
            take = val[ind] + prev[w-wt[ind]];
          }
          prev[w] = max(take, not take);
        }
      }
    return prev[weight];
}
```

Minimum coins | DP on subsequence

arr[] → {1, 2, 3} target = 7

coins may be {3, 3, 1} ans = 3 minimum coins

X (2, 2, 2, 1) ans = 4

Greedy

{1, 2, 3} target = 7

7/3 = (2) {3, 3, 1}

remain = 1

1/2 = 0 3 coins

1/1 = (1)

greedy work for this case but might failed in other case

lets take an example where greedy fails

{9, 6, 5, 1} target = 11

11/9 = (1)

2 remain

2/6 = 0 ans should be 2+1 = 3 but

2/5 = 0 if we choose 6 & 5 ans = 2

2/1 = (2) so greedy fails here.

Approach :- Trying out all combos to form target

↓

take the combo that has min coins

Rules :-

1. Express the recurrence in terms of index.
2. Express or do all stuffs.
3. Min. of all.

Recurrence

@Aashish Kumar Nayak

Whenever there is statement like infinite supply, multiple case's it will stand at same index it will not go back.

T.C. -> ??
O(2^T)

S.C. -> O(N) - recursion

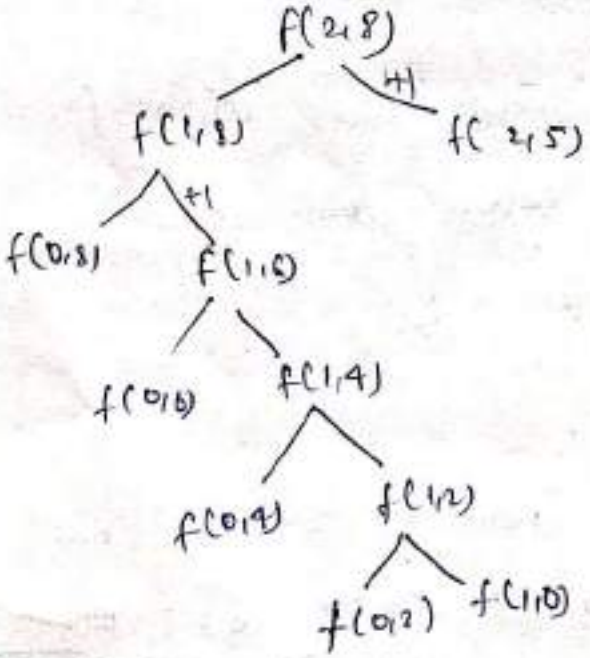
```
f(ind, T)
{
if(ind == 0)
  if(T % arr[0] == 0) return T/arr[0];
  else return 1e7;

int not_take = 0 + f(ind-1, T);
int take = INT_MAX;
if(coins[ind] <= T)
   take = 1 + f(ind, T-coins[ind]);
return min(take, NOTtake);
}
```

X [6] target = 7
arr[0]

✓ [7] target = 7

[4] target = 12

Base case



T.C. -> >> O(2^T)
S.C. -> >> O(N)

Seem Exponential.

Now we will Memoize it,

changing parameters -> ind, target

dp[ind][target] = ?

T.C. : O(N*T)
S.C = O(N*T) + O(T) Auxiliary Stack

@Aashish Kumar Nayak

Tabulation

⇒ Any memoization can be converted into tabulation.

1. Base case
2. find target and write it in opposite fashion.
3. Copy the tarbulance.

for any target

```
for(T=0 → target)
    if(T-1.a[0] == 0)
        dp[0][T] = T/a[0];
    else
        dp[0][T] = INT_MAX;
```

Code :-

```
int f(ind, int T, vector<int> &nums, vector<vector<int>> &dp)
{ if(ind == 0)
  { if(T%nums[0] == 0)
      return T/nums[0];
    return 1e9;
  } if(dp[ind][T] != -1) return dp[ind][T];
int not-take = 0 + f(ind-1, T, nums, dp);
int take = INT_MAX;
if(nums[ind] <= T)
  { take = 1 + f(ind, T-nums[ind], nums, dp);
  }
return min(take, notTake);
  ↳ dp[ind][T] = min(take, not-take);
}
int minimumElements(vector<int> &num, int target)
{ int n = num.size(); vector<vector<int>> dp(n, vector<int>(target+1, -1));
int ans = f(n-1, target, nums, dp);
if(ans >= 1e9) return -1;
return ans;
}
```

@Aashish Kumar Nayak

Tabulation code :-

```
int minimumElements(vector<int>& nums, int target)
{
    int n = nums.size();
    vector<vector<int>> dp(n, vector<int>(target+1, 0));
    for(int T=0; T<=target; T++)
    {
        if(T % nums[0] == 0)
            dp[0][T] = T/nums[0];
        else dp[0][T] = 1e9;
    }
    for(int ind=1; ind<n; ind++)
    {
        for(int T=0; T<=target; T++)
        {
            int notTake = 0 + dp[ind-1][T];
            int take = INT_MAX;
            if(nums[ind] <= T)
            {
                take = 1 + dp[ind][T-nums[ind]];
            }
            dp[ind][T] = min(take, notTake);
        }
    }
    int ans = dp[n-1][target];
    if(ans >= 1e9)
        return -1;
    return ans;
}
```

Space Optimization : (using prev1 & prev2)

```
int minimumElements(vector<int> &nums, int target)
{
    int n = nums.size();
    vector<int> prev(target+1, 0), cur(target+1, 0);
    for(int T=0; T<=target; T++)
    {
        if(T%nums[0] == 0)
            prev[T] = T/nums[0];
        else
            prev[T] = 1e9;
    }

    for(int ind=1; ind<n; ind++)
    {
        for(int T=0; T<=target; T++)
        {
            int notTake = 0 + prev[T];
            int take = INT_MAX;
            if(nums[ind] <= T)
            {
                take = 1 + cur[T-nums[ind]];
            }
            cur[T] = min(take, notTake);
        }
        prev = cur;
    }
    int ans = prev[target];
    if(ans >= 1e9)
        return -1;
    return ans;
}
```

@Aashish Kumar Nayak

Target Sum problem | DP on subsequence

arr[] = {1, 2, 3, 1} Assign +, -

target = 3

+ + - -
1 2 3 1 = 7
1 — 3 — 0 — 1

count all the ways to get target.

- + + -
1 2 3 1 = 3
-1 — 1 — 4 — 3

2 ways

+ - + +
1 2 3 4 = 3

$S_1 - S_2 = D$

sum partition

- + + -
[1 2 3 1]

9 + 2 -1 -1
S_1 S_2
5 - 2 = 3
S_1 S_2

same problem like in DP-19

Code:

```
int targetSum(int n, int target, vector<int>& arr)
{ return countPartition(n, target, arr);
}
int countPartition(int n, int d, vector<int>& arr)
{ int totSum = 0;
  for(auto &it : arr)
      totSum += it;
  if(totSum - d < 0) || (totSum - d) % 2
      return false;
  return findWaysArr, (totSum-d)/2;
}
```

```
int findways(vector<int> &num, int tar)
{
    int n = num.size();
    vector<int> prev(tar+1,0), cur(tar+1,0);
    if(num[0] == 0)
        prev[0] = 2;
    else
        prev[0] = 1;
    if(num[0] != 0 && num[0] <= tar)
        prev[num[0]] = 1;
    for(int sum = 0; sum <= tar; sum++)
    {
        int notTake = prev[sum];
        int take = 0;
        if(num[ind] <= sum)
            take = prev[sum - num[ind]];
        cur[sum] = (notTake + take);
    }
    prev = cur;
}
return prev[tar];
}
```

DP-22 Ways to make coin change

N=3 | 1 | 2 | 3 | target=4

any element can be used any no. of times.

{1, 1, 1, 1} = 4
{1, 1, 2} = 4
{2, 2} = 4
{1, 3} = 4

4 differs

figure out total no. of ways → Try out all the ways
↓
Recursion

```
f( )
{
  Base case — if destination is meet: 1 or 0 else
  f(___
  f(___
  return (__ + __);
}
```

Recurrence :-

1. Express → (ind, T)
2. Explore all possibility → not-take / take } fn all the DP subsequence problem.
3. Sum all possibilities and return.

@ | 1 | 2 | 3 |
0 1 2

f(2, 4) — ind, T

Till index 2, in how many ways can you form 4

@Aashish Kumar Nayak

```
f(ind, T)
{
  if(ind == 0)
  {
    if(T % arr[0] == 0)
      return 1;
    else return 0;
  }
  notTake = f(ind-1, T);
  take = 0
  if(arr[ind] <= T)
    take = f(ind, T-arr[ind]);
  return notTake + take;
}
```

[4] arr[0] — T = 4

[2] — T = 4 ✗

[2] — T = 4 ✓

T = arr[0] pair can be formed.

Memoization will solve this issue →

T.C. = O(N × T)

S.C = O(N × T) + O(target) (auxiliary space ASS)

T.C. ≫ $O(2^n)$ exponential

S.C. ≫ O(N)

standing on the same index.

we are standing at same index.

use Tabulation to optimise ASS

Steps → Tabulation — Bottom-up approach

1. Base case
2. ind (1 — n-1)
   T → (0, T)
3. Copy the recurrence.

```
for(T → 0 → target)
dp[0][T] = (T % arr[0] == 0)
```

Code:

```
long countWaysToMakeChange(int *denomination, int n, int value)
{
  vector<vector<long>> dp(n, vector<long>(value+1, -1));
  return f(n-1, value, denomination, dp);
}
```

```
long f(int ind, int T, int *a, vector<long> &dp){
    if(ind == 0)
        return (T % a[0] == 0); // 1 or 0
    if(dp[ind][T] != -1) return dp[ind][T];
    long notTake = f(ind-1, T, a, dp);
    long take = 0;
        if(a[ind] <= T)
            take = f(ind, T-a[ind], a, dp);
    return dp[ind][T] = take + notTake;
}
```

Tabulation :-

```
long countWaysToMakeChange(int *denomination, int n, int value){
    vector<vector<long>> dp(n, vector<long>(value+1, 0));
    for(int T=0; T<=value; T++){
        dp[0][T] = (T % a[0] == 0);
    }
    for(int ind=1; ind<n; ind++){
        for(int T=0; T<=value; T++){
            long notTake = dp[ind-1][T];
            long take = 0;
            if(a[ind] <= T){
                take = dp[ind][T-a[ind]];
            }
            dp[ind][T] = take + notTake;
        }
    }
    return dp[n-1][value];
}
```

Space optimization

Whenever you see like ind-1, ind. then you can always optimise by using couple of stuffs.

Code :-

```
long countWaysToMake( int *a, int n, int value)
{
    vector<vector<long>> dp(n, vector<long>(value+1, 0));
    vector<vector<
    vector<long> prev(value+1, 0), cur(value+1, 0);
    for(int T=0; T<=value; T++)
    {
        prev[T] = T%a[0]==0;
    }
    for(int ind=1; ind<n; ind++)
    {
        for(int T=0; T<=value; T++)
        {
            long notTake = prev[T];
            long take=0;
            if(a[ind] <= T)
            {
                take = cur[T-a[ind]];
            }
            cur[T] = take + notTake;
        }
        prev = cur;
    }
    return prev[value];
}
```

DP-23 # unbounded knapsack

prerequisite
DP-19 — 0/1 knapsack
| In this there was only one occurrence.

wt → {2, 4, 6}
val → {5, 11, 13}

But here infinite supply of every item. we can pick any one item many times.

Ex ÷
13
wt → 6 11 wt → 4

wt → 6
10 wt → 2 x 2
+ 5x2 = 23

wt → 4 x 2 + wt 2
11x2 + 5 = 27 maximum possible value.

(wt → 2) x 5 = (10) wt
5x5 = (25) val

f(ind, w)
{ if(ind == 0)
return [w/wt[0] x val[0]];

Base case we write for 0

val 10
wt 5
arr[]

w=8 ✓ 2
if w=2 x 0

int (w/wt[0]) x val[0]

notTake = 0 + f(ind-1, w);
take = INT_MIN;
if(arrwt[ind] <= w)
take = val[ind] + f(ind, w - wt[ind]);

return max(take, notTake);

Memoize T.C. — exponential

Tabulation 1. Base case >> O(W)
2. Changing parameter ind
3. copy the recurrance.

@Ashish Kumar Nayak

w can be anything 0, 1, 2, — — — Wt. bag

for(w → 0 to wt.bag)

{ dp[0][w] = $\left(\frac{w}{wt[0]}\right)$ × val[0] ·

}

Code ÷ * Memoized

int f(int ind, int W, vector<int> &val, vector<int> &wt, vector<vector<int>> &dp)

{ int if(ind == 0)

{ return ((int)(W/wt[0])) * val[0];

} if(dp[ind][W] != -1) return dp[ind][W];

int notTake = 0 + f(ind-1, W, val, wt, dp)

int take = 0;

if(wt[ind] <= W)

{ take = val[ind] + f(ind, W - wt[ind], val, wt, dp)

}

return dp[ind][W] = max(take, notTake);

}

int unboundedKnapsack (int n, int w, vector<int> &val,

vector<int> &wt)

{ vector<vector<int>> dp(n, vector<int> (w+1, -1));

return f(n-1, w, val, wt, dp);

}

@Aashish kumar Nayak

Tabulation :-

```
int unboundedknapsack(int n, int w, vector<int> &val,
                      vector<int> &wt)
{
    vector<vector<int>> dp(n, vector<int>(w+1, 0));
    for(int w=0; w<=w; w++)
    {
        dp[0][w] = ((int)(w/wt[0])) * val[0];
    }
    for(int ind=1; ind<n; ind++)
    {
        for(int w=0; w<=w; w++)
        {
            int nottake = 0 + dp[ind-1][w];
            int take = 0;
            if(wt[ind] <= w)
            {
                take = val[ind] + dp[ind][w - wt[ind]];
            }
            dp[ind][w] = max(take, nottake);
        }
    }
    return dp[n-1][w];
}
```

Space optimization

int-1, int → 3D space optimized is possible.

