Alternative Choke –Type Plunger Design

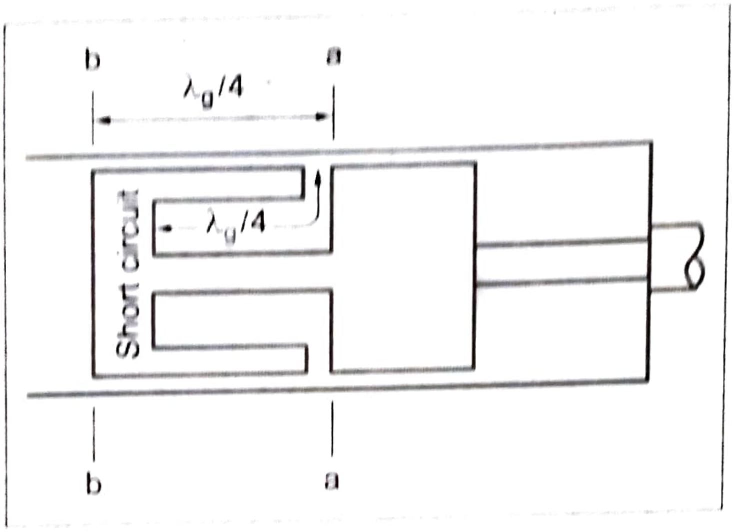


Fig.7.9. Alternative choke-type plunger design

- The alternative choke - type plunger shown in Fig.7.9, in plunger ***a two-section folded quarter-wave transformer*** is used. The inner line transforms the short-circuit impedance to an ideal open circuit at the plane *aa.* At this point, i.e., at an open –circuit or infinite impedance point, the axial current is zero.
- Next, the outer quarter –wave transformer transforms the open-circuit impedance at *aa* into a short-circuit impedance at the front end of the plunger, i.e., at plane *bb.*
- These type of short-circuit plungers give very satisfactory performance and these type of quarter-wave transformers is also used in the construction of choke joints for joining two waveguide sections together, in rotary joints, for plungers used to tune cavity resonators.

7.5. ATTENUATORS

- *An attenuators is basically a passive device which control the amount of microwave power transferred from one point to an another without causing a big distortion to its waveform on a microwave transmission system. Simply it results in decreasing the power level of a microwave signal.*
- Microwave attenuators control the flow of microwave power either reflecting it or absorbing it and it is expressed in ***decibels of relative power***.

- Attenuator which attenuates the RF signal in a waveguide system is referred as ***waveguide attenuator***. There are two main types namely ***fixed*** and ***variable***. They are achieved by insertion of resistive films.

7.5.1. FIXED ATTENUATOR

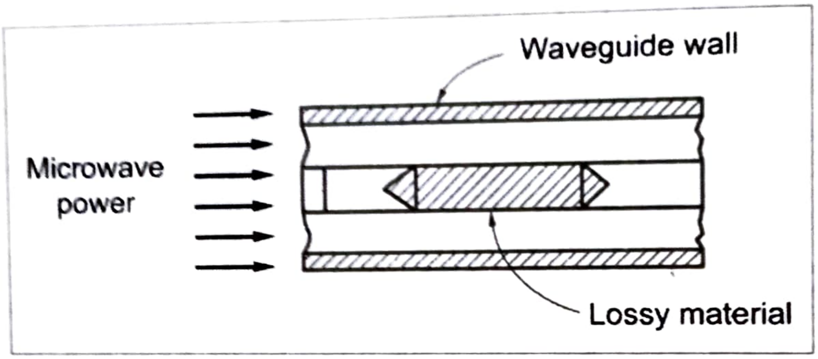


Fig.7.10. Coaxial line attenuator

- Fixed attenuators are used where ***fixed amount of attenuation*** is to be provided. If such a fixed attenuator absorbs all the energy entering into it and it is called as ***waveguide terminator***.

- This Fig.7.10 shows the coaxial line based fixed type of attenuator. Here, resistive film is fixed at the centre of conductor which absorbs the power and as a result of power loss, that is, microwave signal passes through it gets attenuated.

7.5.2. VARIABLE ATTENUATOR

- Variable attenuators provide ***continuous or step wise variable attenuation***. For rectangular waveguides, these attenuators can be flap type or vane type. For circular waveguides the rotary type is normally used.

- The most commonly used variable attenuators are,

  (i) Waveguide variable type, and

  (ii) Rotary-vane attenuator.

Waveguide Variable Attenuator:

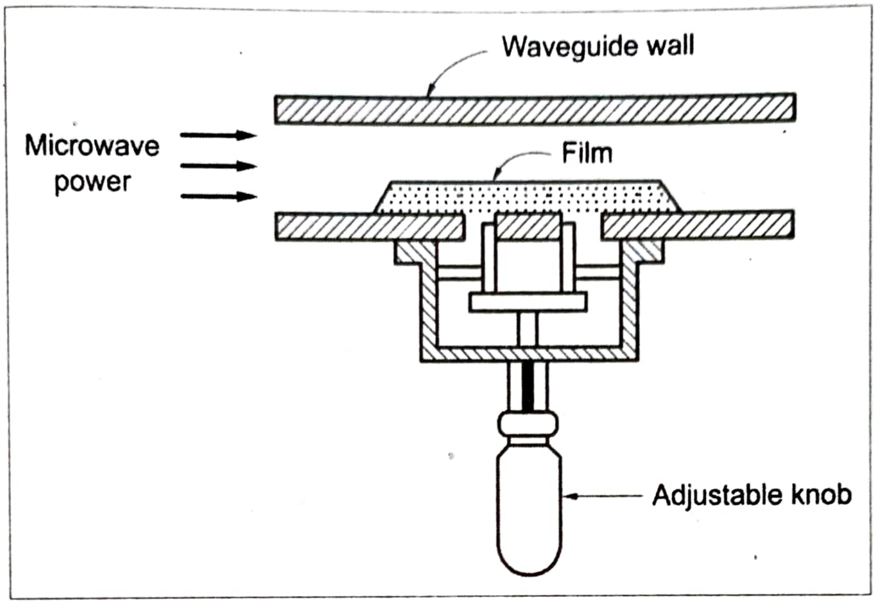


Fig.7.11. Waveguide variable attenuator

- Fig.7.11 shows waveguide variable type attenuator. Here, thin dielectric strip with coated resistive film is placed at the centre of the waveguide. Film is placed in the waveguide parallel to the maximum E field.
- Resistive vane is moved from one side of the wall to the centre by using screw where E field is considered to be maximum. This resistive film is shaped to give linear attenuation variation.

Rotary- Vane Attenuator:

- Rotary–vane attenuator is so accurate that it is used as a calibration standard in most microwave laboratories.

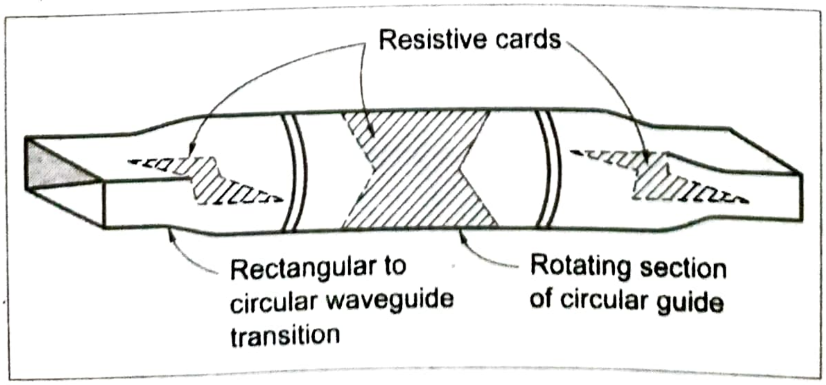


Fig.7.12. Rotary-vane attenuator

- Rotary-vane attenuator shown in Fig.7.12 is a simple form of attenuator consists of a thin tapered resistive card, whose depth of penetration into the waveguide is adjustable.
- The essential parts consists of three circular waveguide sections, two fixed and one rotatable. It also includes input and output transitions that provide low-SWR connections to standard rectangular guide.
- The attenuation is controlled by rotation of the center section. Here, the attenuation is a function of rotation angle θ_m only. The minimum loss occurring with $\theta_m = 0$ and maximum loss when $\theta_m = 90°$. The principle of operation is based upon the interaction between plane-polarized waves and thin resistive cards.

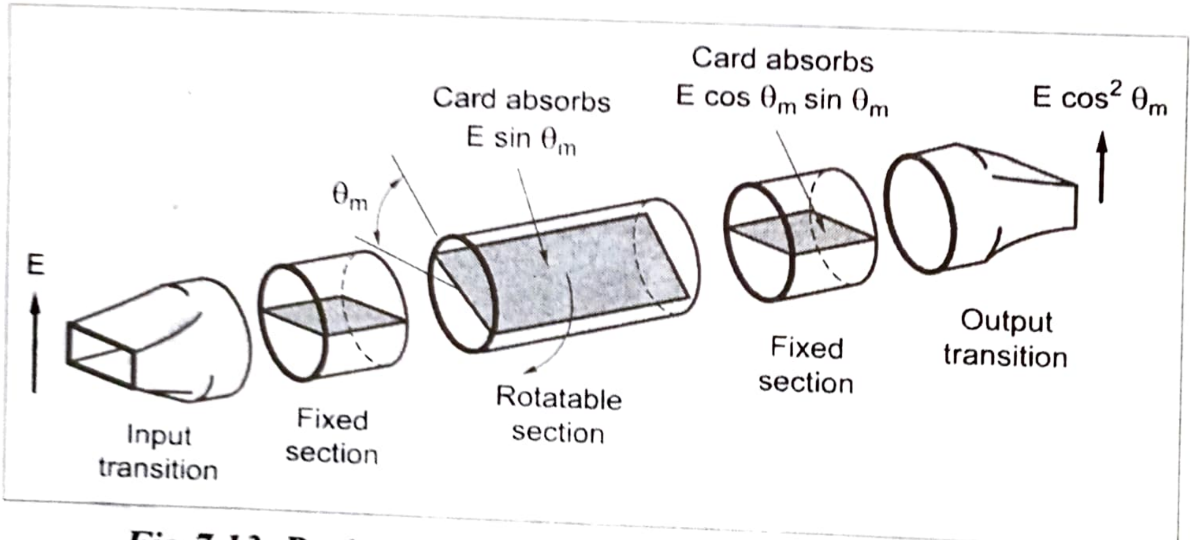


Fig.7.13. Basic construction of a rotary-vane attenuator

- The input transition shown in the Fig.7.13 converts the TE_{10} wave into a vertically polarized TE_{11} wave in circular guide. The electric field associated with the wave is denoted by E.
- In the first fixed section, the resistive card perpendicular to the electric field and the wave propagates without loss.
- When the card in the rotatable section is horizontal ($\theta_m = 0$), the wave passes through it and the output fixed section without loss. Thus for $\theta_m = 0$, the total loss is 0 dB.