```
int unboundedKnapsack( int n, int w, vector<int> & val,
                       vector<int> & wt)
{
    vector<int> prev(w+1, 0), cur(w+1, 0);
    for(int w=0; w<=w; w++)
        { prev[w] = ((int)(w/wt[0])) * val[0];
    }
    for (int ind=1; ind<n; ind++)
        { for (int w=0; w<=w; w++)
            { int notTake = 0 + prev[w];
              int take = 0;
              if(wt[ind] <= w)
              { take = val[ind] + cur[w-wt[ind]]
                                  //prev[w-wt[ind]];
              }
//prev[w]= max(take,notTake);
              cur[w] = max(take, notTake);
        }
R       // prev = cur; //Remove this line
    }
    return prev[w];
}
```

prev: 3 4 5 6 7 1 2

not take = 3 from cur

take = than we rewrite.

1D array optimization

DP-24 Rod cutting problem | DP on subsequences

N = 5 → Rod length

price array[] = | 2 | 5 | 7 | 8 | 10 |
length: 1 2 3 4 5

For length 1 price = 2 — For length 2 price = 5

We need to cut the rod into pieces

price: 2 2 2 2 2
length: 1 1 1 1 1 = 10

2 | 3 | 3 = 12 ↑↑ maximum

5 | 3 = 12 maximum

We need to print maximum cost.

1 2 3 4 5

Collect rod length to make N

↓

Maximize the price — Similar to knapsack

| 2 | 5 | 7 | 8 | 10 |
0 1 2 3 4

means 4 length — means 5 length

Try to pick lengths and sums them up to make the given N.

↳ In all possible ways.

↳ Recurrence

1. Express in terms of index
2. Explore all possibilities ← not Take / Take
3. Maximize the price or possibilities.

2	5	7	8	10
0	1	2	3	4

f(4, N)
↓
Till index 4, what is the minimum price you obtain

n-1 ⟶ f0

✓ [6] ——
Price [0]
rod length = 1

N = 12
↓
looking to make it.

1, 1, 1, 1 . . . 12 times)
Pricing price = N x price[0];

```
f(ind, N)
{
    if(ind == 0)
        return N x price[0];
    }
int notTake = 0 + f(ind - 1, N);
int take = INT_MIN;
int rod_length = ind + 1;
if(rodLength <= N)
    take = price[ind] + f(ind, N - rod_length);
return max(take, notTake);
}
```

Recursion ⟶ Memoization

always greater than 2^n i.e. expon
T.C. - O(Exponential)
S.C. → O(Target)

ind, N
T.C. [N] x [N+1]
T.C. O(N x N)
S.C. → O(N x N) + O(Target)
ASS

Mem **Tabulation**:

1. Base case
2. Changing parameter — ind, N
3. Copy recurrence

Base case.
for (N → 0 to n)
    dp[0][N] = N x price[0];

for ind → 1 → n-1
    N → 0 → N
(Copy the recurrence)

@Aashish Kumar Nayak

```
int f(int ind, int N, vector<int> & price, vector<vector<int>> &dp)
{ if(ind == 0)
    { return N*price[0];
    }
 if(dp[ind][N] != -1) return dp[ind][N];
int notTake = 0 + f(ind-1, N, price, dp);
int take = INT_MIN;
int rodLength = ind+1;
if( rodLength <= N)
  {
    take = price[ind] + f(ind, N-rodLength, price, dp);
  }
return dp[ind][N] = max(take, notTake);
}
int cutRod (vector<int> &price, int n)
{
 vector<vector<int>> dp(n, vector<int>(n+1, -1));
 return f(n-1, n, price, dp);
```

Tabulation :-

```
int cutRod ( vector<int> &price, int n)
{ vector<vector<int>> dp(n, vector<int>(n+1, 0));
   for ( int N=0; N<=n; N++)
     { dp[0][N] = N* price[0];
     }
   for(int ind = 1; ind<n; ind++)
       { for( int N=0; N<=n; N++)
           {
```

```
int notTake = 0 + dp[ind-1][N];
    int take = INT_MIN;
    int rodLength = ind+1;
    if( rodLength <= N)
      { take = price[ind] + dp[ind][N-rodLength];
      }
    dp[ind][N] = max(take, notTake);
    }
  }
  return dp[n-1][n];
}
```

Space optimization :-

```
int cutRod( vector<int> &price, int n)
{ vector<vector<int>> dp(n, vector<int>(n+1,0));
  vector<int> prev(n+1,0), cur(n+1,0);
  for(int N=0; N<=n; N++)
  { prev[N] = N*price[0];
  }
  for(int ind = 1; ind<n; ind++)
    { for(int N=0; N<=n; N++)
      {
        int notTake = 0 + prev[N];
        int take = INT_MIN;
        int rodLength = ind+1;
        if(rodLength <= N)
          { take = price[ind] + cur[N-rodLength];
          }
        cur[N] = max(take, notTake);
      }
    prev = cur;
  }
  return prev[n];
}
```

// prev[N-rodLength]

//prev[N-1]

we can do 1D array optimization like previous one.

@Ashish Kumar Nayak

DP on Strings

[DP on strings] → Comparison of strings
→ replace / edit

longest common [subsequences]

s_1 = "adebc" s_2 = "dcadb"

DP-25 **Longest Common Subsequence :-**

abc → [a, b, c, ab, bc, ac, abc, " "]

power set / ~~Rec~~ or recursion ← we can use these ~~two~~ method to print ~~all~~ the subset.

s_1 = "a d e b c" —— 2^5 subsequences

a, d, e, b, c, ad, ae, ac, de, db, dc, adc, —— adb

s_2 = "dcadb" —— 2^5 subsequences

d, c, a, d, b, dc, da, da, db, — — — adb

we have to find all the subsequences and take out the longest one.

Brute force

Generate all subsequences

take one & compare ↓ so on

T.C. :- O(exponential)

New approach

⇒ Generate all subsequences & compare on way.

↓

Recursion method, we will not generate instead we will use parameter to generate them.

We will write some recurrence which give the answer through the way.

@Aaibish Kumar Nayak

Rules to write the recurrence

(i) Express everything in terms of index

(ii) Explore possibilities on that index.

(iii) take the best among them.

Let's take an example

a c d | c e d

if match: ↑ ind1, ↑ ind2

1 + f(ind1-1, ind2-1);

d == d

a c | c e

ind2, ind1

a c | c e ← if not match

a c | c e ← +1 → f(ind1-1, ind2-1) ← if goes -ve

0 + max[f(ind1-1, ind2), f(ind1, ind2-1)];

f(ind1, ind2) -ve means end of the strings.

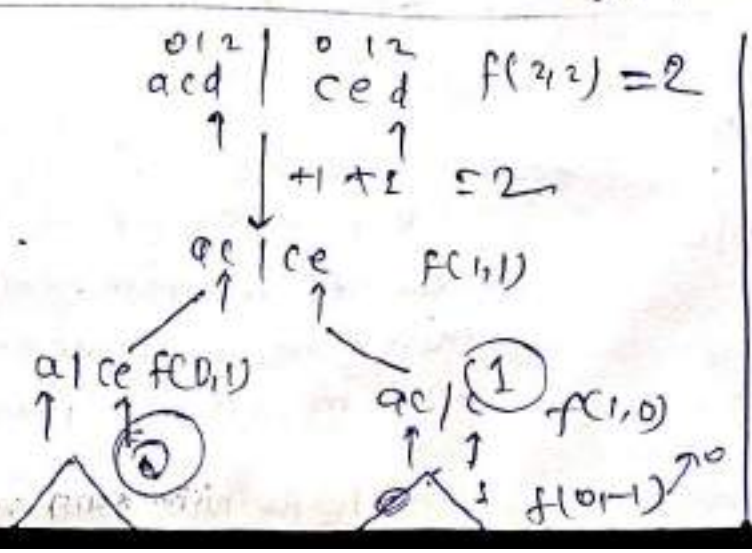
```
{
// Base case
if(ind1 < 0 || ind2 < 0)
    return 0;
```

Match

```
if(s1[ind1] == s2[ind2])
    return 1 + f(ind1-1, ind2-1);
```

No match

```
return 0 + max[f(ind1-1, ind2), f(ind1, ind2-1)];
```



Recursion $2^n \times 2^m$ ≈ exponential

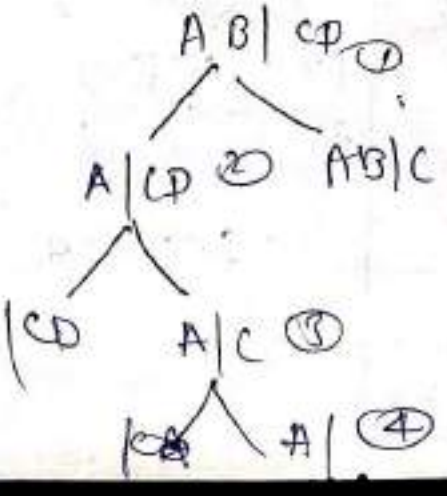
↓ optimised if there are overlapping sorae problem then apply memoization.

Code :-

```
int f(int i, int j, string &s, string &t, vector<vector<int>> &dp)
{ if(i<0 || j<0)
    return 0;                    i==0  j==0
  if(dp[i][j] != -1) return dp[i][j];
  if(s[i] == t[j])
    return 1 + f(i-1, j-1, s, t);
                 i-1  j-1
  return dp[i][j] = max(f(i-1, j, s, t, dp), f(i, j-1, s, t, dp));
}

int lcs(string s, string t)
{
  int n = s.size();
  int m = t.size();
  vector<vector<int>> dp(n, vector<int>(m, -1));
                         n+1          m+1
  return f(n-1, m-1, s, t, dp);
}
```

T.C. - $O(N \times M)$

S.C - $O(N \times M) + O(N+M)$ (A.S.)



$2+2=4$ alternate deletion

if we do alternate deletion then there can be n deletion from s_1 and m deletion from s_2.

Tabulation :-

1. Copy the Base Case
2. changing the parameter
3. Copy the recurrance.

Shifting of indexes.

-1 0 1 2 3 ... n-1

```
if (i==0 || j==0) return 0
    return dp[0][j]

for(j=0 → m-1)  dp[0][j]=0
for(i=0 → n-1)  dp[i][0]=0
```

Code :-

```
int lcs(string s, string t)
{
  int n=s.size();
  int m= t.size();
  vector<vector<int>> dp(n+1, vector<int>(m+1, -1));
for(int j=0; j<=m; j++) dp[0][j]=0;
for(int i=0; i<=n; i++) dp[i][0]=0;
 for(int i=1; i<=n; i++)
    { for(int j=1; j<=m; j++)
      {
        if( s[i-1] == t[j-1]
            dp[i][j] = 1+dp[i-1][j-1];
        else
            dp[i][j] = max(dp[i-1][j], dp[i][j-1]);
      }
    }
    return dp[n][m];
}
```

Space optimization

```
int lcs(string s, string t)
{
    int n = s.size();
    int m = t.size();
    vector<int> prev(m+1, 0), cur(m+1, 0);
    for(int j=0; j<=m; j++)
    {
        prev[j] = 0;
    }
    for(int i=1; i<=n; i++)
    {
        for(int j=1; j<=m; j++)
        {
            if(s[i-1] == t[j-1])
                cur[j] = 1 + prev[j-1];
            else
                cur[j] = max(prev[j], cur[j-1]);
        }
        prev = cur;
        cur = prev;
    }
    return prev[m];
}
```

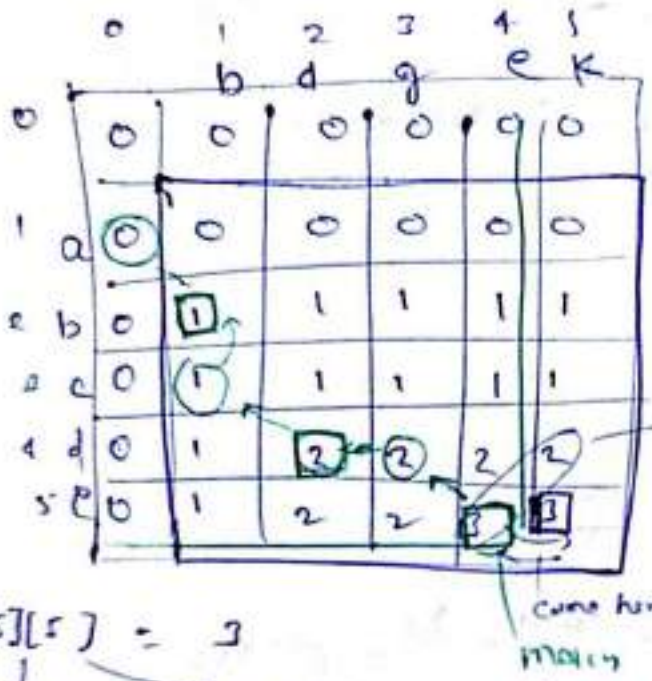
DP-26 print longest common subsequence :-

s_1 = "abcde" s_2 : "bdgek"

len(lcs) = 3 // in previous problem

soln = bde

dp



dp[5][5] = 3

abcde bdgek

if not match then go to max of diagonal [i-1][j] & [i][j-1]

dp[4][2] → 2

bd

dp[i][j] = 1 + dp[i-1][j-1]

only when s1[i-1] == s2[j-1]

dp[i][j] : max(dp[i-1][j], dp[i][j-1])

Code

i = m, j = m

while(i > 0 && j > 0)

{ if(s1[i-1] == s2[j-1])

{

else if(dp[i][j] = max(dp[i-1][j]

}

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~~else~~ i = n, j = m; len = dp[n][m], string s = ""; index = len-1;

while(i > 0 && j > 0)

for(i = 0 → len) s += "g";

$	$	$

{ if(s1[i-1] == s2[j-1])

{ s[index] = s1[i-1];

index--;

i--; j--;

}

else if(dp[i-1][j] > dp[i][j-1])

{

}

else

}

}

Code :-

```
void lcs(string s, string t)
{ int n = s.size();
  int m = t.size();
  vector<vector<int>> dp(n+1, vector<int>(m+1, 0));
for(int j=0; j<=m; j++)
    dp[0][j] = 0;
for(int i=0; i<=n; i++)
    dp[i][0] = 0;
for(int i=1; i<=n; i++)
  { for(int j=1; j<=m; j++)
    { if(s[i-1] == t[j-1])
        dp[i][j] = 1 + dp[i-1][j-1];
      else
        dp[i][j] = max(dp[i-1][j], dp[i][j-1]);
    }
  }
```

```
int len = dp[n][m];
string ans = "";
for(int i=0; i<len; i++)
    ans += '$';
int index = len-1;
int i=n, j=m;
while(i>0 && j>0)
    {
    if(s[i-1] == t[j-1])
        {
        ans[index] = s[i-1];
        index--;
        i--; j--;
        }
    else if(dp[i-1][j] > dp[i][j-1])
        {
        i--;
        }
    else
        {
        j--;
        }
    }
cout<<ans;
}
int main()
{ string s1 = "abcde";
  string s2 = "bdgek";
  lCS(s1,s2);
}
```

Tc. O(n+m)

DP-27 Longest Common Substring :-

S_1 = "abcjklp" S_2 = "acjlqp"

Should be consecutive

Let's
- matching
  $dp[i][j] = 1 + dp[i-1][j-1]$
- not matching
  $dp[i][j] = max(dp[i-1][j], dp[j][j-1])$

This will not work on substring

Let's take an example

acd | axd

This is consecutive so we don't dependent on the previous guys.

Breaking the consecutive rule

+1

ac | ax

a | ax

ac | a

+1

| ax

a | d

c | a

ac |

no need to do a break directly put 0

$dp[i][j] = 0$

abcd

abzd

		a	b	z	d
	0	1	2	3	4
0	0	0	0	0	0
a 1	0	1	0	0	0
b 2	0	0	2	0	0
c 3	0	0	0	0	0
d 4	0	0	0	0	1

The maximum value in the entire matrix.

Code

```
int lcs(string &s, string &t)
{
    int n = s.size();
    int m = t.size();
    vector<vector<int>> dp(n+1, vector<int>(m+1, 0));
    for(int j = 0; j<=m; j++)
        dp[0][j] = 0;
    for (int i=0; i<=n; i++)
        dp[i][0] = 0;
    int ans = 0;
    for (int i = 1; i<=n; i++)
    {
        for (int j = 1; j<=m; j++)
        {
            if( s[i-1] == t[j-1] )
            {
                dp[i][j] = 1 + dp[i-1][j-1];
                ans = max(ans, dp[i][j]);
            }
            else
                dp[i][j] = 0;
        }
    }
    return ans;
}
```

Space optimization :-

Code :-

```
int lcs(string &s, string &t)
{
    int n = s.size();
    int m = t.size();
    vector<int> prev(m+1, 0), cur(m+1, 0);
    int ans = 0;
    for(int i = 1; i <= n; i++)
    {
        for(int j = 1; j <= m; j++)
        {
            if(s[i-1] == t[j-1])
            {
                cur[j] = 1 + prev[j-1];
                ans = max(ans, cur[j]);
            }
            else
                cur[j] = 0;
        }
        prev = cur;
    }
    return ans;
}
```

DP-28 **longest palindromic subsequences :**

S = " bbbab"

→ ab
→ bb
→ bbb
→ bbbb — longest one
→ bab

Ans = 4 (length)

Brute force

Generate all the subsequences → Check for palindrom & pick up the longest.

S_1 = " babcbcab"

S_2 = " bacbcbab" — reverse of S_1 (order)

Just find lcs (S_1, rev(S_1))

Code :

```
int lcs (string s, string t)
{ int n = s.size();
  int m = t.size();
vector<vector<int>> dp(n+1, vector<int>(m+1, 0));
for(int j = 0; j<=m; j++) dp[0][j] = 0;
for(int i=0; i<=n; i++) dp[i][0] = 0;
for(int i = 1; i<=n; i++)
   { for(int j =1; j<=m; j++)
       if(s[i-1] == t[j-1])
          dp[i][j] = 1+dp[i-1][j-1];
       else
          dp[i][j] = max(dp[i-1][j], dp[i][j-1]
       }
     return dp[n][m];
   }
int longestPalindromeSubsequence (string s)
{ string t = s;
  reverse(t.begin(), t.end());
  return lcs(s,t);
}
```

DP-29 Minimum insertions to make a string palindrome:

prerequisite → LCS
→ LPS

S = "abcaa"

abcaa aa cba
reverse

max operation = len(S)

S + rev(S) — becomes palindrome.

aa bc baa ⟶ palindrom and min operation = 2

Hard Approach:-

keep the palindrome portion intact.

S = abcaa - a lot of palindrom so I will try to keep longest palindrome intact.

a b c a a — aaa.

a bc a ba a

we will rotate b not rotate then we will place copy of that in right part.

Code in my mind

n — longest palindromic subsequence

size of string.

Code :-

```
int minInsertion(string &str)
{ return str.size() - longestPalindromeSubsequence(str);
}
int longestPalindromeSubsequence(string s)
{string t = s;
 reverse(t.begin(), t.end());
 return lcs(s, t);
}

int lcs(string s, string t)
{ int n = s.size();
  int m = t.size();
vector<vector<int>> dp(n+1, vector<int>(m+1, 0));
for(int j=0; j<=m; j++) dp[0][j] = 0;
for(int i=0; i<=n; i++) dp[i][0] = 0;
for(int i=1; i<=n; i++)
    {for(int j=1; j<=m; j++)
      {if(s[i-1] == t[j-1])
         dp[i][j] = 1 + dp[i-1][j-1];
       else
         dp[i][j] = max(dp[i-1][j], dp[i][j-1]);
      }
    }
  return dp[n][m];
}
```

DP-80

Minimum insertion/deletion to convert string A to string B

str1 → "abcd" str2 = "anc"

Delete everything from str1 and insert each character from str2 to str1.

it takes str1.size() + str2.size operations.

Intution

what can I not touch

a b c d ⟶ anc
× ×

lets delete b & d

a c — 2 deletion

now insert n

anc — 1 insertion

3 operations

longest common subsequence.

deletions = n - len(lcs)
insertions = m - len(lcs)

n+m - 2*len(lcs) → Ans

Code :

```
int canYouMake(string &str, string &ptr)
{ return str.size() + ptr.size() - 2*lcs(str,ptr);
}
int lcs(string s, string t)
{ int n = s.size();
  int m = t.size();
```

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```
Vector<vector<int>> dp(n+1, vector<int>(m+1,0));
    for(int j=0; j<=m; j++)
        dp[0][j] = 0;
    for(int i=0; i<=n; i++)
        dp[i][0] = 0;

    for(int i=1; i<=n; i++)
    {  for(int j=1; j<=m; j++)
        {  if(s[i-1] == t[j-1])
            dp[i][j] = 1+dp[i-1][j-1];
           else
            dp[i][j] = max(dp[i-1][j], dp[i][j-1]);
        }
    }
    return dp[n][m];
}
```

DP-31 Shortest Common Supersequence :-

S_1 = "brute" S_2 = "groot"

"brutegroot" ← len = 10 this can be called common supersequence.

both string should be present in the ans string. and order has to be same.

~~"broot"~~ "bgruoote" — len = 8

S_1 = "bleed" S_2 = "blue"

"bleued"

fix our lengths

print #

Common guys should be taken only once

common guy in two string means LCS(—,—)

↓

it tells me longest common guys

"bgruoote"

(S1) (S2)

blue (#) goo

(n+m) − len(LCS)

print string

$s_1[i-1] == s[j-1]$

$dp[i][j] = 1 + dp$

we will create dp table.

LCS table

		0	1 (g)	2 (r)	3 (o)	4 (o)	5 (t)
	0	0	0	0	0	0	0
b	1	0	0	0	0	0	0
r	2	0	0	1	1	1	1
u	3	0	0	1	1	1	1
t	4	0	0	1	1	1	2
e	5	0	0	1	1	1	2

if not match max(i,j) = 1

common guys get added by 1

etouorbg

reverse ~~that~~ it

gbrouote → len(s)

```
i=n, j=m
while( i>0 && j>0)
  { if( s1[i-1] == s2[j-1])
    { ans += s1[i-1];
      i--, j--;
    }
  else if (dp[i-1] > dp[i][j-1])
  {
    ans += s1[i-1];
    i--;
  }
  else
    { ans += s2[j-1];
      j--;
    }
```

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i>0 | j>0 | @Aashish Kumar Nayak

String 1 has some length | s2 has some length

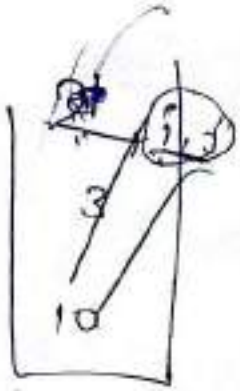
```
while (i>0) ans += s1[i-1]; i--
while (j>0) ans += s2[j-1]; j--
```

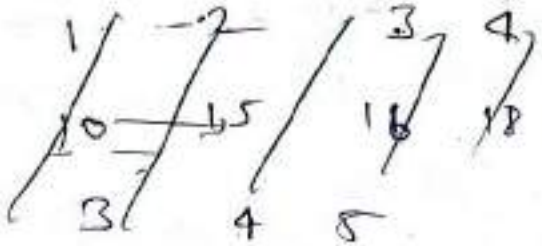
Code :-

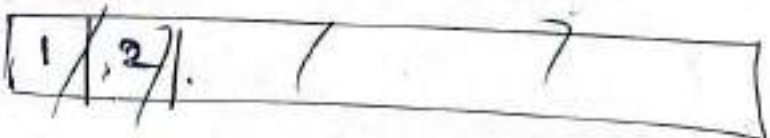
```
string shortestSuperSequence(string s, string t)
{   int n = s.size();
    int m = t.size();
    vector<vector<int>> dp(n+1, vector<int>(m+1, 0));
    for(int j=0; j<=m; j++)
        dp[0][j] = 0;
    for(int i=0; i<=n; i++
        dp[i][0] = 0;
    for(int i=1; i<=n; i++)
    {   for(int j=1; j<=m; j++)
        {   if(s[i-1] == t[j-1])
                dp[i][j] = 1 + dp[i-1][j-1];
            else
                dp[i][j] = max(dp[i-1][j], dp[i][j-1]);
        }
    }
    string ans = "";
    int i = n, j = m;
    while( i>0 && j>0)
    {   if(s[i-1] == t[j-1])
        {   ans += s[i-1]; i--, j--;
        }
        else if( dp[i-1][j] > dp[i][j-1])
```

```
{
    ans += S[i-1];
        i--;
}
else
{
    ans += t[j-1]
        j--;
}

while(i>0)
{
    ans += S[i-1];
    i--;
}
while ( j>0)
    { ans += t[j-1]
        j--;
}}
reverse(ans.begin(), ans.end());
return ans;
}
```







~~DP on~~

Distinct subsequences

S = "babgbag" s_2 = "bag"

occurrance of s_2 in s_1 in babgbag → 5 subs distinct ways

Different methods of comparing

Trying all the ways → Recursion

Count the no. of ways

```
f()
{ Base case ?' go

  f1()
  f2()

  return f1()+f2();
```

// Base case will return 1 or 0 ∴ count ways.

* How to write recurrence
  (i) Express everything in terms of i & j
  (ii) Explore all possibilities
  (iii) Count all ways return the summation of all possibilities
  (iv) Base case

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f(i,j)

S_1 = babgbag (i, n-1)

S_2 = bag (m-1)

i j

f(n-1, m-1) — start call

no. of distinct subsequences of $s_2[0...j]$ in $s_1[0...i]$

if(j<0) return 1;
if(i<0) return 0

if($s_1[i] == s_2[j]$) ~ if they are matching the string reduced to ba from bag

return (f(i-1, j-1) + f(i-1, j));

else

return f(i-1, j) // Reduced and look for someone else.

Space Complexity :-

Tc → exponential

Sc → O(N+m)

s_1 → 2^n, s_2 → 2^m

Optimise → overlapping subproblems

↓ Memoization

T.C. O(N×m)

S.C. O(N×m) + O(N+m)

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Code :-

```
int f(int i, int j, string s, string t, vector<vector<int>> &dp)
{ if(j<0)
    return 1;
  if(i<0)
    return 0;
  if(dp[i][j] != -1) return dp[i][j];
  if(s[i] == t[j])
    return f(i-1, j-1, s, t, dp) + f(i-1, j, s, t, dp);
  return dp[i][j] = f(i-1, j, s, t, dp);
}

int numDistinct(string s, string t)
{ int n = s.size(), int m = t.size();
  vector<vector<int>> dp(n, vector<int>(m, -1));
  return f(n-1, m-1, s, t, dp);
}
```

Tabulation

(i) declare the dp array of same size.

(ii) changing parameter in opposite fashion.

1-based indexing instead of 0-based indexing in order to avoid -1, negative

Base case $\boxed{j==0}$ → 1

no string s2 $i==0$ → 0

```
for(i = 0 → n)
    dp[i][0] = 1
```

Code :-

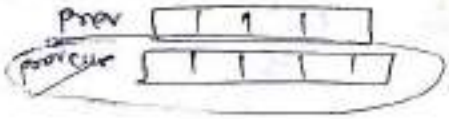
```
int numDistinct(string s, string t)
{ int n = s.size();
  int m = t.size();
  vector<vector<double>> dp(n+1, vector<double>(m+1, 0));
  for(int i=0 ; i<=n; i++)
      dp[i][0] = 1;
  for(int j=0; j<=m; j++)
      dp[0][j] = 0;
  for(int i=1; i<=n ; i++)
  { for(j=1 ; j<=m; j++)
    { if(s[i-1] == t[j-1])
      { dp[i][j] = dp[i-1][j-1] + dp[i-1][j];
      } else {
        dp[i][j] = dp[i-1][j];
      }
  }
}
return (int) dp[n][m];   // converted back
}
```

Space optimization :-

```
int numDistinct(string s, string t)
{ int n = s.size();
  int m = t.size();
  vector<double> prev(m+1,0), cur(m+1,0);
  prev[0] = cur[0] = 1;
  for(int i=1; i<=n; i++)
      { for( int j = 1; j<=m; j++)
          { if( s[i-1] == t[j-1])
              { cur[j] = prev[j-1] + prev[j];
              }
            else {
                  cur[j] = prev[j];
                 }
          }
        prev = cur;
      }
      return (int) prev[m];
}; }
```

single array optimization

```
i = 1 ⟶ n
j = 1 ⟶ m
if() → cur[j] = prev[j-1] + prev[j]
else → cur[j] = prev[j]
```




```
int numDistinct(string s, string t)
{ int n = s.size();
  int m = t.size();
  vector<double> dp(m+1,0);
  prev[0] = 1;
  for(int i=1; i<=n; i++)
  { for(int j=m; j>=1; j--)
      { if(s[i-1] == t[j-1])
          dp[j] = dp[j-1]
                  + dp[j];
      }
  }
  return (int) dp[m];
}
```

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DP-33. # Edit Distance

S_1 = "horse" S_2 = "ros"

1. Insert
2. Remove
3. Replace

The minimum no. of steps by which we can convert S_1 into S_2.

---

(1) Brute method: delete all characters from S_1 and then insert all characters from S_2 to S_1.

It will take $len(S_1) + len(S_2)$ steps

(2) method

replace

S_1 = "horse" S_2 = "ros"

replace h with r — rorse

remove r — rose

remove e — ros

It took 3 steps.

Example

S_1 = "intention" S_2 = "execution"

remove t — inention

replace i → e — enention

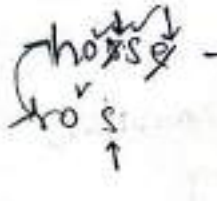
replace n → x — exention

insert u — execution

5 operations

Sol

String matching using recursion



→ If not matching

→ insert of the same character

→ delete & try finding somewhere else

→ replace & match.

Try all ways → Recursion

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How to write Recurrence?

1. Express in terms of (i,j).
2. Explore all paths of matching.
3. Return min(all paths).
4. Base case.

f(i,j)

horse (i)
ros (j)

f(n-1, m-1) → min operation or to convert.

s1[0---i] to s2[0---j]

f(i,j)
{
if(s1[i] == s2[j])
return 0 + f(i-1,j-1); // if they are matching shrink both string and we don't require any steps.
}

1 + f(i, j-1); // insert
// if not matching i can insert

1 + f(i-1, j); // delete

1 + f(i-1, j-1); // replace

s1 gets exhausted

return (j+1) steps

horse
ros

f(-1, j) → min operations to convert empty string to a s2[0---j]

"" "ro"

if(i<0) return j+1;

if(j<0) return i+1; // insert

s2 gets exhausted

return (i+1) operations

s1 = "horse"
0 1 2

s2 = "ros"
-1 0 1 2

f(2,-1) → min operations to convert "hor" → ""

3 operations

Recursion

TC → Exponential

SC → O(N+M)

overlapping subproblem → memoize

After memoization

T.C. → O(N*M)

S.C → O(N×M) + O(N+m)

```
int editDistance(string &str1, string &str2, vector<vector<int>> &dp)
{ if( i<0 ) return j+1; // if( i==0) return j+1;
  if(j<0) return i+1; // if( j==0) return i;
    if(dp[i][j] != -1) return dp[i][j];
  if (s1[i] == s2[j]) return dp[i][j] = f(i-1, j-1, s1, s2,
  // s1[i-1] == s2[j-1]
  return dp[i][j] = 1 + min( f(i-1,j,s1,s2,dp), min(f(i, j-1, s1, s2, dp), f(i-1, j-1, s1, s2, dp));
}

int editdistance(string str1, string str2)
{ int n = str1.size();
  int m = str2.size();
  vector<vector<int>> dp(n, vector<int> (m, -1));
  return f(n-1, m-1, str1, str2, dp);
}
```

Tabulation :-

1. Base case ——— in order to avoid -ve +1 to everywhere
2. changing parameters
3. copy the recurrence

Code : ~~int f(int i~~
try to write Base cases.

f(0,j) i==0 return j — Here j can be anything.

Means for(j = 0 → m) dp[0][j] = j; // Base case 1

j==0 return i

for(i=0→n) dp[i][0] = i;

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2. i = 1 — n

j = 1 — m

3. copy the recurance.

```
int editDistance(string s1, string s2)
{
    int n = s1.size();
    int m = s2.size();
    vector<vector<int>> dp(n+1, vector<int>(m+1, 0));
    for(int i = 0; i<=n; i++) dp[i][0] = i;
    for(int j = 0; j<=m; j++) dp[0][j] = j;
    for(int i = 1; i<=n; i++)
    {
        for(int j = 1; j<=m; j++)
        {
            if(s1[i-1] == s2[j-1])
                return dp[i][j] = dp[i-1][j-1];
            else
                dp[i][j] = 1 + min(dp[i-1][j], min(dp[i][j-1], dp[i-1][j-1]));
        }
    }
    return dp[n][m];
}
```

Note Space optimization

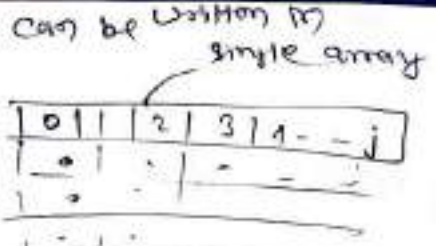
We can definitly optimise the space because there i-1, something like that is involved.