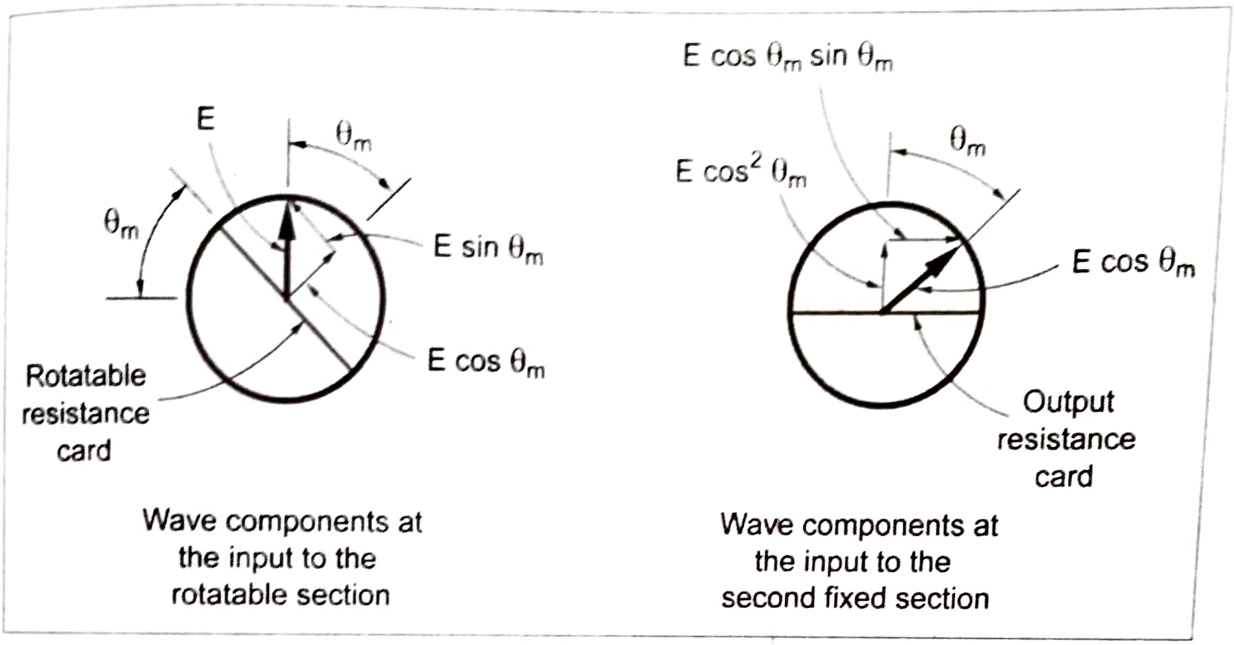


Fig.7.14. Wave components inside the rotary-vane attenuator

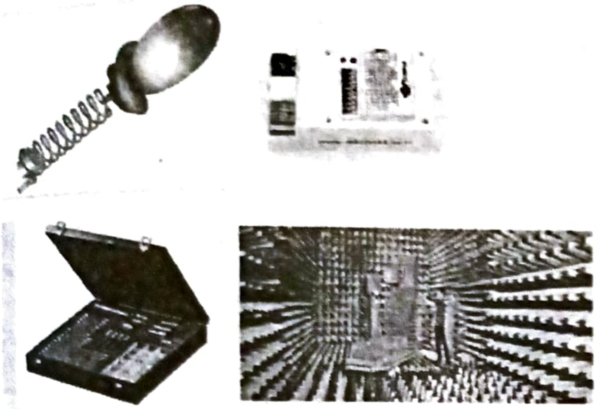
- For any other angle, the component parallel to the rotatable card (E sin θ_m) is absorbed and the perpendicular component (E cos θ_m) arrives at the second fixed section with its polarization at an angle θ_m with respect to the vertical plane.
- The portion of the wave that is parallel to the output card (E cos θ_m sin θ_m) is absorbed, while the perpendicular components (E $\cos^2 \theta_m$) proceeds to the output port via the circular –to-rectangular transition.

7.6. PHASE SHIFTERS

- *A phase shifter is an instrument that produces an adjustable change in the phase angle of the wave transmitted through it.*
- Because of its accuracy, the rotary phase shifter is used as a calibration standard in most microwave laboratories.

7.6.1. ROTARY PHASE SHIFTER

- The Fig.7.15 shows the rotary phase shifter. The essential parts of the phase shifter are three circular waveguide sections, two fixed and one rotatable. The fixed sections are quarter-wave plates, while the rotatable one is half-wave plate.



8

DIRECTIONAL COUPLERS

8.1. DIRECTIONAL COUPLERS

8.1.1. INTRODUCTION

- *A directional coupler is a four port passive device commonly used for coupling a known fraction of the microwave power to a port (coupled port) in the auxiliary line while flowing from input port to an output port in the main line. The remaining port is ideally isolated port and matched terminated.*
- *Here, portions of the forward and reverse traveling waves on a line are separately coupled to two of the other ports.*
- They can be designed to measure ***incident*** and/or ***reflected power, SWR (Standing Wave Ratio)*** values, provide a signal path to a receiver or perform other desirable operations.

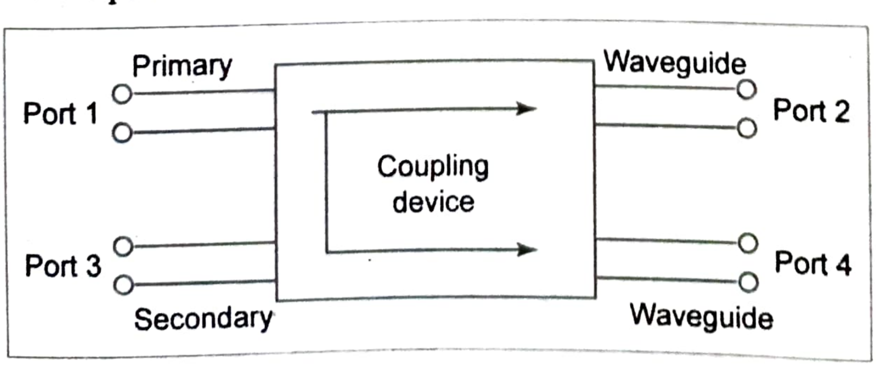


Fig.8.1. A schematic of a directional coupler

- They can be ***unidirectional*** (measuring only incident power) or ***bi – directional*** (measuring both incident and reflected) powers.

8.1.2. PROPERTIES OF A DIRECTIONAL COUPLER

- With matched terminations at all its ports, the ***properties of an ideal directional coupler*** can be summarized as follows:

  (i) *A portion of power traveling from port 1 to port 2 is coupled to port 4 but not to port 3.*

  (ii) *A portion of power traveling from port 2 to port 1 is coupled to port 3 but not to port 4.*

  (iii) *A portion of power incident on port 3 is coupled to port 2 but not to port 1 and a portion of the power incident on port 4 is coupled to port 1 but not to port 2. Also ports 1 and 3 are decoupled as ports 2 and 4.*

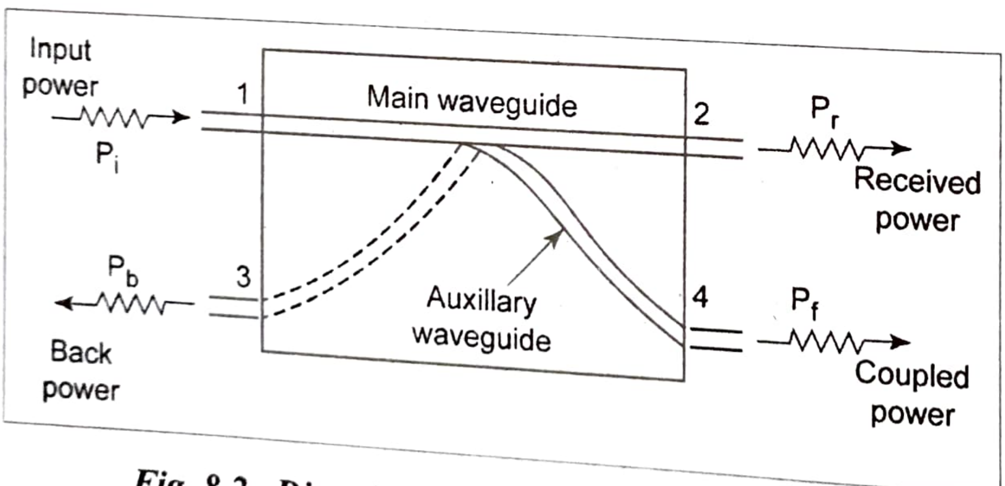


Fig. 8.2. Directional coupler indicating powers

where,

P_1 (or) P_i - Incident power at port 1,

P_2 (or) P_r - Received power at port 2,

P_4 (or) P_f - Forward coupled power at port 4, and

P_3 (or) P_b - Back power at port 3.

8.1.3. COUPLING FACTOR (C), DIRECTIVITY (D) AND ISOLATION (I)

- The performance of a directional coupler is usually described in terms of its coupling, directivity and isolation.

(i) Coupling factor (C):

The coupling factor of a directional coupler is defined as, "*the ratio of the incident power 'P_i' to the forward power 'P_f' measured in dB*".

$$\text{Coupling factor (dB)} = 10 \log_{10} \frac{P_1}{P_4}$$

$$\boxed{C\,(\text{dB}) = 10 \log_{10} \frac{P_i}{P_f}}$$

(ii) Directivity (D):

The directivity of a directional coupler is defined as, "*the ratio of forward power 'P_f' to the backward power 'P_b' expressed in dB*".

$$\text{Directivity (dB)} = 10 \log_{10} \frac{P_4}{P_3}$$

$$\boxed{D\,(\text{dB}) = 10 \log_{10} \frac{P_f}{P_b}}$$

(iii) Isolation (I):

Isolation is defined as, "*the ratio of the incident power 'P_i' to the back power 'P_b' expressed in dB*".

$$\text{Isolation (dB)} = 10 \log_{10} \frac{P_1}{P_3}$$

$$\boxed{I\,(\text{dB}) = 10 \log_{10} \frac{P_i}{P_b}}$$

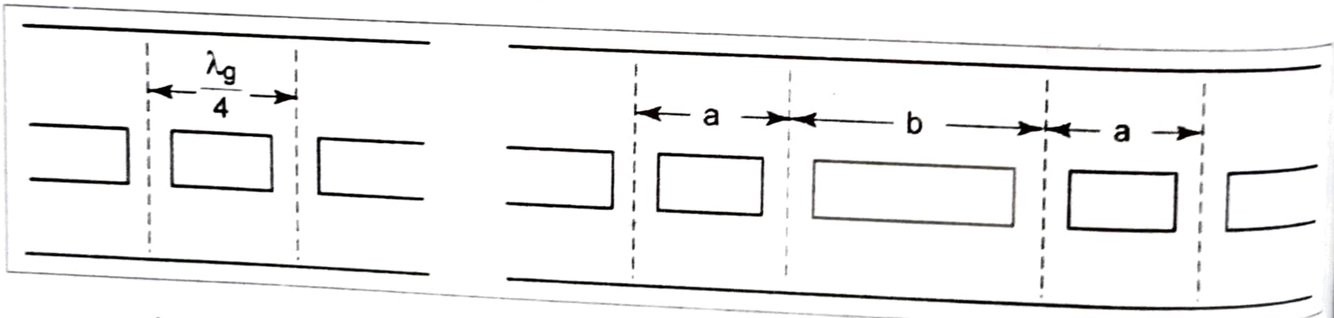
- The coupling factor *is a measure of how much of the incident power is being sampled.*

- Directivity ***is a measure of how well the directional coupler distinguishes between the forward and reverse traveling powers.***
- The term isolation is sometimes used to ***describe the directive properties*** of a coupler. Isolation (dB) equals ***coupling plus directivity.***

Isolation (I) = Coupling factor (C) + Directivity (D)

8.1.4. TYPES OF DIRECTIONAL COUPLERS

- Several types of directional couplers exists, such as a
  - ***(i) Two – hole directional coupler,***
  - ***(ii) Four – hole directional coupler,***
  - ***(iii) Reverse – coupling directional coupler*** (Schwinger coupler), and
  - ***(iv) Bethe – hole directional coupler.***
- A practical waveguide directional couplers are multi-hole couplers in which the desired coupling response Vs frequency can be achieved by a proper selection of the ***number of holes*** and ***size of the holes.***



(a) Two – hole ***(b) Four- hole***

Fig.8.3. Different types of directional couplers

8.1.4.1. TWO – HOLE DIRECTIONAL COUPLERS

- A two-hole directional coupler consists of two waveguides namely, the primary and the secondary with two tiny holes common between them.
- The number of holes can be one (as in ***Bethe cross guide coupler***) or more than two (as in a ***multi hole coupler***).

- The ***degree of coupling*** is determined by the size and location of the holes in the waveguide walls.

- The two holes are at a distance of $\frac{\lambda_g}{4}$ where , λ_g is the guide wavelength.