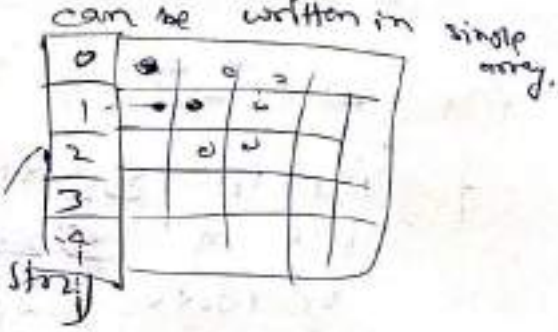
Date —

Base Cases:

dp[0][j] = j

dp[i][0] = i






```
int editDistance(string str1, string str2)
{
    int n = str1.size();
    int m = str2.size();
~~for(vector~~
    vector<int> prev(m+1, 0), cur(m+1, 0);
    for(int j=0; j<=m; j++)
        prev[j] = j;
    for(int i=1; i<=n; i++)
    {   cur[0] = i;
        for(int j=1; j<=m; j++)
        {   if(str1[i-1] == str2[j-1])
            ~~cur~~ cur[j] = prev[j-1];
            else
                cur[j] = 1+min( prev[j],
                                min(cur[j-1],
                                prev[j-1]));
        }
        prev = cur;
    }
    return prev[m];
}
```

DP-34 Wildcard Matching | DP on string (last problem)

s_1 = "?ay"
s_2 = "ray" } all three chars are matching → return true

? → matches with single char.
* → matches with sequence of length 0 or more.

s_1 = "a?b*?d"
s_2 = "acbxdefxd"
return true

s_1 = "a?b?d"
s_2 = "acbxdex"
return false

s_1 = "**abc?d"
s_2 = "abcxd"
return true

1st thing which comes in brain string matching → we can do using recurrance

Problem faced →
ab*?d
abdefxd
for this star we don't know how much of length of other string it will cover

so we have to try all possible ways → means recursion
len = 0, 1, 2, ...

so we will try string matching by recursion.

Rules to write Recurrance :-
1. Express (i, j)
2. Explore comparisons
3. out of all comparisons, if anyone can we return true f(n-1, m-1) - | @ashish kumar Nagar

s1[0...-1]
s2[0...6]

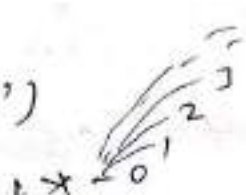
f(m-1, n-1) →
f(4, 6) → (T/F)

0	1	2	3	4
a	b	*	c	d

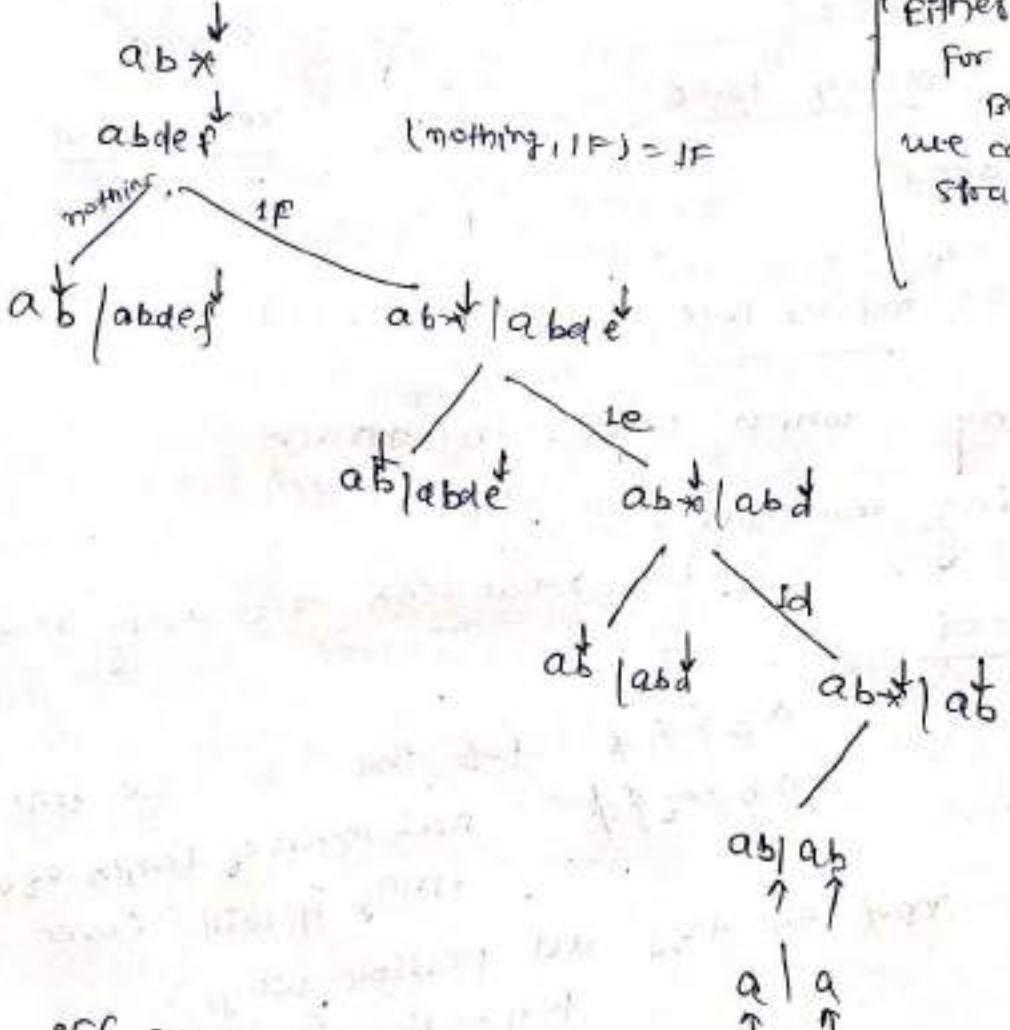
a	b	d	e	f	c	d
0	1	2	3	4	5	6

IF(s1[i] == s2[j] || s1[j] == '?')

return f(i-1, j-1)



Either we can write for loop
But we can do this in straight forward recursion



if(s1[i] == '*')

return f(i-1, j) or f(i, j-1)

return false;

Now analyze the Base case

Base case → return true/false

Either S_1 gets exhausted

S_1 → has no } Both are exhausted
S_2 → has no

if (i<0 && j<0) return true

return true.

only S_1 is get exhausted

if (i<0 && j>=0).

return false

if (j<0 && i>=0) — only S_2 is get exhausted.

if S_1 is left it has to be all "*"

for (ll i=0 → i)

if (s1[ii] != '*')

return false

return true.

} f(i,j)

if(i<0 && j<0) return true

if(i<0 && j>=0) return false

if(j<0 && i>=0) for (ll (——— ..)

Recursion

T.C. — O(exponential)

S.C. — O(N+M)

Auxiliary Stack Space

↓

After memoization

T.C. O(N×m)

S.C. O(N×m) + O(N+m)

↑
Ans

@ Adhish Kumar Nayak

```
int f( int i, int j, string &pattern, string &text, vector
    vector<vector<int>> &dp)

}
i==0 && j==0  if(i<0 && j<0) return true;
i==0 && j>0   if( i<0 && j>=0) return false;
j==0 && i>0   if( j<0 && i>=0)
              {
                for(int ii=0; ii<=i; ii++)
                  { if( pattern[ii-1] != '*')
                      return false;
                  }
                return true;
              }
            if( dp[i][j] != -1) return dp[i][j];
          if ( pattern[i-1] == text[j] || pattern[i-1] == '?')
            { return dp[i][j]=f(i-1, j-1, pattern, text, dp);
            }
          if( pattern[i-1] == '*')
            { return dp[i][j] = f(i-1, j, pattern, text, dp) ||
                                f( i, j-1, pattern, text, dp);
            }
        return dp[i][j] = false;
bool wildMatching(card (string pattern, &string text)
{ int n = pattern.size();
    int m = text.size();
vector<vector<int>> dp(n, vector<int>(m, -1));
return f(n-1, m-1, pattern, text, dp);
}
```

Tabulation :-

@Aashish Kumar Nayak

1. Base case
2. changing the parameter
3. copy the recurance

1st convert it into 1-based indexing. by just increasing +1 everywhere

```
bool wildcardMatching(string pattern, string text)
{   int n = pattern.size();
    int m = text.size();
    vector<vector<bool>> dp(n+1, vector<bool>(m+1, false));
    dp[0][0] = true;

    for(int j=1; j<=m; j++)
        { dp[0][j] = false;
        }

    for(int i=1; i<=n; i++)
        { bool flag = true;
          for(int ii=1; ii<=i; ii++)
            { if(pattern[ii-1] != '*')
                flag = false;
                break;
            }
        }
        dp[i][0] = flag;
    }
for(int i=1; i<=n; i++)
  { for(int j=1; j<=m; j++)
    { if(pattern[i-1] == text[j-1] || pattern[i-1] == '?')
      { dp[i][j] = dp[i-1][j-1];
      }
      else if(pattern[i-1] == '*')
          dp[i][j] = dp[i-1][j] | dp[i][j-1];
```

```
        else
            dp[i][j] = false;
    }
}
return dp[n][m];
```

Space optimization :-

```
bool wildcardMatching(string pattern, string text)
{
    int n = pattern.size();
    int m = text.size();
    vector<bool> prev(m+1, false), cur(m+1, false);
    prev[0] = true;
    for(int j = 1; j <= m; j++)
    {
        prev[j] = false;
    }
    for(int i = 1; i <= n; i++)
    {
        bool flag = true;
        for(int ii = 1; ii <= i; ii++)
        {
            if(pattern[ii-1] != '*')
            {
                flag = false;
                break;
            }
        }
        cur[0] = flag;
        for(int j = 1; j <= m; j++)
        {
            if(pattern[i-1] == text[j-1] || pattern[i-1] == '?')
            {
                cur[j] = prev[j-1];
            }
            else
            {
                cur[j] = false;
            }
        }
        prev = cur;
    }
    return prev[m];
}
```

```
    cur[0] = flag;
    for (int j=1; j<=m; j++)
    {if( pattern[i-1] == text[j-1] || pattern[i-1] == '?')
      { cur[j] = prev[j-1];
      }
      else if( pattern[i-1] == '*')
        { cur[j] = prev[j] | cur[j-1];
        }
       else
          cur[j] = false;
      }
       prev = cur;
    }
    return prev[m];
}
```

DP-35 **Best time to buy and sell Stocks** DP on Stocks

6 Problems

arr[] = { 7, 1, 5, 3, 6, 4 } , n = 6

(1st day price, 2nd day price, 3rd day price)

Buy → 1
Sell → 6
only done once

Buying has to be done before selling.

If we are selling on ith day then we buy on the minimum price from 1st → (i-1)

{ 7, 1, 5, 3, 6, 4 }

keep track of minimum on left

mini = a[0], profit = 0

for 4 min on left is 1 so profit = 3
for 6 min on left is 1 so profit = 5
for 3 ——— 1 ——— 2
for 5 ——— 1 ——— 4
for 1 ——— 7 ——— 6

max = 5 Ans

```
for(i=1; i<n; i++)
{
    cost = a[i] - mini;
    profit = max(profit, cost);
    mini = min(mini, a[i]);
}
```

Code:

```
int maximumprofit(vector<int> &prices)
{ int mini = prices[0];
  int maxprofit = 0;
  int n = prices.size();
  for(int i=1; i<n; i++)
  { int cost = prices[i] - mini;
    maxprofit = max(maxprofit, cost);
    mini = min(mini, prices[i]);
  }
  return maxprofit;
}
```

T.C. - O(N)
S.C. - O(1)

@Aashish Kumar Nayak

DP-36 Buy and sell stocks - II | Recursion to space optimization

Q Best time to Buy & sell stocks - II

In this question we can buy and sell many number of times.

prices[] = { 7, 1, 5, 3, 6, 4 }
B S B S
1 + 3 = 7

Buy and sell has to be done before next buy. i.e. Before buying a stock previous stock should be sold and maximise the profit

7 1 5 3 6 4
↑ ↑
Buy not buy Sell not sell

B S B S B S --- A lot of ways
↓
Try all ways
↓
Recursion
↓
Best answer.

Rule for recurance:

① Express everything in terms of index (ind, buy)
② Explore possibilities on that day
③ Take the max all profits made.
④ Base case.

f(ind, buy)

0	1	2	3	4	5
7	1	5	3	6	4

f(0,1) → start on 0th day with buy, what max profit?

find (ind, buy)

@Ashish Kumar Nayak

if(buy)
profit = max { -price[ind] + f(ind+1, 0) } take
else { 0 + f(ind+1, 1) } not take

sell → profit = max { price[ind] + f(ind+1, 1); , 0 + f(ind+1, 0); }

return profit;

}

Base case

when we exhausted or complete the array

```
if (ind == n)
    return 0;
```

~~T.C. → O(Exponential 2^n)~~

T.C. - O(2^n)

S.C. - O(N)

There is overlapping subproblem

so apply memoization.

dp[ind][buy]

T.C. - O(

```
int f(int ind, int buy, long *values, int n, vector<vector<long>> &dp)
{
    if(ind == n)
        return 0;
    int profit = 0; if(dp[ind][buy] != -1) return dp[ind][buy];
    if(buy)
    {
        profit = max(-values[ind] + f(ind+1, 0),
                     0 + f(ind+1, 1, values, n, dp));
    }
    else
    {
        profit = max(values[ind] + f(ind+1, 1, values, n, dp)
                     0 + f(ind+1, 0, values, n, dp));
    }
    return dp[ind][buy] = profit;
}
long getMaximumProfit(long *values, int n)
{
    vector<vector<long>> dp(n, vector<long>(2, -1));
    return f(0, 1, values, n, dp);
}
```

Tabulation :-

T.C - $O(N \times 2)$

S.C - $O(N \times 2) + \alpha N$

1. Base Case
2. i → (n-1 → 0)
   buy 0 → 1
3. copy the recurance.

dp[n+1]

```
long getmaximumprofit(long *values, int n)
{
 vector<vector<long>> dp(n+1, vector<long>(2,0));
 dp[n][0] = dp[n][1] = 0;
  for(int ind = n-1 ; ind >=0 ; ind--)
      { for(int buy =0; buy <=1; buy++)
        { long profit =0;
       if(buy)
        {
        profit = max(-values[ind] +dp[ind+1][0],
                      0 +dp[ind+1][1]);
        }
       else {
          profit = max(values[ind] + dp[ind+1][1],
                       0 +dp[ind+1][0]);
          }
        dp[ind][buy] = profit;
      }
    }
    return dp[0][1];
}
```

Space optimization :

```
long getmaximumprofit(long *values, int n)
{
  vector<long> ahead(2,0), cur(2,0);
  ahead[0] = ahead[1] = 0;
  for(int ind = n-1; ind >=0; ind--)
  { for(int buy = 0; buy <= 1; buy++)
    { long profit = 0;
      if(buy)
      {
        profit = max(-values[ind] + ahead[0],
                     0 + ahead[1]);
      }
      else
      {
        profit = max(values[ind] + ahead[1],
                     0 + ahead[0]);
      }
      cur[buy] = profit;
    }
    ~~ahead = cur;~~ ahead = cur;
  }
  return ahead[1];
}
```

DP-37 Buy and sell stocks - III

Dream Stocks

Q Best time to Buy & Sell Stock - IV - at max 2 transaction

prices = [3, 3, 5, 0, 0, 3, 1, 4]

at max 2 transaction.

```
f(ind, buy, cap)          // at max transaction
{
  if(ind == n)
    return 0;
  if(cap == 0)
    return 0;

  if(buy)
  {
return max [-price[ind] + f(ind+1, 0, cap),
            0 + f(ind+1, 1, cap)];
  }
  else
  {
selling → return max[ price[ind] + f(ind+1, 1, cap-1),
                      0 + f(ind+1, 0, cap)];
```

Recursion

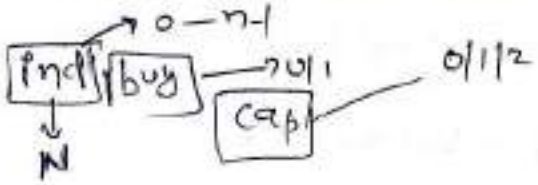
T.C. → exponential
S.C. → O(N) → Ass (Auxiliary stack space)

overlapping subproblems

↓

Memoization

@Ashish Kumar Nayak



three changing parameter | 3D DP

dp[N][2][3] N X 2 X 3

T.C. - N X 2 X 3

S.C. - O(N X 2 X 3) + O(N)

↓ Asc for recursion

Code:-

```
int f(int ind, int buy, int cap, vector<int> &prices
int n, vector<vector<vector<int>>> &dp)
{
  if(ind == n || cap == 0)
      return 0;
  if(buy)
  {
    return dp[ind][buy][cap] = max[-prices[ind] + f(ind+1, 0, cap, prices, n, dp)
                                   0 + f(ind+1, 1, cap, prices, n, dp)];
  }
  return dp[ind][buy][cap] = max[prices[ind] + f(ind+1, 1, cap-1, prices, n, dp)
                                 0 + f(ind+1, 0, cap, prices, n, dp)];
}
int Maxprofit(vector<int> &prices, int n)
{ vector<vector<vector<int>>> dp(n,
  vector<vector<int>>(2, vector<int>(3, -1)));
  return f(0, 1, 2, prices, n, dp);
}
```

Tabulation

① Base case
② changing parameter ind, Buy, cap
③ copy the recurance

④
```
if( cap == 0 || ind == n)
        return 0;
```

if (cap == 0)
ind & Buy can be anything
ind: 0 → n-1
Buy: 0 → 1

if (ind == n)
cap & Buy can be anything
cap: 0 → 2
Buy: 0 → 1

```
for( ind=0 → n-1)
    for (buy → 0 → 1)
        dp[ind][buy][0] = 0

for (buy = 0 → 1)
    for (cap = 0 → 2)
        dp[n][buy][cap] = 0
```

⑤
ind : run in reverse order
ind: n-1 → 0
buy → 0 → 1
cap → 0 → 2

⑥ Answer will be initial call i.e dp[0][1][2]

```
int maxProfit(vector<int> & prices, int n)
{ vector<vector<vector<int>>> dp(n, vector<vector<int>>(2, vector<int>(3,0)));
                                  // already dp is full with 0.
```

So we don't write any base cases. If there would something different like 1, 0 then we may write

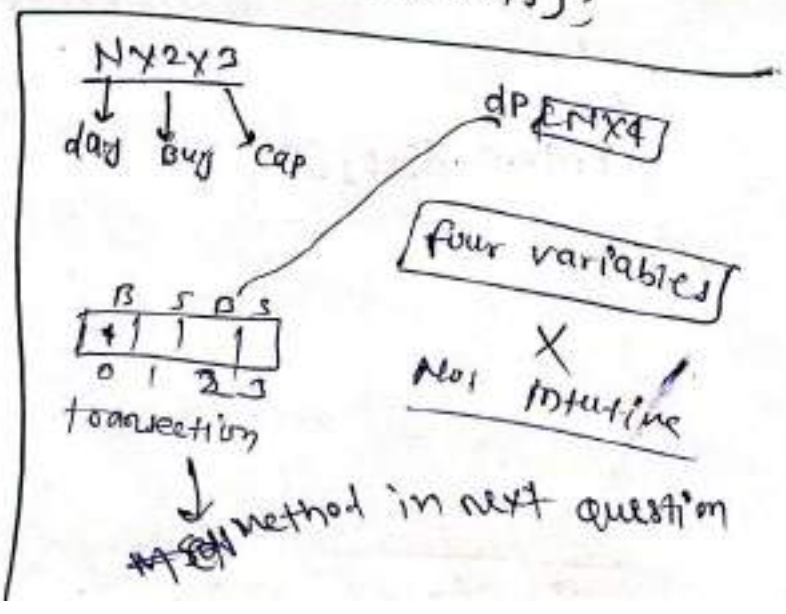
```
for(int ind=n-1; ind>=0; ind--)
    { for(int buy = 0; buy<=1; buy++)
        { for(int cap=1; cap<=2; cap++)   // ignore 0.
            {
              if(buy == 1)
             {
dp[ind][buy][cap] = max(-prices[ind] + dp[ind+1][0][cap],
                        0 + dp[ind+1][1][cap]);
            }
          else
            {
dp[ind][buy][cap] = max( price[ind] + dp[ind+1][1][cap-1],
                         0 + dp[ind+1][0][cap]);
            }
        }
    }
  return dp[0][1][2];
}
```

@Aashish kumar nayak

Space optimization :-

@Ashish Kumar Nayak

```
int maxprofit(vector<int>& prices, int n)
{
  vector<vector<int>> after(2, vector<int>(3,0));
  vector<vector<int>> cur(2, vector<int>(3,0));
   for(ind=n-1; ind>=0; ind--)
   { for(int buy=0; buy<=1; buy++)
       {
         for(int cap=1; cap<=2; cap++)
         { if(buy==1)
           {
cur[buy][cap] = max(-prices[ind]+after[0][cap], 0+after[1][cap]);
           }
         else
           {
cur[buy][cap]= max(prices[ind] + after[1][cap-1],
                                  0+after[0][cap]);
           }
       }
     after = cur;
   }
   return after[1][2];
}
```



DP-38 Buy and Sell Stocks - IV | DP on Stocks

At max K transactions

K can be anything

Code is Similar only change is k

Replace 2 with K

Code (Space optimization)

```
int maxprofit(vector<int> &prices, int n, int k)
{ vector<vector<int>> after(2,vector<int> (k+1,0));
  vector<vector<int>> cur(2, vector<int>(k+1,0));
  for(int ind =n-1; ind >=0; ind--)
     { for(int buy = 0; buy <=1; buy++)
        { for(int cap = 1; cap<=k; cap++)
             { if(buy ==1)
                { cur[buy][cap] =
              max(-prices[ind] + after[0][cap], 0 + after[1][cap]);
                }
        else
     {
  cur[buy][cap] = max(prices[ind] + after[1][cap],
                      0 + after[0][cap]);
        }
     }
  after = cur;
  }
  return after[1][k];
  }
```

@Aashish kumar Nayak

Code

```
int f(int ind, int tranNo, vector<int>& prices, int n, int k,
    vector<vector<int>> &dp)
{ if(ind == n || tranNo == 2*k)
      return 0;
  if(dp[ind][tranNo] != -1)
      return dp[ind][tranNo];
  if(tranNo % 2 == 0)
  {
    return dp[ind][tranNo] = max(-prices[ind] +
                f(ind+1, tranNo+1, prices, n, k, dp),
            0 + f(ind+1, tranNo, prices, n, k, dp));
  }
  return dp[ind][tranNo] = max(prices +
f(ind+1, tranNo+1, prices, n, k, dp), 0+f(ind+1, tranNo,
prices, n, k, dp));
}
int maximumProfit(vector<int> &prices, int n, int k)
{ vector<vector<int>> dp(n, vector<int>(2*k, -1));
  return f(0, 0, prices, n, k, dp);
}
```

ie Tabulation

```
int maxiprofit(vector<int> &prices, int n, int k)
{
 vector<vector<int>> dp(n+1, vector<int>(2*k+1,0));
 for(int ind = n-1; ind>=0; ind--)
   { for(int tranNO = 2*k-1; tranNO >=0; tranNO--)
      { if(tranNO %. 2 == 0)
          {
     dp[ind][tranNO] = max(-prices[ind] + dp[ind+1][tranNO+1],
                           dp[ind+1][tranNO]);
      }
   else
   {
    dp[ind][tranNO] = max(prices[ind] + dp[ind+1][tranNO+1],
                          dp[ind+1][tranNO]);
   }
 }
 return dp[0][0];
} Space
```

Space optimization :-

```
int maxProfit(vector<int> &prices, int n, int k)
{
    vector<int> after(2*k+1, 0);
    vector<int> cur(2*k+1, 0);
    for(int ind = n-1; ind >=0 ; ind--)
    {
        for(int tranNo = 2*k-1; tranNo >=0; tranNo--)
        {
            if(tranNo%2==0)
            {
                cur[tranNo] = max(-prices[ind] + after[tranNo+1],
                                  0 + after[tranNo]);
            }
            else
            {
                cur[tranNo] = max(prices[ind] + after[tranNo+1],
                                  0 + after[tranNo]);
            }
        }
        after = cur;
    }
    return after[0];
}
```

T.C. ≈ O(2K)
≈ O(K)

S.C.

@Ashish Kumar Nayak

DP-39 **Buy and sell stocks with cooldaon**

5th problem
DP on stocks

Cooldown → can't Buy right after sell.

price [] → {4, 9, 0, 4, 10}.

5 + 10 = 15

Cooldown {4, 9, 0, 4, 10}
B S ✗ B
B ✓

```
f(ind, Buy)
{
  if(ind == n) return 0

  if(Buy)
return max { -prices[ind] + f(ind+1, 0),
             0 + f(ind+1, 1) }.
  else
return max ( price[ind] + f(ind+1, 1),
             0 + f(ind+1, 0) );
```

Buy and stock - II
unlimited time
attach cooldown condition
on this problem

ind+2 → that day we can buy.

only this minor change on DP on stocks II problem we solved in DP on stock ✓

@Aashish Kumar Nayak

```
int f(int ind, int buy, vector<int> &prices, vector<vector<int>> &dp)
{
    if(ind >= prices.size())
        return 0;
    if(dp[ind][buy] != -1) return dp[ind][buy];
    if(buy == 1)
        return dp[ind][buy] = max(-prices[ind] + f(ind+1, 0, prices, dp),
                                  0 + f(ind+1, 1, prices, dp));
    else
        return dp[ind][buy] = max(prices[ind] + f(ind+2, 1, prices, dp),
                                  0 + f(ind+1, 0, prices));
}
int maxProfit(vector<int> &prices)
{
    int n = prices.size();
    vector<vector<int>> dp(n, vector<int>(2,-1));
    return f(0, 1, prices, dp);
}
```

Tabulation :-

```
int maxProfit(vector<int> &prices)
{
    int n = prices.size();
    vector<vector<int>> dp(n+2, vector<int>(2,0));
    for(int ind=n-1; ind>=0; ind--)
    {
        for(int buy=0; buy<=1; buy++)
        {
            if(buy==1)
            dp[ind][buy] = max(-prices[ind] + dp[ind+1][0],
                               0 + dp[ind+1][1]);
            else
            dp[ind][buy] = max(prices[ind] + dp[ind+2][1],
                               0 + dp[ind+1][0]);
        }
    }
    return dp[0][1];
}
```

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more optimation


```
int maxProfit(vector<int>& prices)
{
    int n = prices.size();
    vector<int> front2(2,0);
    vector<int> front1(2,0);
    vector<int> cur(2,0);
    for(int ind = n-1; ind>=0; ind--)
    {
        cur[1] = max(-prices[ind] + front2[0],
                     0 + front1[1]);
        cur[0] = max(prices[ind] + front2[1],
                     0 + front1[0]);
        front2 = front1;
        front1 = cur;
    }
    return cur[1];
}
```

T.C. $O(N)$
S.C. $O(6) \approx O(1)$

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DP-40 Buy and sell stocks with Transaction fee

DP on stocks
6th problem

prices[] = {1, 3, 2, 8, 4, 9} fee = 2

7-2 + 5-2 = 12-4 = (8) Net profit.

```
f( ind, buy)
{
 if( ind == 0 ) return 0;
 if(buy == 1)
   max( -price[ind] -fee + f(ind+1, 0), 0 + f(ind+1, 1));
 else
   max( price[ind] - fee + f(ind+1, 1), 0 + f( ind+1, 0));
```

OR

After the sell is done deduct the fee or we can also deduct the fee during buying.

Code:

```
int maximumprofit( int n, int fee, vector<int> &values)
{ int aheadNotBuy, aheadBuy, curBuy, curNotBuy;
  aheadbuy aheadNotbuy = aheadbuy = 0;
 for( int ind= n-1; ind>=0; ind--)
  {  //sell
    curNotbuy = max(values[ind] + aheadbuy, 0 + aheadNotbuy);
 // buy
 curbuy = max(-values[ind] - fee + aheadBuy, 0 + aheadBuy);
    aheadBuy = curBuy;
    aheadNotBuy = curNotBuy;
  }
 return aheadBuy;
}
```

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DP-41 Longest increasing Subsequence

arr[] : [10, 9, 2, 5, 3, 7, 101, 18]

{2, 3, 7, 101} → len=4

{2, 3, 7, 18} → len=4

arr[] = {8, 8, 8}

{8} → len = 1

Brute force

Let's try out various subsequences → power set (or) Recursion

(2^n) exponential

Check for increase

Store the longest

In order to generate all subsequences.

Trying all ways → Recursion

[10, ✗]

prev=10 to check

Rearrange

(i) Express everything in terms of indexes

(ii) Explore all possibilities in subsequence — ✓ ✗

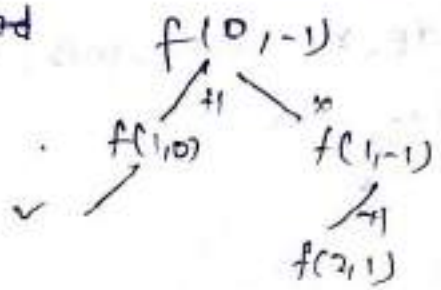
(iii) Take the max length

f(ind, prev_ind)

f(0,-1) → length of LIS starting from 0.

f(0,0) → length of LIS starting from 3rd index whose previous index is 0. → prev

f(ind)



Base case :-

```
if( ind == n)
    return 0;
```

~~Recu~~ Recursion -

T.C. $O(2^n)$ for every index we have 2 options i.e $O(2^n)$ .. S.C - $O(N)$

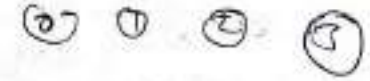
take / not-take

↓ overlapping sub-problem

Memoization

f(ind, prev_ind)

Co-ordinate change :-



T.C. - $O(N \times N)$

S.C - $O(N \times N) + O(N)$

Code :-

```
int f(int ind, int prev_ind, int arr[], int n, vector<vector<int>>& dp)
{
    if( ind == n) return 0;
    if( dp[ind][prev_ind+1] != -1 ) return dp[ind][prev_ind+1];
    int len = 0 + f(ind+1, prev_ind, arr, n, dp);
    if( prev_ind == -1 || arr[ind] > arr[prev_ind])
    {
        len = max( len, 1 + f(ind+1, ind, arr, n, dp));
    }
    return dp[ind][prev_ind+1] = len;
}
int longestIncreasingSubsequence( int arr[], int n)
{
    vector<vector<int>> dp(n, vector<int>(n+1, -1));
    return f(0, -1, arr, n, dp);
}
```

Tabulation :-

Rules.

① Base case ~~~ dp[n][n+1] = 0
∵ we already assign 0 to our so no need to write base case.

② Changing parameter
- ind = n-1 → 0
- prev_ind = ind-1 → -1

③ copy the recurrence follow coordinated shift.