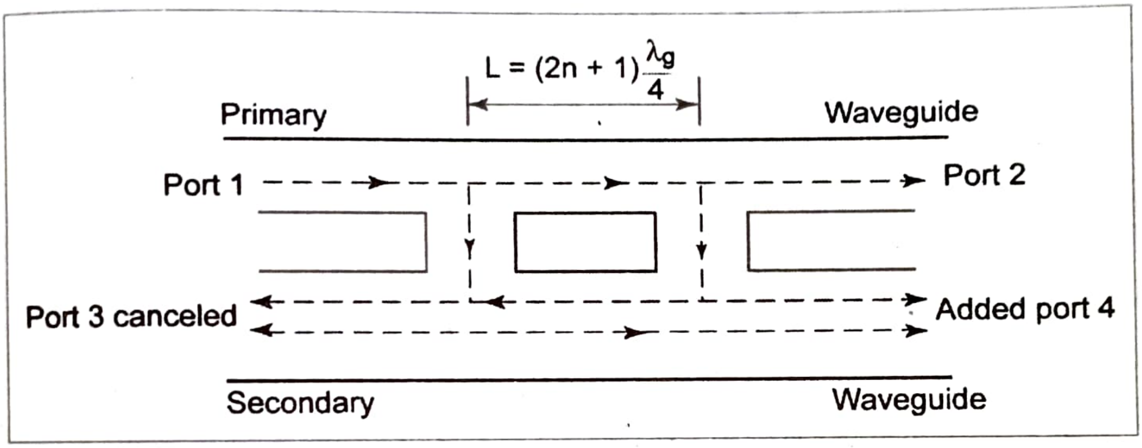


Fig.8.4. Two – hole directional coupler

- The Fig.8.4 shows the traveling waves propagation in a two – hole directional coupler. A fraction of the wave energy entered into port 1 passes through the holes and is radiated into the secondary guide as the holes act as ***slot antennas***. The spacing between the centers of two holes must be,

$$\boxed{L = (2n+1)\frac{\lambda_g}{4}}$$

where, n is any positive integer, and

λ_g is the guide wavelength.

- The forward waves in the secondary guide are in same phase, regardless of the hole space, and are added at port 4. The coupling is given by,

$$\boxed{C = -20 \log 2 \,|\, B_f \,|}$$

where, B_f – Amplitude in the forward direction

- The backward waves in the secondary guide (waves are progressing from right to left) are out of phase by 180° at the position of the 1st hole and are canceled at port 3.

8.1.5. SCATTERING MATRIX OF A DIRECTIONAL COUPLER

- Directional coupler is a ***four port network.*** Hence [S] is a 4 × 4 matrix and it is expressed as,

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix} \quad \ldots (1)$$

- In a directional coupler all ***four ports*** are ***perfectly matched*** to the junction. Hence, all the diagonal elements are zero.

$$S_{11} = S_{22} = S_{33} = S_{44} = 0 \quad \ldots (2)$$

- From symmetric property, $S_{ij} = S_{ji}$

$$\boxed{S_{12} = S_{21}, S_{23} = S_{32}, S_{13} = S_{31}, S_{24} = S_{42}, S_{34} = S_{43}, S_{41} = S_{14}} \quad \ldots (3)$$

There is ***no coupling*** between port 1 and port 3.

$$\boxed{S_{13} = S_{31} = 0} \quad \ldots (4)$$

Also there is no coupling between port 2 and port 4.

$$\boxed{S_{24} = S_{42} = 0} \quad \ldots (5)$$

- By substituting the values of scattering parameters as per equations (2) to (5) in equation (1), we get

$$[S] = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{12} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{14} & 0 & S_{34} & 0 \end{bmatrix} \quad \ldots (6)$$

- By applying an unity property of [S] matrix for equation(6), we get

$$[S]\,[S^*] = I$$

$$\begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{12} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{14} & 0 & S_{34} & 0 \end{bmatrix} \begin{bmatrix} 0 & S_{12}^* & 0 & S_{14}^* \\ S_{12}^* & 0 & S_{23}^* & 0 \\ 0 & S_{23}^* & 0 & S_{34}^* \\ S_{14}^* & 0 & S_{34}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

R_1C_1: $$|S_{12}|^2 + |S_{14}|^2 = 1 \quad \text{... (7)}$$

R_2C_2: $$|S_{12}|^2 + |S_{23}|^2 = 1 \quad \text{... (8)}$$

R_3C_3: $$|S_{23}|^2 + |S_{34}|^2 = 1 \quad \text{... (9)}$$

Then, using zero property of [S] matrix,

R_1C_3: $$S_{12}S_{23}^* + S_{14}S_{34}^* = 0 \quad \text{... (10)}$$

- By comparing equations (7) and (8), we get

$$|S_{12}|^2 + |S_{14}|^2 = |S_{12}|^2 + |S_{23}|^2$$

$$\boxed{S_{14} = S_{23}} \quad \text{... (11)}$$

- Similarly, by comparing equations (8) and (9), we get

$$|S_{12}|^2 + |S_{23}|^2 = |S_{23}|^2 + |S_{34}|^2$$

$$\boxed{S_{12} = S_{34}} \quad \text{... (12)}$$

- Let us assume that S_{12} is real and positive = 'p'

$$S_{12} = S_{34} = P = S_{34}^* \quad \text{... (13)}$$

- By substituting equations (13) and (11) in equation (10), we get

$$S_{12}S_{23}^* + S_{14}S_{34}^* = 0 \qquad \because |S_{14} = S_{23}|$$

$$p\left(S_{23} + S_{23}^*\right) = 0$$

$$S_{23} + S_{23}^* = 0$$

$$S_{23} = -S_{23}^* \quad \text{... (14)}$$

From equation (14), it is clear that S_{23} must be ***imaginary,***

$$S_{23} = jq$$

$$S_{23}^{*} = -jq$$

From equations (11) and (12),

$$\boxed{S_{12} = S_{34} = p} \quad \ldots (15a)$$

and

$$\boxed{S_{23} = S_{14} = jq} \quad \ldots (15b)$$

By substituting equations (15a) and (15b) in equation (7) we get,

$$\boxed{p^2 + q^2 = 1} \quad \ldots (16)$$

- By substituting equations (15a) and (15b) in equation (6), then [S] matrix of a directional coupler is reduced to

$$[S] = \begin{bmatrix} 0 & p & 0 & jq \\ p & 0 & jq & 0 \\ 0 & jq & 0 & p \\ jq & 0 & p & 0 \end{bmatrix} \quad \ldots (17)$$

8.2. MICROWAVE HYBRID CIRCUITS (OR) JUNCTIONS

8.2.1. MICROWAVE T- JUNCTIONS

- A T-junction is an *intersection* ***of three*** waveguides in the form of English alphabet 'T'. There are several types of Tee junctions.

***Tee** – Junction*

*In microwave circuits a waveguide or coaxial-line with three independent ports is commonly referred to as a **Tee junction.***

- *They are used to connect branch (or) section of the waveguides* ***in series*** *or* ***parallel*** *with the main waveguide transmission line for providing means of* ***splitting*** *and also of* ***combining power*** *in a waveguide system.* The ***two basic types*** are

  (i) E – plane Tee (Series), and

  (ii) H – plane Tee (Shunt).

8.2.1.2. CHARACTERISTICS OF THREE-PORT TEE JUNCTIONS

- The ***characteristics*** of a three – port tee junctions are,

  (i) A short circuit may always be placed in one of the arms of a three – port junction in such a way that no power can be transferred through the other two arms.

  (ii) If the junction is symmetric about of its arms, a short circuit can always be placed in that arm so that no reflections can occur in power transmission between the other two arms.

  (iii) It is impossible for a general three – port junction of arbitrary symmetry to present matched impedances at all three arms.

- Waveguide tees are ***poorly matched devices,*** adjustable matching can be done by tuning the screws which is placed at the centre.

8.2.2. E-PLANE TEE (SERIES TEE)

- An E – plane tee is a waveguide tee in which the axis of its ***side arm*** is ***parallel*** to the ***E – field*** of the ***main guide. Ports 1 and 2*** are the ***collinear arms*** and ***port 3*** is the ***E – arm (side arm).***

- A rectangular slot is cut along the ***broader dimension*** of a long waveguide and a side arm is attached as shown in Fig.8.5.

- When the waves are fed into the side arm (port 3), the waves appearing at port 1 and port 2 of the collinear arm will be in opposite phase and in the same magnitude.

$$S_{13} = -S_{23} \qquad \ldots (1)$$

- In general, the power out port 3 (side or E arm) is proportional to the difference between instantaneous powers entering from ports 1 and 2.

Difference Arm:

If two input waves are fed into port 1 and port 2 of the collinear arm, the output wave at port 3 will be opposite in phase and subtractive. Sometimes, this third port is called the difference arm.

8.2.2.1. SCATTERING MATRIX FOR E-PLANE TEE

Scattering matrix for E – plane tee can be derived as follows:

- For E-plane tee, [S] is a ***3 × 3*** matrix since there are 3 ports,

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \quad \ldots (1)$$

The scattering coefficient due to difference arms are,

$$S_{23} = -S_{13} \quad \ldots (2)$$

- The equation (2) represents that the outputs at ports 1 and 2 are ***out of phase*** by ***180°*** with an input at port 3.

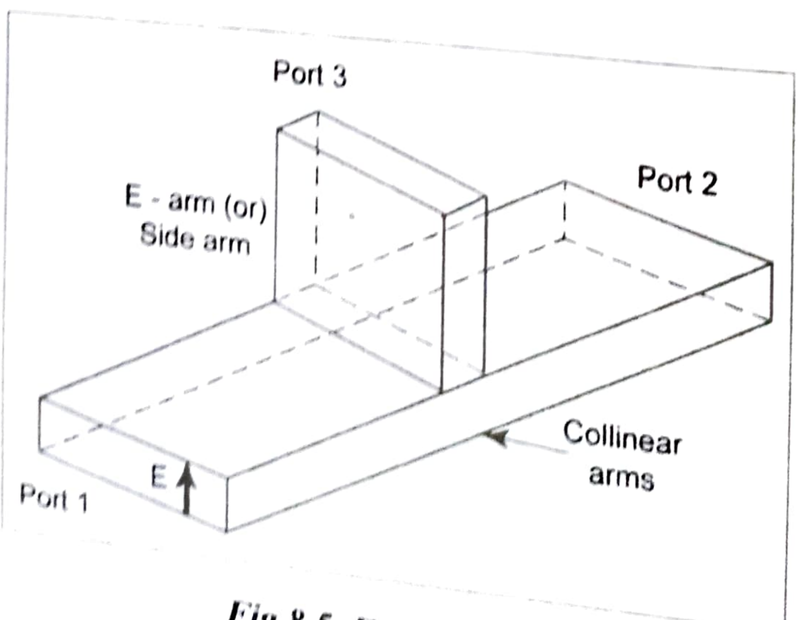


Fig.8.5. E-plane Tee

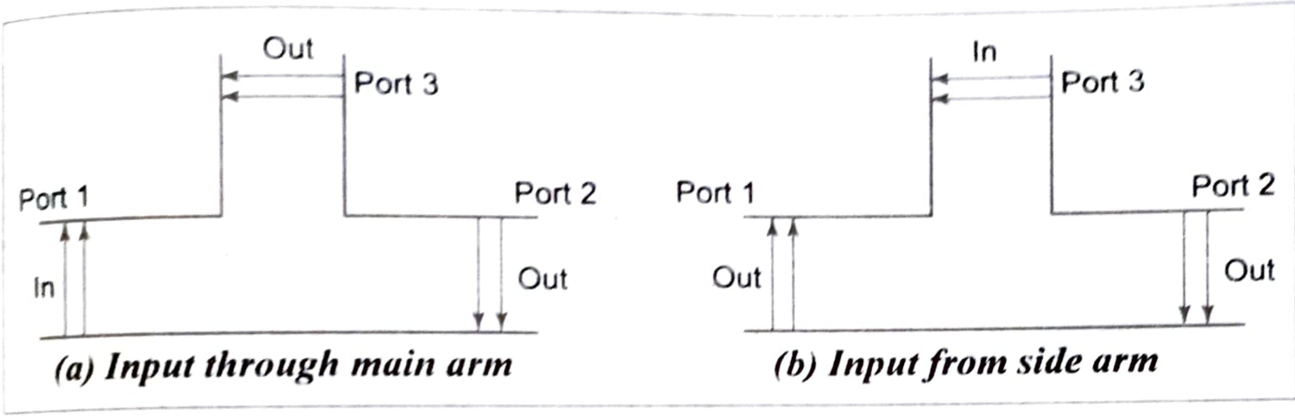


Fig.8.6. Two-way transmission of E-plane Tee

- If port 3 is perfectly matched to the junction,

$$\boxed{S_{33} = 0} \quad \ldots (3)$$

Using symmetric property, $S_{ij} = S_{ji}$

$$\boxed{\begin{aligned} S_{12} &= S_{21} \\ S_{13} &= S_{31} \\ S_{23} &= S_{32} \end{aligned}} \quad \ldots (4)$$

- By substituting equations (3) and (4) in equation (1) and using equation (2) also, we get

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & -S_{13} \\ S_{13} & -S_{13} & 0 \end{bmatrix} \quad \ldots (5)$$

- Apply unity property of [S] matrix for equation(5), we get

$$[S] \cdot [S]^* = [I]$$

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & -S_{13} \\ S_{13} & -S_{13} & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* \\ S_{12}^* & S_{22}^* & -S_{13}^* \\ S_{13}^* & -S_{13}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_1C_1: \quad |S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = 1 \qquad \ldots (6)$$

$$R_2C_2: \quad |S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 = 1 \qquad \ldots (7)$$

$$R_3C_3: \quad |S_{13}|^2 + |S_{13}|^2 = 1 \qquad \ldots (8)$$

- Using zero property of [S] matrix, we get

$$R_3C_1: \quad S_{13}.S_{11}^* - S_{13}S_{12}^* = 0 \qquad \ldots (9)$$

By equating equations (6) and (7), we get

$$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = |S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2$$

$$\boxed{S_{11} = S_{22}} \qquad \ldots (10)$$

From Equation (8), we get the value of S_{13} as

$$2|S_{13}|^2 = 1$$

$$|S_{13}|^2 = \frac{1}{2}$$

$$\boxed{|S_{13}| = \frac{1}{\sqrt{2}}} \qquad \ldots (11)$$

From equation (9),

$$S_{13}\left(S_{11}^* - S_{12}^*\right) = 0$$

$$S_{11}^* - S_{12}^* = 0$$

$$\boxed{S_{11} = S_{12} = S_{22}} \qquad \ldots (12)$$

- By using equations (10), (11) and (12) in equation (6), we get

$$|S_{11}|^2 + |S_{11}|^2 + \frac{1}{2} = 1$$

$$2|S_{11}|^2 = \frac{1}{2}$$

$$\boxed{S_{11} = \frac{1}{2}} \qquad \ldots (13)$$

- By substituting the values from equations(11) ,(12) and (13) in equation (5), we get

$$[S] = \begin{bmatrix} 1/2 & 1/2 & 1/\sqrt{2} \\ 1/2 & 1/2 & -1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 \end{bmatrix} \quad \ldots (14)$$

Thus the equation (14) represents the scattering matrix for E-plane tee.

8.2.3. H-PLANE TEE (SHUNT TEE)

- A H – plane tee junction is formed by cutting a rectangular slot along the width of a main waveguide and attaching another waveguide the side arm called the ***H-arm***.
- The port 1 and port 2 of the main waveguide are called ***collinear ports*** and port 3 is the ***H-arm*** (or) ***Side arm.***
- An H – plane tee is a waveguide tee in which the axis of its side arm is "shunting" the E – field (or) parallel to the H – field of the main guide.

Sum Arm:

In a H – plane tee if two input waves are fed into port 1 and port 2 of the collinear arm, the output wave at port 3 will be ***in phase*** *and* ***additive.*** *Hence, the third port is called the* ***sum arm.***

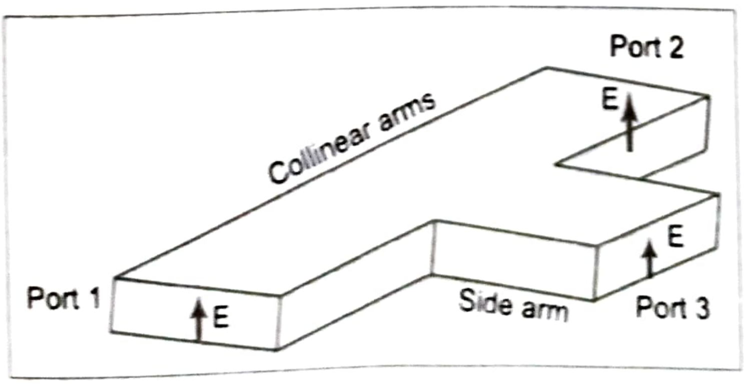


Fig.8.7. H – plane Tee

8.2.3.1. SCATTERING MATRIX FOR H-PLANE TEE

Scattering matrix for H – plane tee can be derived as follows:

- For E-plane tee, [S] is a *3 × 3* matrix since there are 3 ports.

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \quad \ldots (1)$$

- If the input is fed into the port 3, then the wave will split equally into port 1 and port 2 in phase and in the same magnitude. Here, the scattering coefficients S_{13} and S_{23} must be equal.

$$\boxed{S_{13} = S_{23}} \quad \ldots (2)$$

- From the symmetric property, $S_{ij} = S_{ji}$

$$\boxed{\begin{aligned} S_{12} &= S_{21} \\ S_{23} &= S_{32} = S_{13} \\ S_{13} &= S_{31} \end{aligned}} \quad \ldots (3)$$

Since port 3 is perfectly matched to the junction.

$$\boxed{S_{33} = 0} \quad \ldots (4)$$

- With the properties from equations(3) and(4), the [S] matrix of equation (1) becomes,

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{13} \\ S_{13} & S_{13} & 0 \end{bmatrix} \quad \ldots (5)$$

- From the unitary property,

$$[S] \cdot [S]^* = [I]$$

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{13} \\ S_{13} & S_{13} & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* \\ S_{12}^* & S_{22}^* & S_{13}^* \\ S_{13}^* & S_{13}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

R_1C_1: $$S_{11} S_{11}^* + S_{12} S_{12}^* + S_{13} S_{13}^* = 1$$

$$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = 1 \quad \ldots (6)$$

Similarly,

R_2C_2: $$|S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 = 1 \quad \ldots (7)$$

R_3C_3: $$|S_{13}|^2 + |S_{13}|^2 = 1 \quad \ldots (8)$$

- Using zero property of [S] matrix, we get

R_3C_1: $$S_{13} . S_{11}^* + S_{13} S_{12}^* = 0 \quad \ldots (9)$$

From equation (8),

$$2 \, |S_{13}|^2 = 1$$

$$|S_{13}|^2 = \frac{1}{2}$$

$$\boxed{S_{13} = \frac{1}{\sqrt{2}}} \quad \ldots (10)$$

- Comparing equations (6) and (7), we get

$$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = |S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2$$

$$|S_{11}|^2 = |S_{22}|^2$$

$$\boxed{S_{11} = S_{22}} \quad \ldots (11)$$

- From equation (9),

$$S_{13} \left(S_{11}^* + S_{12}^* \right) = 0, \qquad \text{since } S_{13} \neq 0,$$

$$S_{11}^{*} + S_{12}^{*} = 0$$

$$S_{11}^{*} = -S_{12}^{*} \quad \dots (12)$$

- Using above value in equation (6), we get

$$|S_{11}|^2 + |S_{11}|^2 + \frac{1}{2} = 1$$

$$2|S_{11}|^2 = \frac{1}{2}$$

$$\boxed{S_{11} = \frac{1}{2}} \quad \dots (13)$$

- From equations (11), (12) and (13),

$$\boxed{\begin{aligned} S_{12} &= -\frac{1}{2} \\ S_{22} &= \frac{1}{2} \end{aligned}} \quad \dots (14)$$

- By substituting values of S_{13}, S_{11}, S_{12} and S_{22} from equations (10),(13) and (14) in equation (5) we get,

$$[S] = \begin{bmatrix} 1/2 & -1/2 & 1/\sqrt{2} \\ -1/2 & 1/2 & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix} \quad \dots (15)$$

Thus, the equation (15) represents the scattering matrix for H-plane tee.

8.2.4. MAGIC TEES (HYBRID OR E-H PLANE TEES OR MAGIC-T)

- *A hybrid junction is a four – port network in which a signal incident on any one of the port divides between two output ports with the remaining port being isolated.*

8.2.4.1. INTRODUCTION

- Here, rectangular slots are cut both along the width and breadth of a long waveguide and side arms are attached as shown in Fig.8.8.
- *A magic tee is a* ***combination of the E – plane tee*** *and* ***H – plane tee.*** Ports 1 and 2 are collinear arms, port 3 is the H – arm, and port 4 is the E – arm.

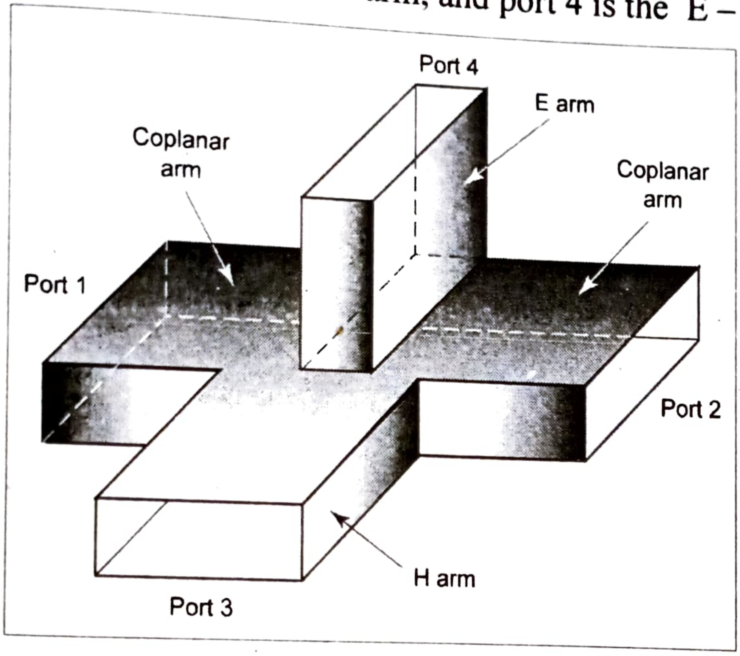


Fig.8.8. Magic Tee

8.2.4.2. CHARACTERISTICS OF MAGIC TEE

- The magic – T has the following ***characteristics*** when all the ports are terminated with matched load.

(i) If two in phase waves of equal magnitude are fed into ports 1 and 2, the output at port 4 is ***subtractive*** and hence zero and the total output will appear additively at port 3. Hence, port 4 is called the ***difference (or) E – arm*** and port 3 is the ***sum (or) H – arm.***

(ii) A wave incident at port 4 (E – arm) divides equally between ports 1 and 2 but opposite in phase with **no coupling** to port 3 (H – arm).

(iii) A wave incident at port 3 (H – arm) divides equally between ports 1 and 2 are in phase with no coupling to port 4 (E – arm).

$$\boxed{S_{43} = S_{34} = 0} \quad \ldots (1)$$

(iv) A wave fed into one collinear port 1 or 2 will not appear in the other collinear port 2 or 1. Hence, two collinear ports 1 and 2 are isolated from each other.

$$\boxed{S_{12} = S_{21} = 0} \quad \dots (2)$$

- A magic – T can be matched by putting screws suitably in the E and H arms without destroying the symmetry of the junction. For an ideal, lossless magic – T matched at ports 3 and 4.

$$\boxed{S_{33} = S_{44} = 0} \quad \dots (3)$$

8.2.4.3. S-MATRIX FOR MAGIC TEE

Using the properties of E – H plane Tee, its scattering matrix can be obtained as follows:

- [S] is a ***4 × 4 matrix*** since there are 4 ports.