Code :-

```
int longestIncreasingSubsequence(int arr[], int n)
{
  vector<vector<int>> dp(n+1, vector<int>(n+1, 0));
  for(int ind = n-1; ind>=0; ind--)
  {
    for(int prev_ind = ind-1; prev_ind>=-1; prev_ind--)
    {
      int len = 0 + dp[ind+1][prev_ind+1];
      if(prev_ind==-1 || arr[ind] > arr[prev_ind])
      {
        len = max(len, 1+dp[ind+1][ind+1]);
      }
      dp[ind][prev_ind+1] = len;
    }
  }
  return dp[0][-1+1];
}
```

Space optimization :-

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```
int longestIncreasingSubsequence(int arr[], int n)
{
    vector<int> next(n+1,0), cur(n+1,0);
    for(int ind = n-1; ind >= 0; ind--)
    {
        for(int prev_ind = ind-1; prev_ind >= -1; prev_ind--)
        {
            int len = 0 + next[prev_ind+1];
            if(prev_ind == -1 || arr[ind] > arr[prev_ind])
            {
                len = max(len, 1 + next[ind+1]);
            }
            cur[prev_ind+1] = len;
        }
        next = cur;
    }
    return next[-1+1];
}
```

T.C. - $O(N^2)$

S.C. - $O(N) \times 2$

Example

5	4	11	1	16	8

dp[n]

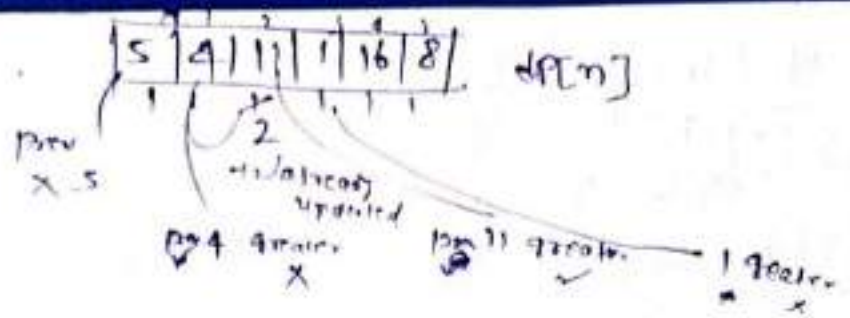
dp[i] → signifies the longest increasing subsequence that ends at index i.

{5, 16}

{3, 11, 16}

Ans → max of all dp indexes.

$\max_{i=0 \text{ to } n-1} (dp[i])$




```
for(ind = 0 → n-1)
    for(prev = 0 → ind-1)
        if(arr[prev] < arr[ind])
            dp[ind] = max(1+dp[prev], dp[ind]);
```

Code :-

```
fn: longestIncreasingSubsequence(int arr[], int n)
{
    vector<int> dp(n,1);
    int maxi = 1;
    for(int i = 0; i<n; i++)
    {
        for(int prev = 0; prev<i; prev++)
        {
            if(arr[prev] < arr[i])
            {
                dp[i] = max(dp[i], 1+dp[prev]);
            }
        }
        maxi = max(maxi, dp[i]);
    }
    return maxi;
}
```

T.C. - O(N²) — This solution will be required if we want to trace the LIS.

S.C. - O(N)

DP-42

print the subsequence

5	7	11	1	16	8
1	2	2	1	3	3	dp

| 0 | 1 | 2 | 0 | 4 | 5 | hash

4 → 2 → 0

16 11 5

reverse it → 5, 11, 16

Code :-

```
int longestIncreasingSubsequence(int arr[], int n)
{
  vector<int> dp(n,1), hash(n);
  int maxi = 1;
  int lastIndex = 0;
  for(int i=0; i<n; i++)
  {
    hash[i] = i;
    for(int prev=0; prev<i; prev++)
    {
      if(arr[prev] < arr[i] && 1+dp[prev] > dp[i])
      {
        dp[i] = 1+dp[prev];
        hash[i] = prev;
      }
    }
    if(dp[i] > maxi)
    {
      maxi = dp[i];
      lastIndex = i;
    }
  }
}
```

```
vector<int> temp;
temp.push_back(arr[lastIndex]);
while(hash[lastIndex] != lastIndex)
    {
        lastIndex = hash[lastIndex];
        temp.push_back(arr[lastIndex]);
    }
reverse(temp.begin(), temp.end());
for(auto it: temp)
    cout<<it<<" ";
cout<<endl;
return maxi;
}
```

T.C. - $O(N^2)$

$O(N)$

DP-43 Longest Increasing Subsequence

In this lecture we will use Binary search

LIS using Binary search

[1, 7, 8, 4, 5, 6, -1, 9]

Increasing subsequences are

1, 4, 5, 6, 9 — longest

Recursion
Memoization
space optimization
Tabulation
Algorithmic approach

T.C — $O(N^2)$
S.C — $O(N)$

Approach :- 1 ~~, 4, 6, 9~~

Try to go every element and form subsequence

[1, 7, 8, 4, 5, 6, -1, 9]

[1, 7, 8, 9] len=4

[1, 4, 5, 6, 9] len=5 → longest

[-1, 9] len=2

But a lot of space will be consumed

Intuition

- To avoid space we will overwrite instead of creating new array.

[1, 7, 8, 4, 5, 6, -1, 9]

[1, 4, 5, 6, 9]

[1, 4 — instead of making [1, 4 we will replace new overwrite in 1st array Replace 4 with 7

arr[] = {1, 4, 5, 4, 2, 8}

[1, 4, 5, 8] len = 4 — space has been optimized.

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Binary Element el -> arr[i]

if(find arr[i] .

else find 1st greater of arr[i]

ed.

lower_bound( )

→ it gives you 1st pointer of arr[i]
or
first index > arr[i]

Code :-

```
int longestIncreasingSubsequence(int arr[], int n)
  {
    vector<int> temp;
    temp.push_back(arr[0]); int len = 1;
    for(int i=1; i<n; i++)
      { if(arr[i] > temp.back())
          { temp.push_back(arr[i]);
            len++;
          }
        else
          {
            int ind = lower_bound(temp.begin(), temp.end(),
                                  arr[i]);
            temp[ind] = arr[i];
          }
      }
    return temp.size();
  }  return len; // len will take less space than temp
```

lower bound. size()

T.C. - O(N log N)
S.C. - O(N)

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DP-44 Largest Divisible Subset | longest increasing subsequence

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arr[] = {1, 16, 7, 8, 4}

{1, 16, 8} — subsequence

{1, 8, 16}
{1, 16, 8}
{1, 7, 16}
} subset not necessary to follow the order.

arr[i] % arr[j] == 0
or arr[j] % arr[i] == 0

lets take an example

{16, 8, 4} (16,8) (8,4) (16,4) Every pair is divisible

{1, 16, 8, 4} (1,16), (1,8), (1,4), (16,8), (16,4), (8,4)
All the pairs are divisible this can be possible ans

len = 4

print any answer you can jumble it and return.

{1,16,8,4} ✓ or {4, 16, 1, 8} ✓ ...

intution

sort the array

{1, 4, 7, 8, 16}

{1, 4, 8, 16}

4 is divisible by 1 so store it
7 is not divisible by 4
8 is divisible by 4 & and also it is divisible by 4 then it will automatically divisible by 1 or previous elements.

len=4 this

we will get longest divisible increasing subsequence.

LIS code :-

```
for(i=1; i<n; i++)
  { for(j=0; j<i; j++)
      { if(arr[i] % arr[j] == 0)
          if(~~arr[i] < arr[j]~~ && dp[i] < dp[j]+1)
              dp[i] = dp[j]+1
              hash[i] = j
      }
```

because it is already sorted just change arr[i] % arr[j] or arr[j] % arr[i];

Code :-

```
vector<int> divisibleSet(vector<int> &arr)
{ int n = arr.size();
  vector<int> dp(n,1), hash(n);
  int maxi = 1;
  int lastIndex = 0;
  sort(arr.begin(), arr.end());
  for(int i=0; i<n; i++)
  { hash[i] = i;
    for(int prev=0; prev<i; prev++)
    { if(arr[i] % arr[prev] == 0 && 1+dp[prev] > dp[i])
      { dp[i] = 1 + dp[prev];
        hash[i] = prev;
      }
    }
    if(dp[i] > maxi)
    { maxi = dp[i];
      lastIndex = i;
    }
  }
  vector<int> temp;
  temp.push_back(arr[lastIndex]);
  while(hash[lastIndex] != lastIndex)
  { lastIndex = hash[lastIndex];
    temp.push_back(arr[lastIndex]);
  }
  reverse(temp.begin(), temp.end());
  return;
}
```

T.C. $O(N^2)$
S.C - $O(N)$

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DP-45 Longest string Chain | longest increasing subsequence

words[] = {"a", "b", "ba", "bca", "bda", "bdca"}

{a, ba, bda} add b, add d

{b, ba, bca} add a, add c

{a, ba, bca}

for(i=1 → n)
{ for (j=0; j<i; j++) Compare(arr[i], arr[j])
{
if(~~arr[i] > arr[j]~~ && dp[j]+1 > dp[i])
{ dp[i] = dp[j]+1;
}
maxi = max(maxi, dp[i]);
}
return maxi;

one line change.

bdca

bda

assume

arr[i] bcda ← ta → arr[j] bcd

bcd

if both pointers reaches its end means match.

In this question there is sequence NOT sub-sequence so we can pick from any where.

["xbc", "pcxbcr", "xb", "[xbc", "pcxbc"]
(2) (5) (1) (3) (4)

We will sort this array according to the length.

write comparator

bool comp(string &s1, string &s2)
return s1.size() < s2.size();

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Code :-

```
int longestStrChain(vector<string> &arr)
{
    sort(arr.begin(), arr.end(), comp);
    int n = arr.size();
    vector<int> dp(n, 1);
    int maxi = 1;
    for(int i=0; i<n; i++)
    {
        for(int prev=0; prev<i; prev++)
        {
            if(checkpossible(arr[i], arr[prev])
               && 1+dp[prev] > dp[i])
            {
                dp[i] = 1+dp[prev];
            }
        }
        if(dp[i] > maxi)
        {
            maxi = dp[i];
        }
    }
    return maxi;
}

bool comp(string &s1, string &s2)
{
    return s1.size() < s2.size();
}

bool checkpossible(string &s1, string &s2)
{
    if(s1.size() != s2.size()+1) return false;
    int first = 0;
    int second = 0;
    while(first < s1.size())
    {
        if(s1[first] == s2[second])
        {
            first++; second++;
        }
        else { first++; }
    }
    if(first == s1.size() && second == s2.size()) return true; else return false;
}
```

length of the string

T.C. - $O(N^2) \times l$ + $O(n \log n)$ sorting.

DP-46 Longest Bitonic Subsequence | LIS

arr[] = {1, 11, 2, 10, 4, 5, 2, 1}

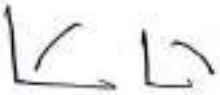
Bitonic → increasing then decreasing. or it can be just increase or just decrease

{1, 2, 10, 4, 2, 1}

{1, 2, 10, 5, 2, 1}

{1, 11, 10, 4, 2, 1}

{1, 11, 10, 5, 2, 1}



arr[] = {1, 10, 2, 3, 4, 5, 6}

because of subsequence {1, 2, 3, 4, 5, 6} len=6

arr[] = {5, 4, 3, 100, 2, 1}

{5, 4, 3, 2, 1} — len = 5

Both of them are also Bitonic

```
for(i=0; i<n; i++)
{ dp[i]=1
  for(j=0; j<i; j++)
    { if(arr[i]>arr[j] && dp[i]+1<dp[j])
        dp[i] = 1+dp[j];
    }
}
```

↑ Longest increasing subsequence till index i.

arr[] = {1, 11, 2, 10, 4, 5, 2, 1}

dp1[] = {1, 2, 2, 3, 3, 4, 2, 1}

dp2[] = {1, 2, 5, 2, 4, 3, 3, 2, 1}

2 → common

6 3 6 5 6 3 1

3 + 4 = 7 Ans

1 is common

↓

1 = 6 Ans

Code:

```
int longestBitonicSequence(vector<int> &arr, int n)
{ vector<int> dp1(n,1);
  for(int i=0; i<n; i++)
  { for(int prev=0; prev<i; prev++)
      { if(arr[prev] < arr[i] && 1+ dp1[prev] > dp1[i])
          { dp1[i] = 1+ dp1[prev];
          }
  }   }
                int maxi=0;
  vector<int> dp2(n,1);
   for(int i=n-1; i>=0; i--)
     { for(int prev=n-1; prev>i; prev--)
         { if(arr[prev] < arr[i] && 1+dp2[prev] > dp2[i])
           {
              dp2[i] = 1+dp2[prev];
           }
     }    } -> maxi = max(maxi, dp1[i] + dp2[i] -1);
                  avoid this by using this ↑
   int maxi=0;
   for(int i=0; i<n; i++)
     { maxi = max(maxi, dp1[i] + dp2[i] -1);
     }
   return maxi;
}
```

Save O(N) T.C.

T.C. - O(N²)

S.C. -

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DP-47 Number of longest increasing subsequence

arr[] = {1, 3, 5, 4, 7}

{1, 3, 4, 7}
{1, 3, 5, 7}
— ② subsequences of longest length

arr[] → {1, 5, 4, 3, 2, 8, 7, 10, 8, 9}
(8 attached with 1)

dp[] → 1 x2 x2 x2 x2 x2 3
cnt[] → 1 1 1 1 1 2 1 3 4 5

1, 5

4 itself of length 1 and cnt is 1, I can be attached with 4 so increase cnt by 1 and dp remain 1

1, 5, 7 → 1, 6, 7
1, 4, 7 → 5, 6, 7
1, 3, 7 → 4, 6, 7
1, 2, 3 → 3, 6, 7
3, 6, 7 → 2, 6, 1

{1, 3, 5, 4, 7}
dp[] = 1, 2, 3, 3, 4
cnt[] = 1, 1, 1, 1, 2

Code :-

```
int findNumberOfLIS(vector<int> &arr)
{ vector<int> dp(n,1), cnt(n,1);
  int maxi = 1; int n = arr.size();
  for(int i = 0; i<n; i++)
  { for(int prev = 0; prev<i; prev++)
    { if(arr[prev] < arr[i] && 1+dp[prev] > dp[i])
      { dp[i] = 1+dp[prev];
        cnt[i] = cnt[j];
      }
```

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```
else if (arr[prev] < arr[i] && 1 + dp[prev] == dp[i])
{
    cnt[i] += cnt[prev];
}
}
maxi = max(maxi, dp[i]);
}
int cnt = 0; int nos = 0;
for(int i = 0; i < n; i++)
{ if(dp[i] == maxi)
    nos += cnt[i];
}
return nos;
}
```

DP - 48 Matrix chain Multiplication [MCM] Partition DP starts

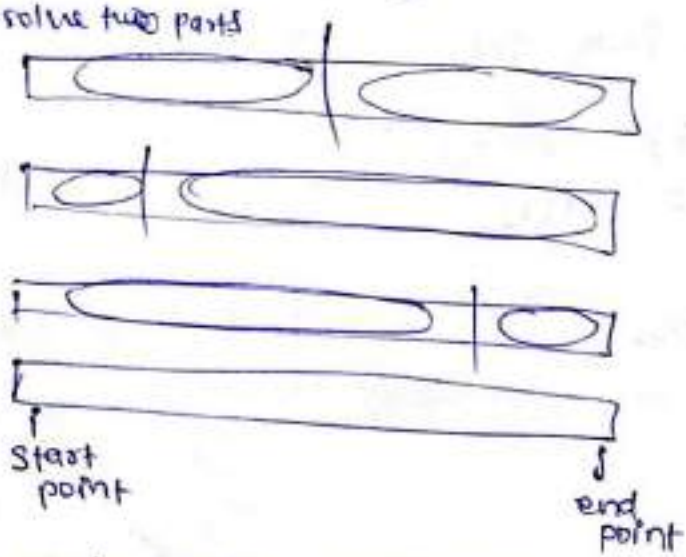
New pattern - [pattern DP] Tough pattern

Solve a problem in a pattern

$(1+2+3)\times(5)$

$(1+2)+(3\times5)$

We have to solve two parts



MCM → matrix chain multiplication

$B = 30\times5$

$L = 5\times60$

A B

$[\ ]_{10\times30}$ $[\ ]_{30\times5}$

$= 10\times30\times5 = 1500$ operations required to multiply the matrix.

$\begin{bmatrix}1 & 2\\ 3 & 1\end{bmatrix}_{2\times2} \times \begin{bmatrix}2\\ 3\end{bmatrix}_{2\times1} = \begin{bmatrix}1\times2 + 2\times3\\ 3\times2 + 1\times3\end{bmatrix}$

$2\times2\times1 = 4$ operations

$(A\ B\ C)$

$(A\ B)\ C$

$10\times30\ \ 30\times5$

$A = 10\times30$

$B = 30\times5$

$C = 5\times60$

$[\ ]_{10\times5}\ [\ ]_{5\times60} \Rightarrow (10\times5\times60)$

$= 1500 + 3000 = 4500$

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A(BC)

30x60 → 5x60

9000

[]10x30 []30x60

= 10x30x60 + 9000

= 18000 + 9000

= 27000

The minimum operation required given the N matrix dimension

ABCD

A(B(CD)) AB(CD)

Given arr[] → [10, 20, 30, 40, 50] N = 5

↓ dimensions of N-1 matrixes

A → 10x20, B → 20x30, C → 30x40, D → 40x50

1st → A[0]xA[1] 2nd A[1]xA[2] 3rd → A[2]xA[3] ith → A[i-1]xA[i]

Tell me the no. of operations required to multiply the Matrices.

Approach Partition DP

take the 4 guys and give one-the ans.