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix} \quad \dots (4)$$

- From symmetric property, $S_{ij} = S_{ji}$

$$\boxed{S_{12} = S_{21},\ S_{13} = S_{31},\ S_{14} = S_{41},\ S_{23} = S_{32},\ S_{24} = S_{42},\ S_{34} = S_{43}} \quad \dots (5)$$

Port 3 has H-plane tee section,

$$S_{23} = S_{13} \quad \dots (6)$$

Similarly, port 4 has E-plane tee section

$$S_{24} = -S_{14} \quad \dots (7)$$

- By substituting equations (1), (3), (5),(6) and (7) in equation (4), the S – matrix for a magic – T matched at ports 3 and 4 is given by,

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{13} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix} \quad \dots (8)$$

- Using unitary property on equation(8),

$$[S].[S]^* = [I]$$

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{13} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* & S_{14}^* \\ S_{12}^* & S_{22}^* & S_{13}^* & -S_{14}^* \\ S_{13}^* & S_{13}^* & 0 & 0 \\ S_{14}^* & -S_{14}^* & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

R_1C_1: $$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1 \quad \dots (9)$$

R_2C_2: $$|S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1 \quad \dots (10)$$

R_3C_3: $$|S_{13}|^2 + |S_{13}|^2 = 1 \quad \dots (11)$$

R_4C_4: $$|S_{14}|^2 + |S_{14}|^2 = 1 \quad \dots (12)$$

- By equating equations (9) and (10), we get

$$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 + |S_{14}|^2 = |S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 + |S_{14}|^2$$

$$\boxed{|S_{11}| = |S_{22}|} \quad \dots (13)$$

- From equation (11),

$$|S_{13}|^2 + |S_{13}|^2 = 1$$

$$2|S_{13}|^2 = 1$$

$$|S_{13}|^2 = \frac{1}{2}$$

$$\boxed{|S_{13}| = \frac{1}{\sqrt{2}}} \quad \dots (14)$$

- From equation (12),

$$|S_{14}|^2 + |S_{14}|^2 = 1$$

$$2|S_{14}|^2 = 1$$

$$\boxed{|S_{14}| = \frac{1}{\sqrt{2}}} \quad \text{... (15)}$$

- By substituting equations (14) and (15) in equation (9), we get

$$|S_{11}|^2 + |S_{12}|^2 + \frac{1}{2} + \frac{1}{2} = 1$$

$$|S_{11}|^2 + |S_{12}|^2 = 0$$

which is valid if,

$$\boxed{S_{11} = S_{12} = 0} \quad \text{... (16)}$$

- From equations (13) and (16), we get

$$\boxed{S_{22} = 0} \quad \text{... (17)}$$

- The [S] of magic tee is obtained by substituting the scattering parameters from equations (16) and (17) in equation (8), we get

$$[S] = \begin{bmatrix} 0 & 0 & S_{13} & S_{14} \\ 0 & 0 & S_{13} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix} \quad \text{... (18)}$$

- Using the equations(14) and (15), the scattering matrix for an ideal hybrid tee may be stated in [S]- matrix in the following form as,

$$\boxed{[S] = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \\ 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix}} \quad \text{... (19)}$$

where,

$$|S_{13}| = \frac{1}{\sqrt{2}} = |S_{14}|$$

- Hence, in any four ports junction, if any two ports are perfectly matched to the junction, then the remaining two ports are automatically matched to the junction. Such a junction where in all the four ports are perfectly matched to the junction is called a ***magic Tee.***

Applications of Magic Tee:

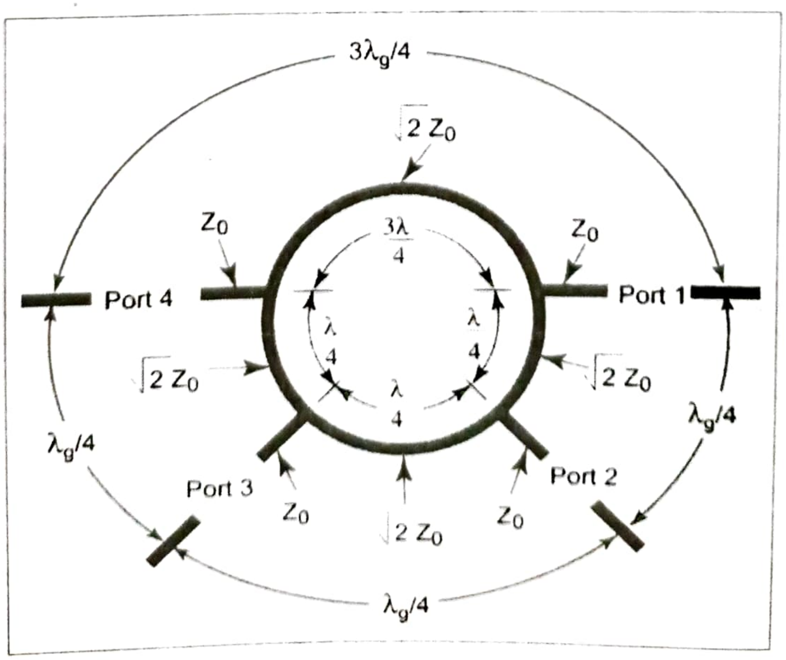
A magic tee has several applications as,

(i) Measurement of impedance,
(ii) As duplexer,
(iii) As mixer, and
(iv) As an isolator.

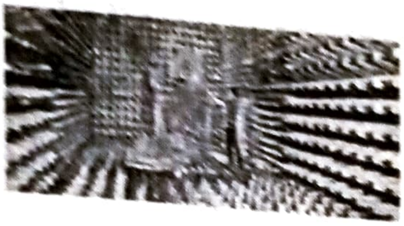
8.2.5. HYBRID RINGS (RAT-RACE CIRCUITS)

8.2.5.1. INTRODUCTION

- One of the first hybrid junctions developed for microwave use was the hybrid ring, sometimes referred to as the ***rat race.***
- It is a four port junction, which is obtained by adding a fourth port to the normal three port tee junction.





11

GUNN DIODE OSCILLATOR

11.1. INTRODUCTION

- Transferred electron oscillator or Gunn diode oscillator make use of two terminal devices based on the phenomenon known as ***"transferred electron effect"***.

Transferred Electron Effect:

Some materials like gallium arsenide (GaAs) exhibit a negative differential mobility (i.e., a decrease in the carrier velocity with an increase in the electric field) when biased above a threshold value of the electric field. The electrons in the lower – energy band will be transferred into the higher – energy band. The behaviour is called ***transferred electron effect or Gunn effect*** *and the device is also called as* ***Transferred Electron Device (TED) or transferred electron oscillator or Gunn diode or Gunn oscillator.***

- Thus the material behave as negative resistance device over a range of applied voltages and can be used in microwave oscillators.
- In the high – energy band, the effective electron mass is larger and hence the electron mobility is lower than low – energy band.

- The conductivity is ***directly proportional*** to the mobility and hence the current decreases with an increase in electric field strength.

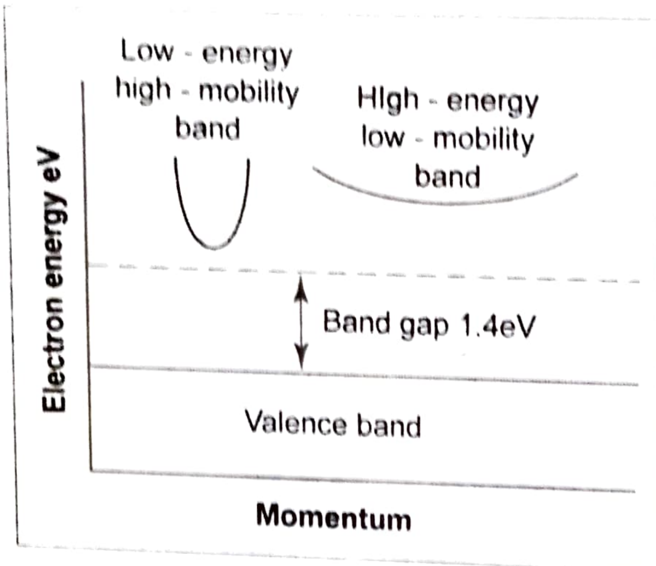


Fig.11.1. Energy conduction band for a Gunn material such as GaAs

Applications of Gunn Diode:

The various applications of Gunn diodes are:

(i) *Gunn diodes are negative resistance devices which are normally used as* ***low power oscillator*** *at microwave frequencies in transmitter and also as* ***local oscillator*** *in receiver front ends.*

(ii) *Used in parametric amplifiers as pump source.*

(iii) *Used in radar transmitters (Police radar, CW Doppler radar).*

(iv) *In broadband microwave amplifiers.*

(v) *Pulsed Gunn diode oscillators used in transponders for air traffic control (ATC) and in industry telemetry systems.*

(vi) *Fast combinational and sequential logic circuits.*

(vii) *Low and medium power oscillator in microwave receivers.*

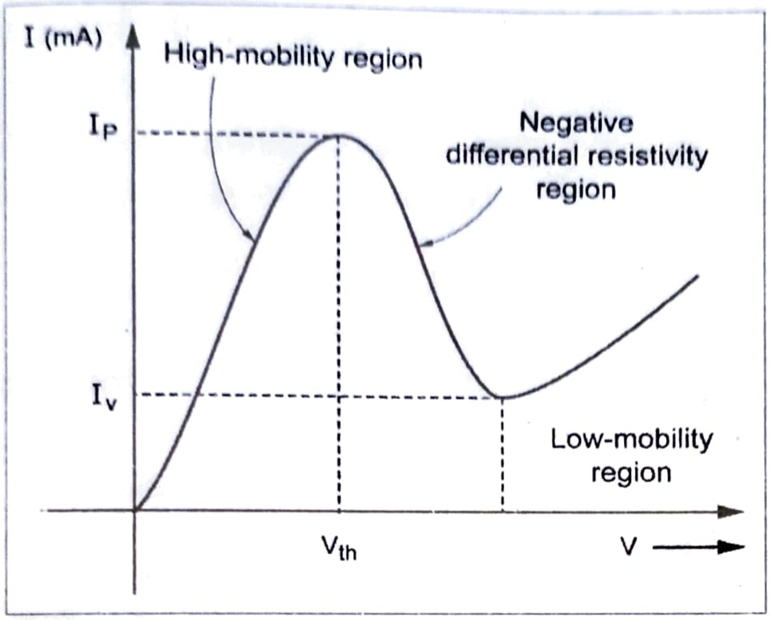


Fig.11.2. Current – voltage characteristics for a Gunn device.

- TEDs are fabricated from compound semiconductors, such as gallium arsenide (GaAs), indium phosphide (InP), or Cadmium telluride (CdTe).
- The ***positive resistances absorb power*** (passive devices), whereas ***negative resistances generate power*** (active devices).

Features of TEDs

The important features of TEDs are,

(i) TEDs are bulk devices without junctions,

(ii) TEDs are operate with hot electrons having more thermal energy, and

(iii) TEDs are tunable over a wide frequency range with low a noise characteristics.

Difference between Microwave Transistors and TEDs

The main difference between the microwave transistors and TEDs are as follows:

(i) TEDs are bulk devices having ***no junction or gates*** as compared to microwave transistors which operate with either junction or gates.

(ii) The majority of transistors are fabricated from elemental semiconductors, such as ***silicon*** or ***germanium***, whereas TEDs are fabricated from compound semiconductors, such as ***gallium arsenide (GaAs), indium phosphide (InP), or a Cadmium telluride (CdTe).***

(iii) TEDs operate with ***hot electrons*** whose energy is very much greater than the thermal energy. Transistors operate with ***warm electrons*** whose energy is not much greater than their thermal energy (0.026eV at room temperature).

- Because of these fundamental difference in the theory and technology the transistors cannot be applied to TEDs.

11.1.1. WORKING PRINCIPLE

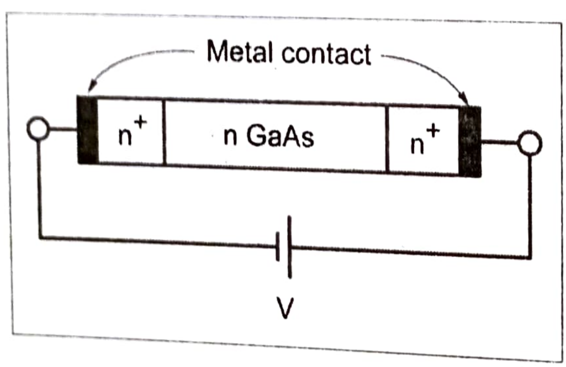


Fig.11.3. A simple Gunn Oscillator

- The basic structure of a Gunn diode is shown in Fig.11.3 which consists of n – type GaAs semiconductor with regions of high doping (n^+).
- Eventhough there is no junction this is called a diode with reference to the positive end (anode) and negative end (cathode) of the dc voltage (V) applied across the device.
- If a dc (or) diode voltage (or) an electric field at low level is applied to the GaAs, an electric field is established across it. Initially, the current will increase with a rise in the voltage.
- At low E-field in the material, most of the electrons will be located in the lower energy band.

- When the diode voltage exceeds a certain threshold value (V_{th}), a high electric field (3.2 kV/m for GaAs) is produced across the active regions and ***electrons*** are excited from their ***initial lower valley*** to the ***higher valley*** where they become virtually immobile.
- If the rate at which electrons are transferred is very high, the current will decrease with an increase in voltage, resulting in an equivalent ***negative resistance effect***.

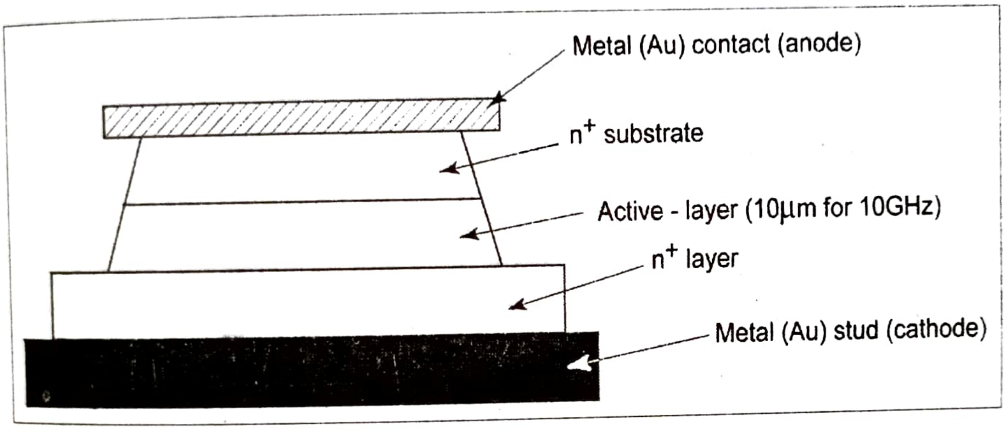


Fig.11.4. Basic construction of Gunn diode

Negative Resistance:

The carrier drift velocity is linearly increased from zero to a maximum when the electric field is varied from zero to a threshold value. When the electric field is beyond the threshold value of 3000V/cm, the drift velocity is decreased and the diode exhibits negative resistance.

- The electrical equivalent circuit of Gunn diode is shown in Fig.11.5.

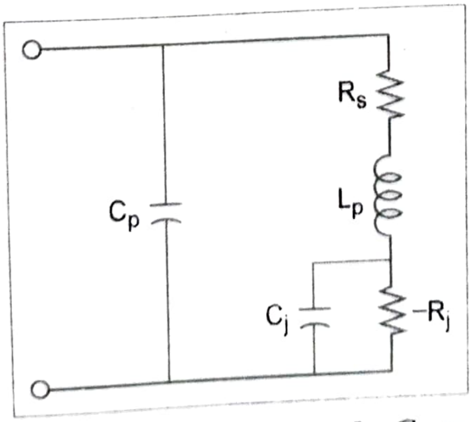


Fig.11.5. Equivalent circuit of a Gunn diode

where,

C_j – Diode capacitance,

$-R_j$ – Diode resistance,

R_s – Total resistance of leads, ohmic contact, bulk resistance of diode,

L_p – Package inductance, and

C_p – Package capacitance.

- GaAs is a poor conductor, considerable heat is generated in the diode. The diode should be well bonded into a heat sink. The negative resistance has a value that typically lies in the range -5 to -20 ohm.

11.2. RIDLEY-WATKINS-HILSUM (RWH) THEORY

11.2.1. DIFFERENTIAL NEGATIVE RESISTANCE

- The fundamental concept of the RWH theory is the ***differential negative resistance*** developed in a ***bulk solid – state III – V compound,*** when either a voltage (or electric field) or a current is applied to the terminals of the sample.
- Two modes are available in the negative resistance devices:
  - (i) Voltage controlled negative-resistance mode, and
  - (ii) Current controlled negative-resistance mode.

(i) Voltage Controlled Mode:

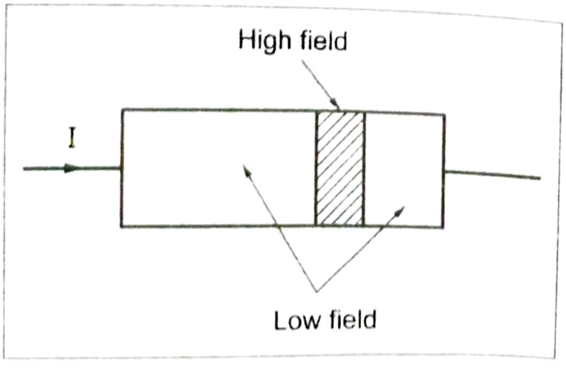


Fig.11.6. High – field domain

The current density can be multivalued. ***High field domains*** are formed by separating a two low field regions.

(ii) Current controlled mode:

The voltage value can be ***multivalued***. The mode splitting the sample results in high – current filaments running along the field direction.

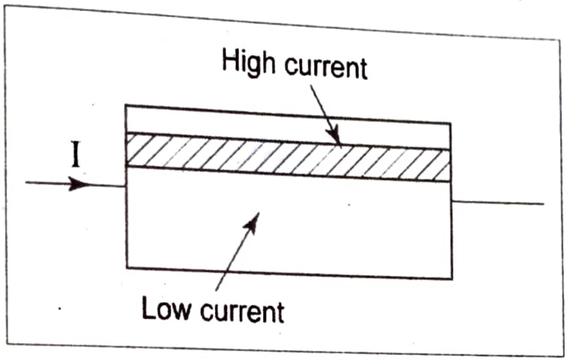
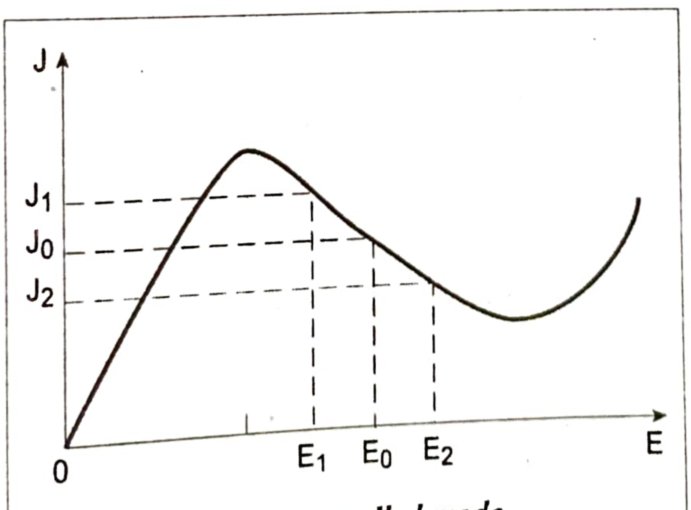


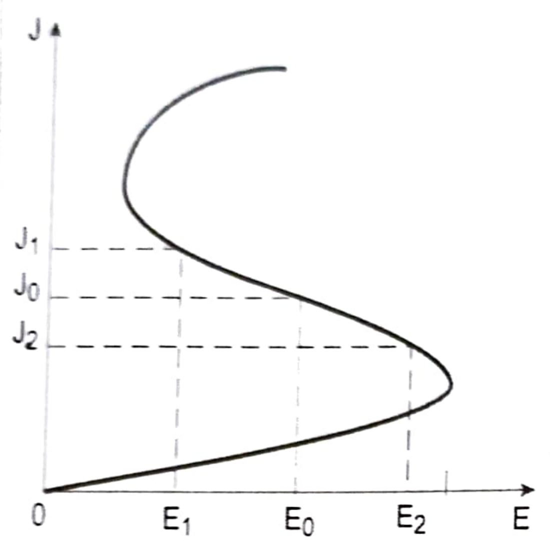
Fig.11.7. High-current filament

The ***negative resistance*** of the sample at a particular region is,

$$\text{Negative resistance} = \frac{dI}{dV} = \frac{dJ}{dE} \qquad \dots (1)$$



(a) Voltage - controlled mode



(b) Current - controlled mode

Fig.11.8. Multiple values of current density for negative resistance.

Explanation:

If an electric field E_0 (or voltage V_0) is applied to the sample, then the current density J_0 is generated. When the applied field is increased to E_2, then the current density is decreased to J_2. When the field is decreased to E_1, the current density is increased to J_1.

11.2.2. TWO-VALLEY MODAL THEORY

- According to the energy band theory of the n – type GaAs, a high – mobility lower valley is separated by energy of ***0.36eV*** from a low – mobility upper valley.

Data for two valleys in GaAs

Valley	Effective Mass M_e	Mobility μ	Separation ΔE
Lower	$M_{el} = 0.068$	$\mu_l = 8000 cm^2 / V\text{-}sec$	$\Delta E = 0.36eV$
Upper	$M_{eu} = 1.2$	$\mu_u = 180\ cm^2 / V\text{-}sec$	$\Delta E = 0.36eV$

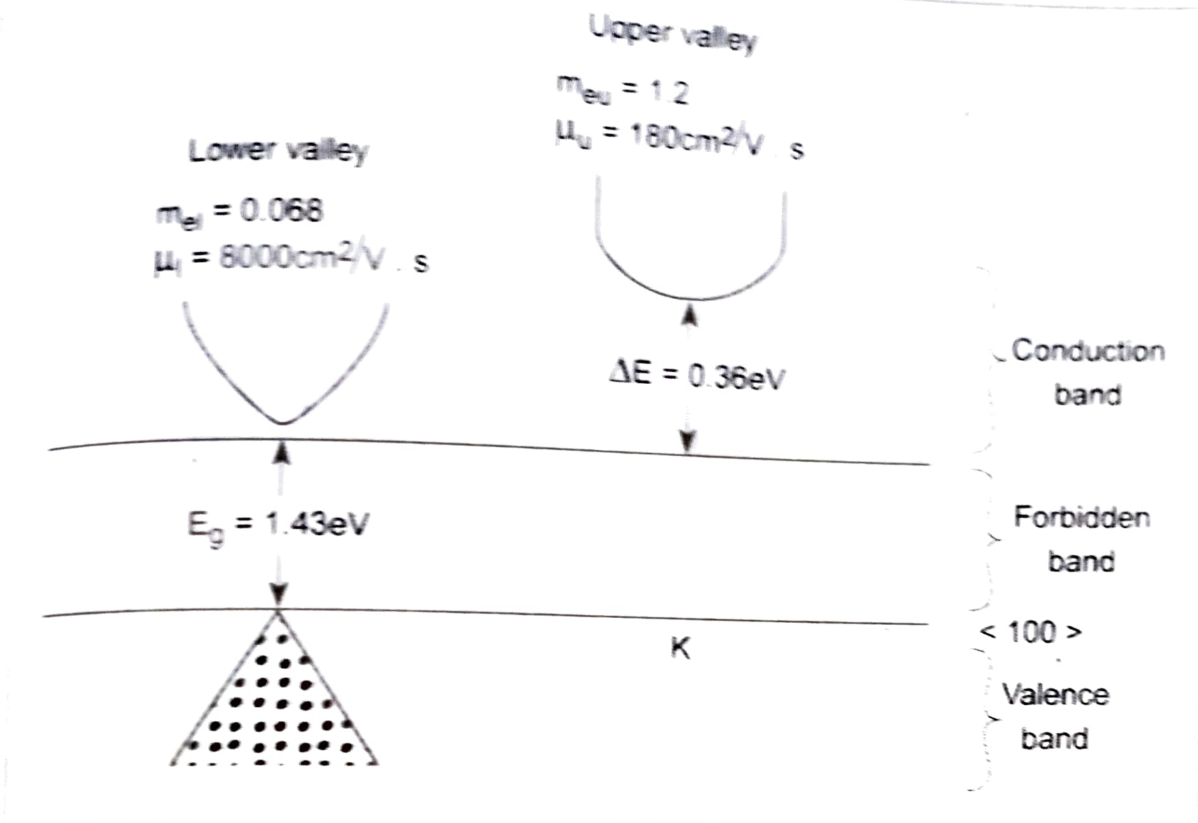


Fig.11.9. Two – valley model for n – type GaAs.