ABCD

(AB)(CD) (A)(BCD) (ABC)(D) → take the minimal of them

1. Start with entire block/array F[i,j] start point, end point
2. Try all partition → Run a loop to try all partitions
3. Return the best possible 2 partition

[10, 20, 30, 40, 50]

A B C D

i j

1 n-1

f(1,4) → return the minimum multiplications to multiply Matrix 1 → 4

Base case :-

```
f(i,j)
{ if(i==j)
    return 0;
```

Try all partions

{ 10, 20, 30, 40, 50 }
(A) (B C D)

( ) ( )

( ) ( )

$K = (i \to j-1)$

K=1 F(i, K), F(K+1, j)

K=2 f(1,2), f(3,4)

K=3 f(1,3), f(4,4)

K = (i+1, → j)

f(i, k-1), f(k, j)

for(k → i → j-1)

{ steps = A[i-1] x arr[k] x arr[j]) + f(i,k) + f(k+1,j)

f(1,1) f(2,4) (BCD)

A = 10x20

B = 20x30

C = 30x40

D = 40x50

(10x20) (20x50) (10x50) 10x20x50 operation or steps

B C D

(20x30) (30x40) (40x50)

B C D

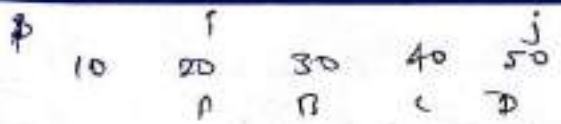
20x30 30x50

(20x50)

(20x40) (40x50)

(20x50)

Aashish Kumar Nayak



f(A B) + f(C D) +

10×20 30×30 40×50

20×30

30×30

10×30

10×50

No. of steps
= 10×50×50

A[i-1] × A[k] × A[j]

Recursion Tree

(10, 20, 30, 40, 50)
A B C D
i j

+ single matrix No multiply

A [20] [30, 40, 50]
i i j
B C D

AB CD A B C D
20 30 40 30 10 30 40 50 30
i j i j i j k j i j

(10 * 20 * 30) + f + f

B C D
30 40 50
i j

[B] [CD] [BC] [D]

then take min of all.

Cont

Code :=

```
int f(int i, int j, vector<int> &arr)
{ if(i==j) return 0;
int mini = 1e9;
for(int k=i; k<j; k++)
  { int steps = arr[i-1] * arr[k] * arr[j] + f(i,k,arr,dp)
                + f(k+1,j,arr,dp);
    if(steps<mini)
      mini = steps;
  }
  return mini;  return dp[i][j] = mini;
}

int matrixmultiplication(vector<int> &arr, int N)
{ vector<vector<int>> dp(N, vector<int>(N,-1));
  return f(1,N-1,arr,dp);
}
```

T.C. – Exponential

Apply Memoization

changing variables f(i,j)

DP 49 Matrix chain multiplication | Bottom-up | Tabulation

Rules :-

1. Copy the base case
2. write the changing states.

dp[N][N]

for i = (n-1 → 1)

j :- ___

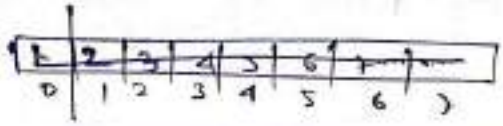
Code :-

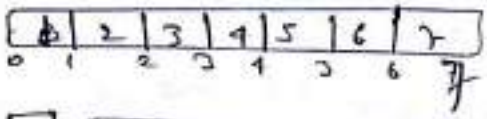
```
int matrixmultiplication(vector<int> &arr, int N)
{
  int dp[N][N];
  for(int i=N-1; i>=1; i--)
  {
    for(int j= i+1; j<N; j++)
    {
      int mini = INT_MAX;
      for(int k= i; k<j; k++)
      {
        int steps = arr[i-1]*arr[k]*arr[j]
                    + dp[i][k]
                    + dp[k+1][j];
        if(steps < mini)
        {
          mini = steps;
        }
      dp[i][j] = mini;
    }
  }
  return dp[1][N-1];
}
```

DP-50 Minimum cost to cut the stick

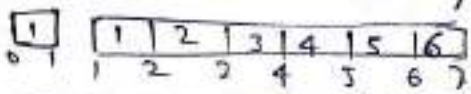
arr[] = [1, 3, 4, 5] n = 7

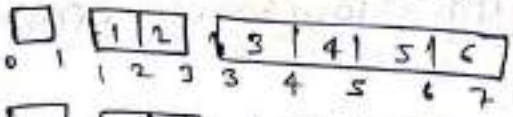
cost = length of the stick you are cutting

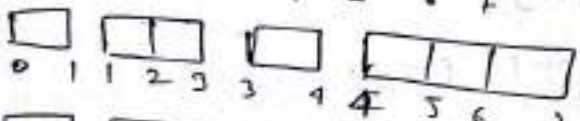


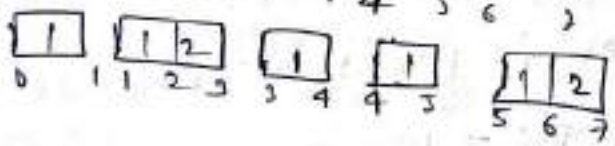


7 + 6 + 4 + 3 = 20

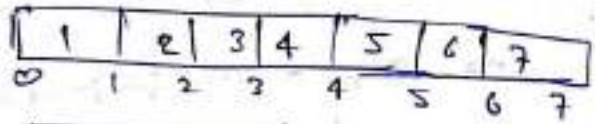


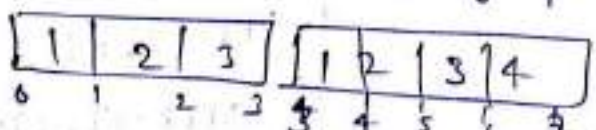




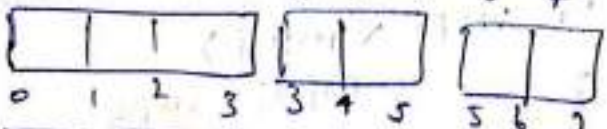


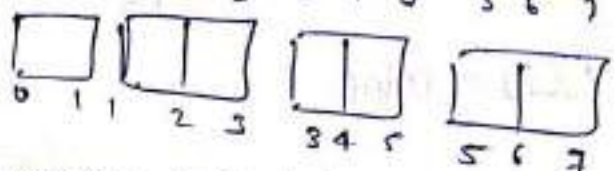
{ 3, 5, 1, 4 }



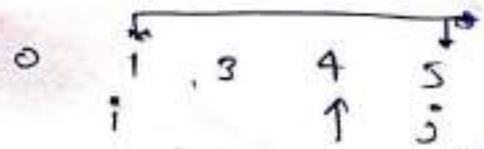


7 + 4 + 3 + 2 = 16





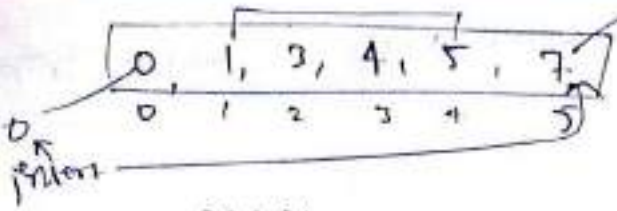
Array is sorted



cuts[j+1] − cuts[i−1]

7 − 0 = 7

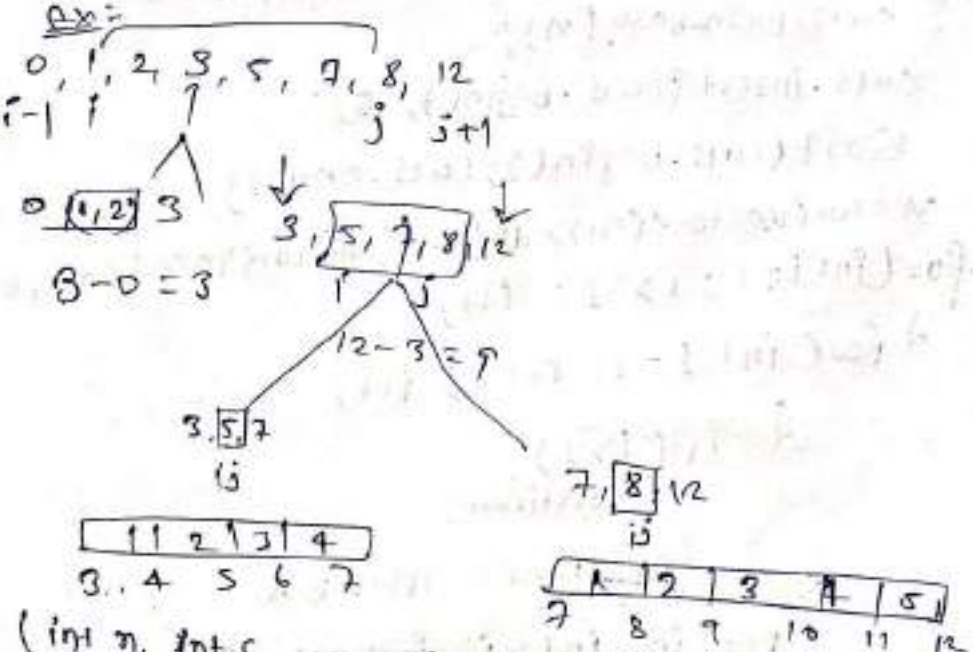
cost = length of that stick in which 4 is.....



f(i,j)

f(1,4)

```
if(i>j)
  return 0;
mini = INT_MAX;
for(ind = i → j)
{ cost = cuts[j+1] - cuts[i-1]
       + f(i, ind-1) + f(ind+1, j)
  mini = min(mini, cost);
}
return mini;
}
```



Code

```
int cost(int n, int c, vector<int> &cuts,
{ cuts.push_back(n);
  cuts.insert(cuts.begin(), 0);
  sort(cuts.begin(), cuts.end());
  return vector<vector<int>> dp(c+1, vector<int>(c+1,-1));
  return f(1, c, cuts, dp);
}
```

@Aashish Kumar Nayak

```
int f(int i, int j, vector<int>& cuts, vector<vector<int>>& dp)
{ if(i > j) return 0;
    if(dp[i][j] != -1)
        return dp[i][j];
    int mini = INT_MAX;
    for(int ind = i; ind <= j; ind++)
    { int cost = cuts[j+1] - cuts[i-1] + f(i, ind-1, cuts, dp)
                 + f(ind+1, j, cuts, dp);
      mini = min(mini, cost);
    }
    return dp[i][j] = mini;
}
```

T.C. $(M^2 \times M) \approx O(M^3)$

S.C. $= O(M^2) + \text{stack space} \approx O(M^3)$ Ans

Tabulation :-

```
int cost(int n, int c, vector<int>& cuts)
{ cuts.push_back(n);
  cuts.insert(cuts.begin(), 0);
  sort(cuts.begin(), cuts.end());
  vector<vector<int>> dp(c+2, vector<int>(c+2, 0));
  for(int i = c; i >= 1; i--)
  { for(int j = 1; j <= c; j++)
    { if(i > j)
        continue;
      int mini = INT_MAX;
      for(int ind = i; ind <= j; ind++)
      {
        int cost = cuts[j+1] - cuts[i-1] + dp[i][ind-1] +
                   dp[ind+1][j];
        mini = min(mini, cost);
      }
      dp[i][j] = mini;
    }
  }
  return dp[1][c];
}
```

@Aashish Kumar Nayak

DP-51 Brust Ballons | partion DP

arr[] = {3, 1, 5, 8} n = 4

Summation of those coins ↑↑

if we burst 5 → 1 (Left) × 5 × 8 (right) = 40

1 × 3 × 1 = 3

for 1, now 3 is already burst

1 × 1 × 5 = 5

for 5

1 × 5 × 8 = 40

[8] 1 × 8 × 1 = 8

1	[3, ~~1~~, 5, 8] ↓	3 × 1 × 5 = 15
5	[3, ~~5~~, 8]	3 × 5 × 8 = 120
3	[~~3~~, 8]	1 × 3 × 8 = 24
8	[8]	1 × 8 × 1 = 8
		167 → maximum

~~Maximal~~ ~~start from~~ ~~min~~ value ~~to burst~~ ~~last~~ ~~Ballon~~.

[3, 1, 5, 8]

i j can be anyone. so we can think as MCM

[b1, b2, b3, b4, b5, b6]

(b1 × b4 × b5) + (b1 b2 b3) + (b5 b6)

Subproblems

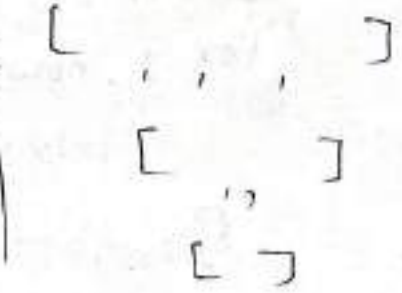
This will not work if solving independently

[(b1, b2, b3), [b4], b5, b6]

{ b1, b2, b3, b4, b5, b6 }

if we take subpart as an individual subproblem and try to solve for b_3 then we can't become it. depends on other portion b_4 i.e. b_4.

Start thinking in opposite direction



[3, X, 8]

[X, 8]

[8]

$3 \times 1 \times 5 = 15$

$3 \times 5 \times 8 = 120$

$\rightarrow 1 \times 3 \times 8 = 24$

$\rightarrow 1 \times 8 \times 1 = 8$

{ b1, b2, b3, b4, b5, b6 }

i ... X ... j

ind

$= \left(a[i-1] \times a[ind] \times a[j+1] \right) + f(i, ind-1) + f(ind+1, j)$

{b1, b4} or {b2, b4} or {b3, b4} or 2nd last Basis

[ ]

[ ]

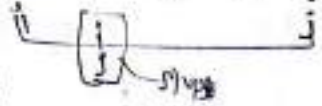
[b4]

$= 1 \times b_4 \times 1$

(last Basis)

! [b1, b2, b3, b4, b5] !

0 1 2 3 4 5



f(1,5) or f(1,4)

```
f(i,j)
{
  if(i >= j)          // i == j - 1 subproblems (last subproblem)
      return 0;
  for(ind = i -> j)  maxi = INT_MIN;
  {
    cost = a[i-1] x a[ind] x a[j+1] + f(i, ind-1) + f(ind+1, j)
    mini = MIN(mini, cost);
  }
  return mini;
```

T.C ⟹ (Recursion) T.C — Exponential

↓ Memoization

T.C. → N×N×M ≃ N^3

S.C. → O(N^2) + O(N)
Auxiliary Space

i j
[N+1]

Code :-

```
int f(int i, int j, vector<int> &a, vector<vector<int>> &dp)
{  if(i>j) return 0;
   if(dp[i][j] != -1)
        return dp[i][j];
   int maxi = INT_MAX;
   for(int ind = i; ind <= j; ind++)
      {int cost = a[i-1] * a[ind] * a[j+1] +
                  f(i, ind-1, a, dp) + f(ind+1, j, a, dp);
       maxi = max(maxi, cost);
      }
   return dp[i][j] = maxi;
}
```

@Aashish Kumar Nayak

```
int n = a.size();
a.push_back(1);
a.insert(a.begin(), 1);
vector<vector<int>> dp(n+1, vector<int>(n+1, -1));
return f(1, n, a, dp);
}
```

Tabulation :-

④

```
int maxCoins(vector<int>& a)
{
    int n = a.size();
    a.push_back(1);
    a.insert(a.begin(), 1);
    vector<vector<int>> dp(n+2, vector<int>(n+2, 0));
    for(int i = n; i >= 1; i--)
    {
        for(int j = 1; j <= n; j++)
        {
            if(i > j)
                continue;
            int maxi = INT_MIN;
            for(int ind = i; ind <= j; ind++)
            {
                int cost = a[i-1] * a[ind] * a[j+1]
                    + dp[i][ind-1] + dp[ind+1][j];
                maxi = max(maxi, cost);
            }
            dp[i][j] = maxi;
        }
    }
    return dp[1][n];
}
```

TC:- $O(N^3)$

S.C. $O(N^2)$

DP-52

Boolean Expression to True | partion DP

Compression = "T|T&F"

"T&F"

"F"

or → |
& → and
XOR → ^

You have to perform in such a way you will get True

"T|T&F"

"T|F" ① → no. of ways

"T"

"T^F|T&F^T|F"
(i j)

Similar to MCM

(T^F|T)&(F^T|F)

(T^F|T) & (F^T|F)

f(i, ind-1) — partition (ind) — f(ind+1, j)

left & right

both has to be true.