Transfer of Electron Densities:

- Electron densities in the lower and upper valleys remain the same under an equilibrium condition.

(i) When the applied electric field is ***lower than*** the electric field of the lower valley ***($E < E_l$). No electrons*** will ***transfer*** to the ***upper valley*** as shown in Fig.11.10(a).

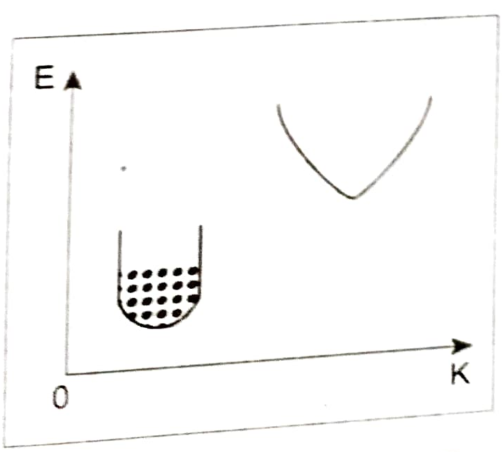


Fig.11.10(a). $E < E_l$

(ii) When the applied electric field is higher than that of the lower valley and lower than that of the upper valley ***($E_l < E < E_u$)*. *Electrons*** will begin to ***transfer*** to the ***upper valley*** as shown in Fig.11.10(b).

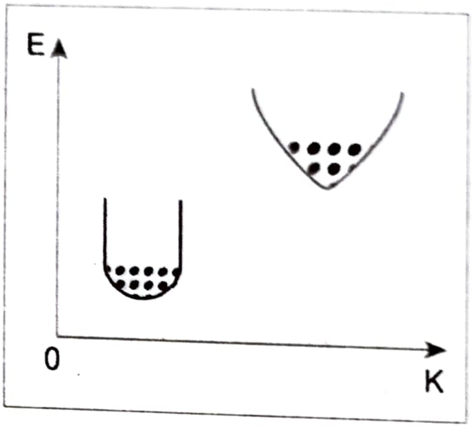


Fig.11.10(b). $E_l < E < E_u$

(iii) When the applied electric field is higher than that of the upper valley ***($E_u < E$), all electrons*** will ***transfer*** to the ***upper valley*** as shown in Fig.11.10(c).

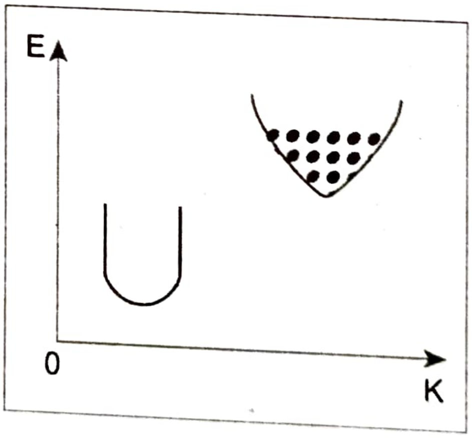


Fig.11.10(c). $E_u < E$

- If an electron densities in the lower and upper valleys are n_l *and* n_u, the ***conductivity*** of the n – type GaAs is expressed as,

$$\boxed{\sigma = e(\mu_l n_l + \mu_n n_n)} \quad \ldots (2)$$

where, e - Electron charge,

μ - Electron mobility, and

n = $n_l + n_u$ is the electron density.

- The ***electron density 'n'*** and ***mobility 'μ'*** are both ***functions*** of an ***electric field E***. Differentiate conductivity (σ)with respect to E as,

$$\frac{d\sigma}{dE} = e\left(\mu_l \frac{dn_l}{dE} + \mu_u \frac{dn_u}{dE}\right) + e\left(n_l \frac{d\mu_l}{dE} + n_u \frac{d\mu_u}{dE}\right) \qquad \dots (3)$$

If the ***total electron density*** is given by,

$$n = n_l + n_u \qquad \dots (4)$$

It is assumed that μ_l ***and*** μ_n are ***proportional*** to E^p, where p is a constant, then

$$\frac{d}{dE}(n_l + n_u) = \frac{dn}{dE} = 0 \qquad \dots (5)$$

$$\frac{dn_l}{dE} = -\frac{dn_u}{dE} \qquad \dots (6)$$

and

$$\frac{d\mu}{dE} \propto \frac{dE^p}{dE} = pE^{p-1}$$

$$= \frac{pE^p}{E} \propto \frac{p.\mu}{E}$$

$$\boxed{\frac{d\mu}{dE} = \frac{p.\mu}{E}} \qquad \dots (7)$$

From equations (6) and (7),

$$\frac{dn_u}{dE} = -\frac{dn_l}{dE} \qquad \dots (8a)$$

$$\frac{d\mu}{dE} = \mu.\frac{P}{E}\ ; \quad \frac{d\mu_l}{dE} = \frac{\mu_l p}{E} \qquad \dots (8b)$$

- By substituting equation (8a) and (8b) in equation (3), we get

$$\frac{d\sigma}{dE} = e\left[\mu_l \frac{dn_l}{dE} + \mu_u\left(\frac{dn_u}{dE}\right)\right] + e\left[n_l\left(\frac{d\mu_l}{dE}\right) + n_u\left(\frac{d\mu_u}{dE}\right)\right]$$

$$= e\,\mu_l \frac{dn_l}{dE} - e\mu_u \frac{dn_l}{dE} + en_l\,\mu_l \frac{p}{E} + en_u\,\mu_u \frac{p}{E}$$

$$\frac{d\sigma}{dE} = e(\mu_l - \mu_u)\frac{dn_l}{dE} + e\,(n_l\,\mu_l + n_u\,\mu_u)\frac{p}{E} \qquad \dots (9)$$

Ohm's law is,

$$J = \sigma E \qquad \dots (10)$$

- Differentiate Ohm's law with respect to 'E', as

$$\frac{dJ}{dE} = \sigma + E.\frac{d\sigma}{dE} \qquad \dots (11)$$

Equation (11) can be rewritten as,

$$\frac{\frac{dJ}{dE}}{\sigma} = 1 + \frac{E}{\sigma}\frac{d\sigma}{dE}$$

$$\frac{1}{\sigma}\frac{dJ}{dE} = 1 + \frac{d\sigma/dE}{\sigma/E} \qquad \dots (12)$$

- For ***negative resistance,*** the ***current density*** 'J' must be ***decrease*** with ***increasing*** field ***E*** or the ratio of $\frac{dJ}{dE}$ must be ***negative.*** The condition for ***negative resistance*** is,

$$\boxed{-\frac{d\sigma/dE}{\sigma/E} > 1} \qquad \dots (13)$$

From equation (9),

$$\frac{d\sigma/dE}{\sigma/E} = \frac{e(\mu_l - \mu_u)\frac{dn_l}{dE}}{\frac{\sigma}{E}} + \frac{e(n_l\mu_l + n_u\mu_u)\frac{p}{E}}{\frac{\sigma}{E}}$$

$$= \frac{Ee}{\sigma}(\mu_l - \mu_u)\frac{dn_l}{dE} + \frac{Eep}{\sigma E}(n_l\mu_l + n_u\mu_u) \qquad \text{... (14)}$$

- By substituting equation (2) in equation (14)

$$= \frac{Ee}{e}\left(\frac{\mu_l - \mu_u}{\mu_l n_l + \mu_u n_u}\right)\frac{dn_l}{dE} + \frac{ep}{e}\frac{(n_l\mu_l + n_u\mu_u)}{(n_l\mu_l + n_u\mu_u)}$$

$$= \frac{E\,dn_l}{dE}\left(\frac{\mu_l - \mu_u}{\mu_l n_l + \mu_u n_u}\right) + p$$

$$\frac{d\sigma/dE}{\sigma/E} = E.\frac{dn_l}{dE}\frac{1}{n_l}\left(\frac{\mu_l - \mu_u}{\mu_l + \mu_u f}\right) + p \qquad \text{... (15)}$$

where, $f = \frac{n_u}{n_l}$

By substituting equation (15) in equation (13) we get,

$$\frac{-d\sigma/dE}{\sigma/E} > 1 \qquad \text{... (16a)}$$

$$\left[\left(\frac{\mu_l - \mu_u}{\mu_l + \mu_u f}\right)\left(\frac{-E}{n_l}\frac{dn_l}{dE}\right) - p\right] > 1 \qquad \text{... (16b)}$$

- The field exponent "p" is a ***function of the scattering mechanism*** and should be negative and large. Equation (16a) and consequently (16b) is the required condition for negative resistance in Gunn diode.

- On the basis of the Ridley – Watkins – Hilsum theory, the band structure of a semiconductor must satisfy three criteria in order to exhibit negative resistance
  - (i) The separation energy between the bottom of the lower valley and the bottom of the upper valley must be several times larger than the thermal energy (about 0.026 eV) at room temperature. This means that $\Delta E > kT$ or $\Delta E > 0.026$ eV.
  - (ii) The separation energy between the valleys must be smaller than the gap energy between the conduction and valence bands.
  - (iii) Electrons in the lower valley must have high mobility, small effective mass, and a low density of state, whereas those in the upper valley must have low mobility, large effective mass, and a high density of state.

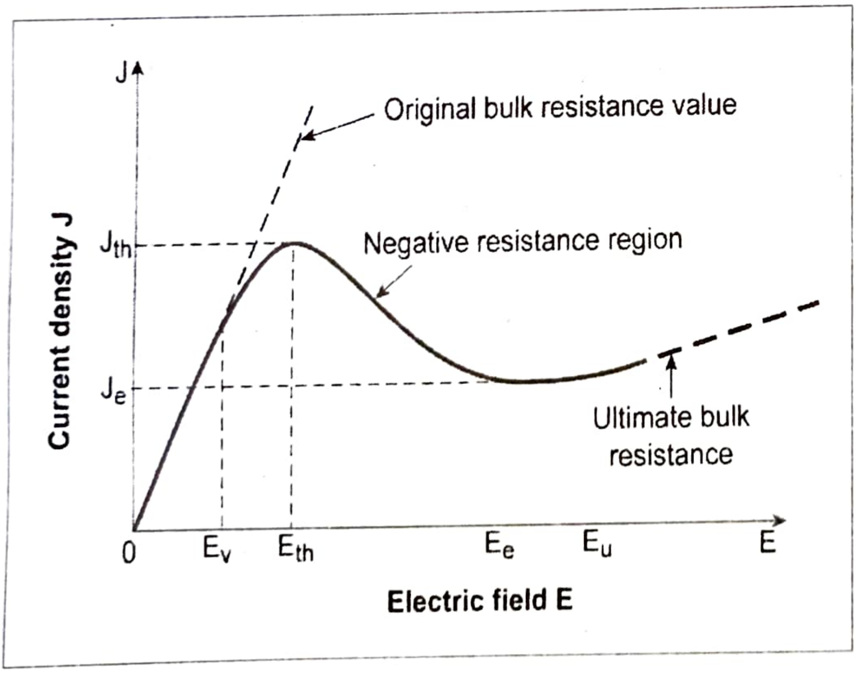


Fig.11.11. Current versus field characteristic of a two – valley semiconductor

- From electric field theory the ***magnitude*** of the current density in a semiconductor is given by,

$$J = q\, n\, v \qquad \dots (17)$$

where, q – Electric charge,

n – Electron density, and

v – Average electron velocity.

Differentiation of equation (17) with respect to an electric field 'E' as,

$$\frac{dJ}{dE} = qn\frac{dv}{dE} \qquad \dots (18)$$

- The condition for a negative differential conductance is,

$$\frac{dv_d}{dE} = \mu_n < 0 \qquad \dots (19)$$

where, μ_n denotes the ***negative mobility***.