If I'm looking for no. of ways true left comes true or no. of ways in which right is true.

x + y.
left right

left | Right

T = x1 T = x2 = x1 x x2

F = x3 = x1 x x3

T ^ T = F

F ^ F = F

T ^ F = T

F ^ T = T

@Ashish Kumar Nayak

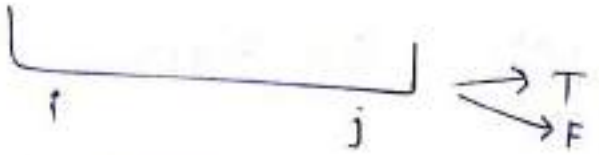
left ∧ right

T = x_1 T = x_2

F = x_3 F = x_4

$x_1 \vee x_4$

$x_3 \vee x_2$

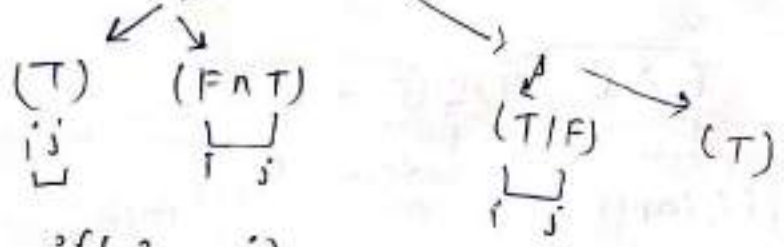


How many ways we can make it T or F.

~~f(i,j) is true~~

```
f(i,j, isTrue)
{ if(i>j)
    return 0;
```

T | F ^ T
i j




```
if(i==j)
  { if(isTrue==1) return a[i]=='T';
    else return a[i]=='F';
```

looking for false.

1 if false
0 if true.

```
  do and try to make partition
  int ways=0
for(ind i+1, ind<=j; ind=ind+2)
  { lT = f(i, ind-1, 1);
    lF = f(i, ind-1, 0);
    rT = f(ind+1, j, 1);
    rF = f(ind+1, j, 0);
```

ways = 0

if(a[ind] == 'T')

if(isTrue) ways += LTxRT; else ways += (LTxLF)+(LFxRT)+(RFxLF);

else if '&'

{

}

else if '|'

{

}

else

{

return ways;

So we are applying recursion

T.C. — O(exponential)

f(i, j, isTrue)

dp[N][N][2]

Code :-

```
int mod = 1000000007;
long long f(int i, int j, int isTrue, string &exp, vector<vector<vector<long long>>> &dp);
{ if(i>j) return 0;
  if(i==j)
  { if(isTrue)
    { return exp[i]=='T';
    }
    else return exp[i] == 'F';
  }
  long long ways = 0;
  if(dp[i][j][isTrue] != -1) return dp[i][j][isTrue];
for(int ind = i+1; ind<=j-1; ind+=2)
{ long long lT = f(i, ind-1, 1, exp, dp);
  long long lF = f(i, ind-1, 0, exp, dp);
  long long rT = f(ind+1, j, 1, exp, dp);
  long long rF = f(ind+1, j, 0, exp, dp);
```

```
if(exp[ind] == '&')
{ if(isTrue) ways = (ways + (lT*rT)%.mod;
  else ways = ways + (lt*lF)%.mod + (lF*lT)%.mod +
}                    (lF*rF)%.mod)%.mod;
else if(exp[ind] == '|')
{
if (isTrue)
ways = (ways + (lT*rT)%.mod + (lT*rF)%.mod +
                    (lF*rT)%.mod)%.mod;
}
else{
    ways = (ways + (lF*lF)%.mod)%.mod;
}
}
else
{ if(isTrue)
  { ways = (ways + (lT*rF)%.mod + (lF*rT)%.mod)
  }                                          %.mod;
  else
  { ways = (ways + (lT*rT)%.mod + (lF+rF)%.mod)
  }                                          %.mod.
}}
return dp[i][j][isTrue] = ways;
int evaluateExp(string & exp)
{ int n = exp.size();
vector<vector<vector<long long>>> dp(n, vector<vector<long long>>
(n, vector<ll>(2, -1)));
return f(0, n-1, 1, exp, dp);
}
```

DP-53 Palindrome partitioning - a | Front partition

Given String = bababcbadcede

Small string str = aabb n=4

a|a|b|b n-1 partition

3 partition

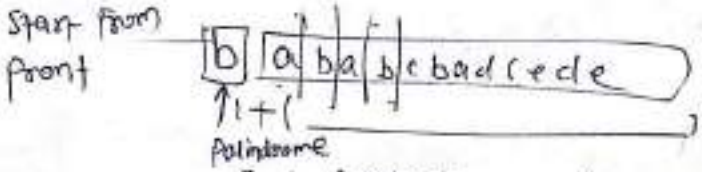
Every individual substring will be palindrome.

aa|bb

a|a|b|b

pal

Such problems are generally solved using front partition



1 + (abcb_ _)

1 + (ababc_b _ _)

1 + (cbadcede)

3 subproblems take the minimum no. of cuts.

take this one

1 + (abcbadcede)

1 + (dcede)

Getting a recurrence :-

1. Express everything in terms of index.
2. Express all the ~~ways~~ possibilities.
3. Take the min of all possibilities
4. Write the base case.

Give me the min. cut to make all substring palindrome.

@Ashish Kumar Nayak

```
f(i)
{ if( i == n) return 0;
        temp = "";   if(dp[i] != -1) return dp[i];
for(j=i ; j<n; j++)
    {
      temp += s[j];
          if( isPalindrome(temp))
              cost = 1+f(j+1);
              minCost = min(minCost, cost);
    }
}       return minCost;
```

temp = b
temp = ba
temp = bab

dp[i] =>

Recursion → T.C. = O(Exponential)

↓ Memoization

Changing parameter
0 → N
dp[N]

Code :

```
int palindromePartitioning(string str)
{ int n = str.size();
  vector<int> dp(n,-1);
  return f(0, n, str, dp)-1;
}
int f(int i, int n, string &str, vector<int> &dp)
{ if(i == n) return 0;
  if(dp[i] != -1) return dp[i];
  int minCost = INT_MAX;
  for(int j=i; j<n; j++)
    { if(ispalindrome(i,j,str))
      { int cost = 1+ f(j+1, n, str, dp);
        minCost = min(minCost, cost);
      }
  }return dp[i] = minCost;
}
```

```
bool ispalindrome(int i, int j, string s)
{ while(i<j)
  { if(s[i] != s[j])
      return false;
    i++;
    j--;
  }
  return true;
}
```

T.C. $O(N) \times O(N) = O(N^2)$

S.C. $O(N) + O(N)$ Ass.

Tabulation :-

int (1) Base case (i == n) → 0

(2) i = n-1 → 0 dp[n] = 0;

(3) Copy the recurrence

```
int palindromepartitioning(string str)
{ int n = str.size();
  vector<int> dp(n+1);
  dp[n] = 0;
  for(int i=n-1; i>=0; i--)
  { int mincost = INT_MAX;
    for(int j=i; j<n; j++)
    { if(ispalindrome(i, j, str))
      { int cost = 1 + dp[j+1];
        mincost = min(mincost, cost);
      }
    }
    dp[i] = mincost;
  }
  return dp[0]-1;
}
```

T.C. - $O(N^2)$

S.C - $O(N)$

@Aashish kumar Nayak

DP-54 Partition Array for Maximum Sum / Front Partition

arr[] = [1, 15, 7, 9, 2, 5, 10] K = 3
(0 1 2 3 4 5 6)

[15, 15, 15, 9, 9, 9, 10, 10]

Sum = 77

all elements of partition converted into max element

[1 15 7 | 9 | 2 5 10]
[15 15 15 9 10 10 10]

sm = 84 ↑↑

front partition logic ⟷ Maximum

We have various ways → Try recursion

Rules :

(i) Express everything in terms of index.
(ii) Try partition possible from that index.
(iii) Take the best partition.

[0: 1, 1: 15, 2: 7, 3: 9, 4: 2, 5: 5, 6: 10]
↑ ind

f(3)

Base case : Try out all partitions from that index

f(0)
give me the max sum if we have the array from 0.

Base case :
Whenever we are crossing the n we will stop

```
f(ind)
{
    Base case:
    maxAns = INT_MIN;
    len = 0; maxi = INT_MIN;
    for(j = ind; j < min(n, ind+k); j++)
    {
        len++;
        maxi = max(maxi, arr[j]);
        sum = (len x max) + f(j+1);
        MaxAns = max(MaxAns, sum);
    }
    return MaxAns;
}
```

15, 15 7 5
j=1 j=2 j=3

T.C. O(exponential)
↓ Memoization
1 changing parameter
T.C. - O(N) x O(K)
S.C. - O(N) + O(N)

```
int f(int ind, vector<int> &num, int k, vector<vector<int>> &dp)
{
    if(ind == num.size())
        return 0;
    int len = 0; if(dp[ind] != -1) return dp[ind];
    int maxi = INT_MAX;
    int maxAns = INT_MIN;
    for(int j = ind; j < min(ind+k, n); j++)
    {
        len++;
        maxi = max(maxi, num[j]);
        int sum = len * maxi + f(j+1, num, k, dp);
        maxAns = max(maxAns, sum);
    }
    return dp[ind] = maxAns;
}

int maximumSubarray(vector<int> &num, int k)
{ int n = num.size();
  vector<int> dp(n, -1);
  return f(0, num, k, dp);
}
```

Tabulation :-

① Base case

if(ind == n) return 0

i.e dp[n] = 0

② Changing Variable

0 to n (this time opposite) ind : n-1 to 0

③ copy the recursion.

@Ashish Kumar Nayak

```
int maximumSubarray(vector<int> &num, int k)
{  int n = num.size();
   vector<int> dp(n+1,0);
   for(int ind = n-1; ind>=0; ind--)
      {  int len = 0;
         int maxi = INT_MIN;
         int maxAns = INT_MIN;
       for(int j= ind; j< min(ind+k, n); j++)
           {  len++;
            maxi = max(maxi, num[j]);
            int sum = len * maxi + dp[j+1];
            maxAns = max(maxAns, sum);
           }
         dp[ind] = maxAns;
      }
      return dp[0];
}
```

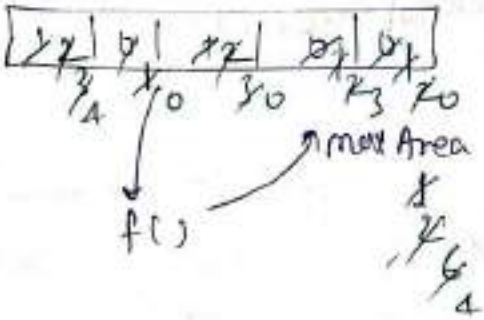
@Aashish Kumar Nayak

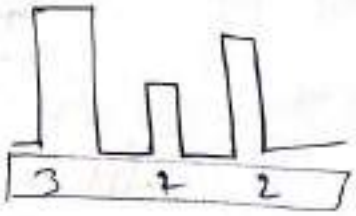
DP-55 Maximum Rectangle area with all 1's / pp on Rectangle

1	0	1	0	0
1	0	1	1	1
1	1	1	1	1
1	0	0	1	0

$2 \times 3 = 6$ (maximum)

We will solve this question using Histogram





if we find 0 then it becomes 0
else previous height will be added.

max = 6

Code :-

```
int maximalAreaOfSubmatrixOfAll(vector<vector<int>> &mat,
                                int n, int m)
{
    int maxArea = 0;
    vector<int> height(m, 0);
    for(int i=0; i<n; i++)
    {
        for(int j = 0; j<m; j++)
        {
            if(mat[i][j] == 1)
                height[j]++;
            else
                height[j] = 0;
        }
        int area = largestRectangleArea(height);
        maxArea = max(area, maxArea);
    }
    return maxArea;
}
```

@ Ashish Kumar Nayak

```
int largestRectangleArea(vector<int>& histo)
{
    stack<int> st;
    int maxA = 0;
    int n = histo.size();
    for(int i = 0; i <= n; i++)
    {
        while(!st.empty() && (i == n || histo[st.top()] >= histo[i]))
        {
            int height = histo[st.top()];
            st.pop();
            int width;
            if(st.empty())
                width = i;
            else
                width = i - st.top() - 1;
            maxA = max(maxA, width * height);
        }
        st.push(i);
    }
    return maxA;
}
```

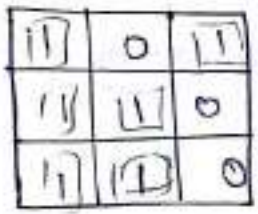
T.C. O(N X (m+n))

S.C. O(N)

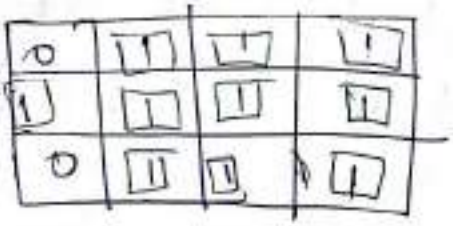
We are trying to remember the heights that why this is kept in dp.

@Ashish Kumar Nayak

DP-56 Count No. of square Submatrices

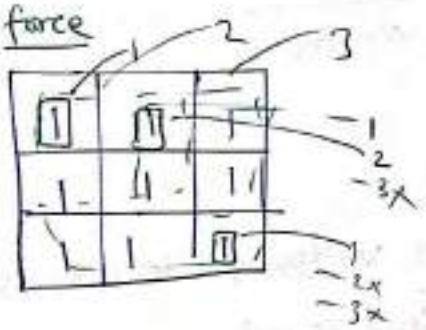


Size 1 — 6
Size 2 — 1



size 1 — 10
size 2 — 4
size 3 — 1

Brute force



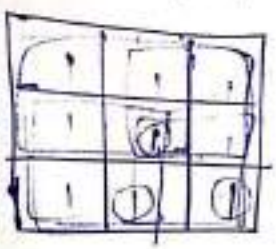
We can do better using dp.

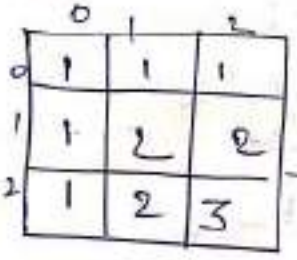
Try to create dp of similar size

dp[i][j]

How many squares end at (i,j) Right Bottom at (i,j)

Try

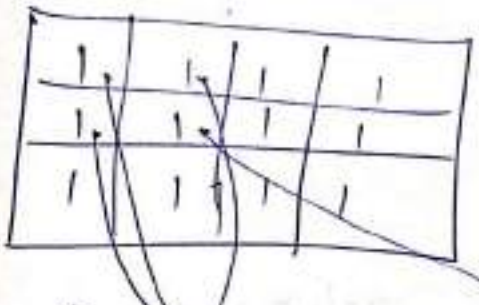




1 sq he himself
+ he is right most bottom two
= 2

sum all of them we will get our ans.

but How we will fill this array.



take minimal of three and add 1

first row and first column they themselves are sq. of size 1.

min = 2+1 = 3

if there 3 are 1 and if this is 1.
then it will form square of size 2.

Now add of of them.
and return the ans.

@Aashish Kumar Nayak

1	1	0
1	1	1
1	1	0

→

1	1	0
1	2	1
1	2	0

min:0 or 1 — Now add all of them.

min(1,1,1) = 1+1 = 2

dp[i][j] = min[dp[i-1][j], dp[i-1][j-1], dp[i][j-1]] + 1

upper. diagonal left.

Code:-

```
int countSquares( int n, int m, vector<vector<int>> &arr)
{ vector<vector<int>> dp(n, vector<int>(m,0));
   for (int j=0; j<m; j++)
       dp[0][j] = arr[0][j];
   for(int i=0 ; i<n; i++
       dp[i][0] = arr[i][0];
   for( int i=1; i<n;  i++)
     { for(int j=1; j<m; j++)
        { if(arr[i][j] == 0)
            dp[i][j] = 0;
          else
            {
            dp[i][j] = 1 + min(dp[i-1][j], min(dp[i-1][j-1],
                                               dp[i][j-1]));
            }
        }
     }
   int sum = 0;
   for(int i=0; i<n; i++) {
      for(int j=0; j<m; j++) {
        sum += dp[i][j]; } } return sum; }
```

add all of them in dp and return.