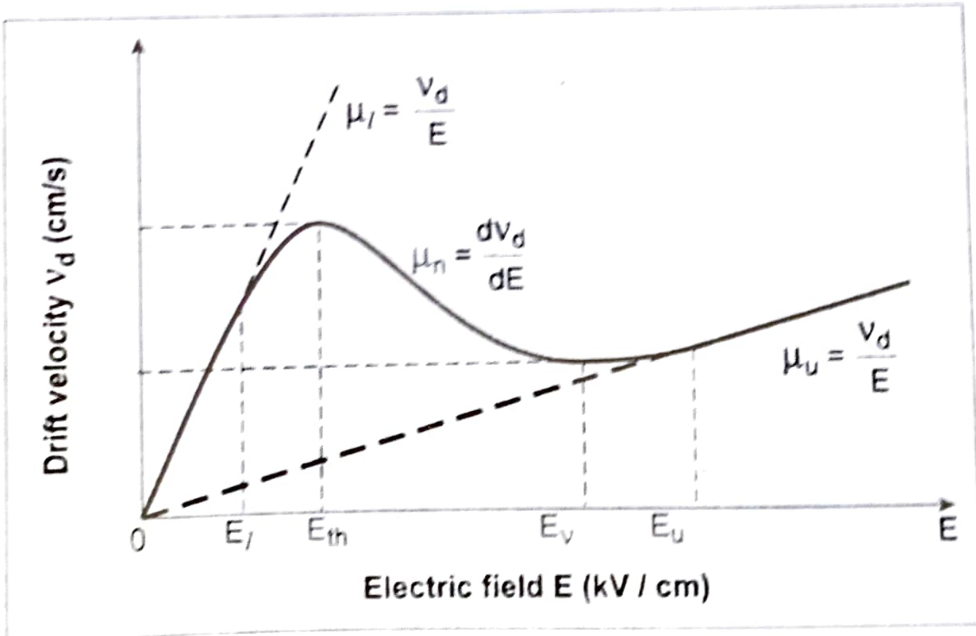
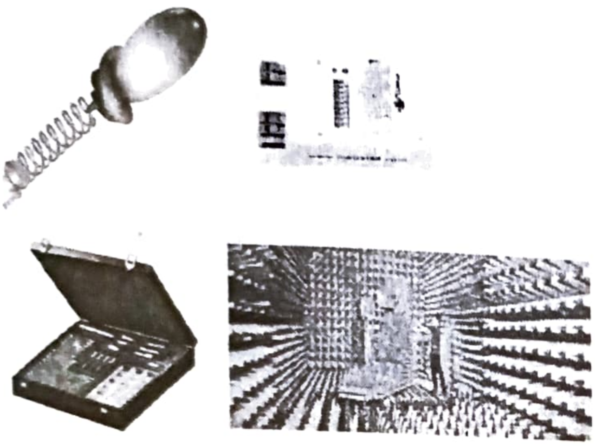


Fig.11.12. Electron drift velocity versus electric field.



12

IMPATT, VARACTOR DIODE & MIC

12.1. AVALANCHE TRANSIT – TIME DEVICES

- Avalanche transit – time devices (W.T.Read, 1958) are p-n junction diode with the ***highly doped p and n regions.*** They could produce a ***negative resistance*** at microwave frequencies by using a ***carrier impact ionization avalanche breakdown*** and ***carriers drift*** in the high field intensity region under the reverse biased condition.

Modes of Avalanche Device:

- There are ***three distinct modes*** of avalanche oscillators.

(i) IMPATT: Impact Ionization Avalanche Transit Time Device

In this mode, the typical ***dc – to – RF*** conversion efficiency is ***5 to 10%*** and frequencies are as high as ***100GHz*** with silicon diodes.

(ii) TRAPATT: Trapped Plasma Avalanche Triggered Transit Device

Its typical conversion efficiency is from ***20 to 60%***.

(iii) BARITT: Barrier Injected Transit – Time Device

It has ***long drift regions*** similar to those of IMPATT diodes. The carriers traversing the ***drift regions*** of BARITT diodes. However, they are generated by minority carrier injection from forward – biased junctions rather than being extracted from the plasma of an avalanche region.

BARITT diodes have ***low noise figures*** of ***15dB,*** but their bandwidth is relatively narrow with low output power.

12.2. IMPATT DIODE OSCILLATOR AND AMPLIFIER

12.2.1. PHYSICAL STRUCTURES

- The word IMPATT is acronym for ***Impact Ionization Avalanche Transit Time***. These diodes employ ***impact ionization*** and transit time properties of semiconductor structure to produce negative resistance at microwave frequencies.
- IMPATT diodes have many forms. $n^- pi\, p^+$ or $p^+ ni\, n^+$ read device. $p^+ n\, n^-$ ***abrupt junction***, and $p^+ i\, n^-$ ***diode.***
- Many IMPATT diodes consist of a ***high doping avalanche region*** followed by a ***drift region*** where the field is low enough that the carriers can traverse through it without avalanching.
- IMPATT diodes can be manufactured from ***Ge, Si, GaAs,*** or ***InP.*** Among these, **GaAs** provides the ***highest efficiency***, the ***highest operating frequency***, and ***least noise figure***. But the fabrication process is more difficult and is more expensive than *Si*.
- A typical construction and package of a simple IMPATT diode is shown in Fig.12.1. An n-type expitaxial layer is formed over the n^- substrate. On top of this is the diffused p^- layer. A metallised cathode and plated heat sink as anode are also included.

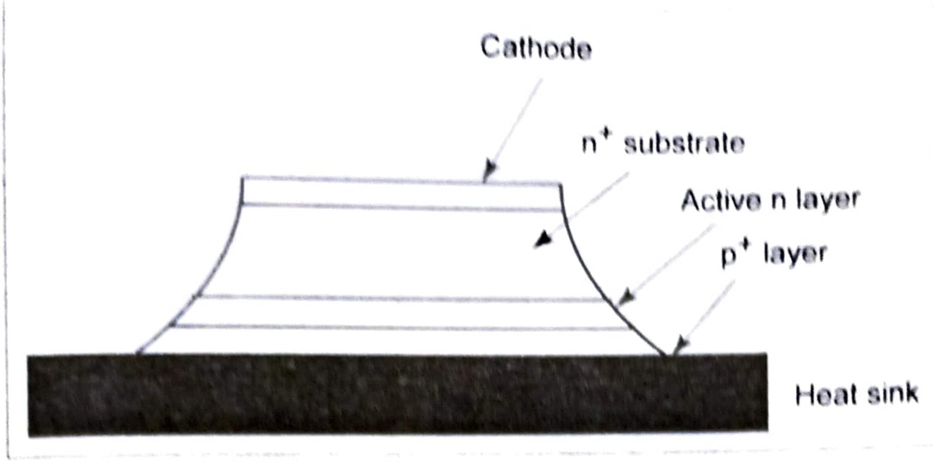


Fig.12.1. Construction and package of p^+nn^+ IMPATT diode

12.2.2. PRINCIPLES OF OPERATION

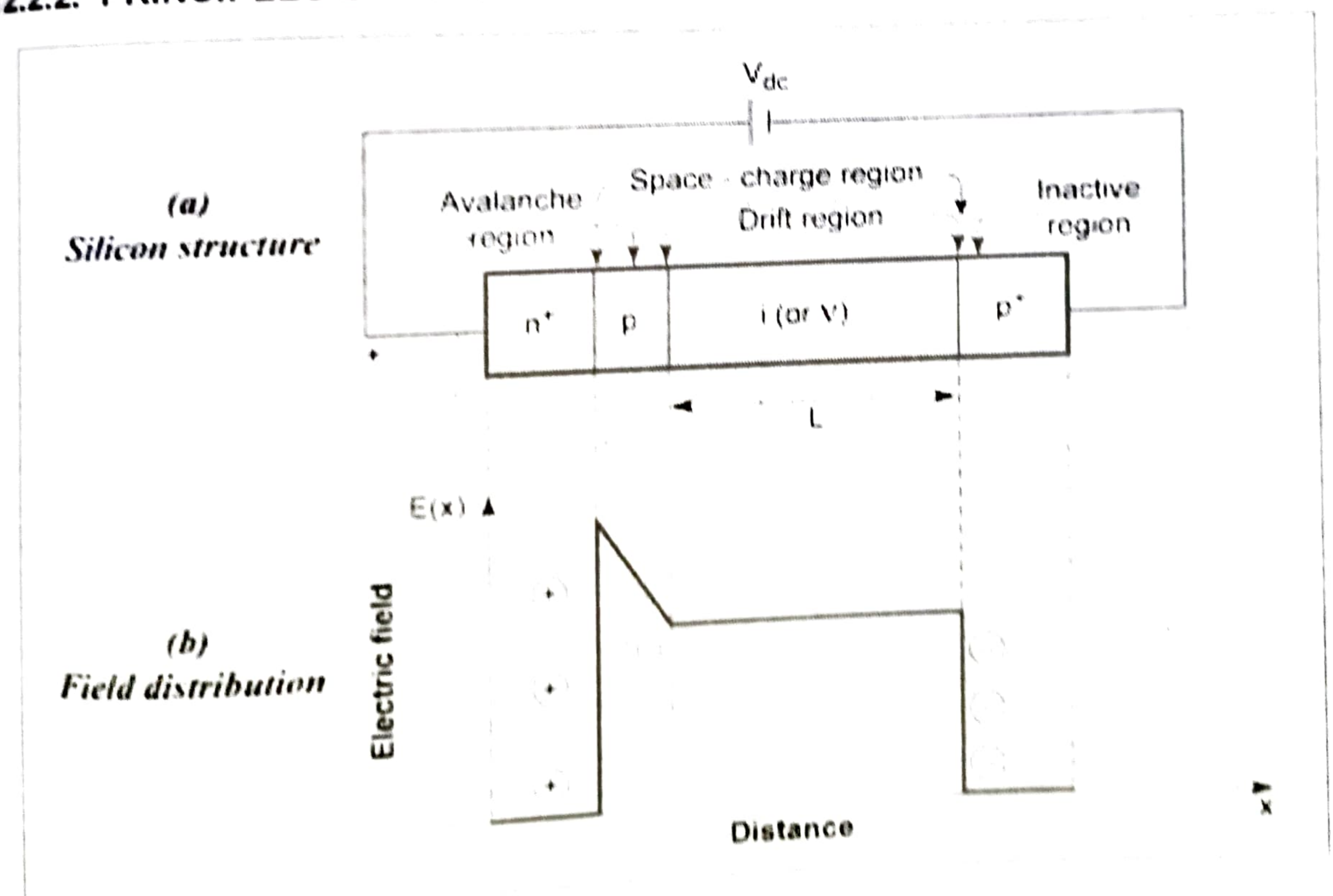


Fig.12.2. IMPATT diode operation

- From the Fig.12.2, IMPATT diode is an $n^+ - p - i - p^+$ structure, where the superscript ***plus sign*** denotes ***very high doping*** and 'i or v' refers to ***intrinsic material.***

- The devices consists essentially two regions.
  - (i) The thin 'p' region at which avalanche multiplication occurs. This region is also called the ***high – field region*** or the ***avalanche region.***
  - (ii) The ***i or v region*** through which the generated holes must drift in moving to the P^+ contact. This region is also called as the ***intrinsic region*** or the ***drift region***.
- The space between the $n^+ - p$ junction and $i - p^+$ junction is called the ***space – charge region.***

Avalanche Multiplication

- When the reverse bias voltage (V_{dc}) exceeds the breakdown voltage V_b, a maximum electric-field of very high value (MV/m) appears at the $n^+ p$ junction.
- The space – charge region always extends from the n^+ - p junction through the p and i regions to the $i - p^+$ junction.
- A positive charge gives a rising electric field in moving from left to right. The maximum field, which occurs at the $n^+ - p$ junction, is about ***several hundred kilovolts*** per centimeter.
- Carriers (holes) moving in the high field near the $n^+ - p$ junction acquire energy to knock valence electrons into the conduction band, thus producing hole – electron pairs. The ***rate of pair production,*** or ***avalanche multiplication***, is a sensitive nonlinear function of the field.

The transit time of a hole across the drift i-region is expressed as,

$$\tau = \frac{L}{v_d} \qquad \ldots (1)$$

where,

τ – Transit time,

v_d – Drift velocity of the holes, and

L – Length of the drift region.

Then the ***avalanche multiplication*** factor M is expressed as,

$$M = \frac{1}{1 - \left(\frac{V_{dc}}{V_b}\right)^n} \quad \ldots (2)$$

where, n = 3-6 for silicon is a numerical factor depending on the doping of p^+-n or n^+-p junction.

12.2.3. MECHANISM OF OSCILLATIONS

- When the IMPATT diode is mounted in a microwave resonant circuit, an *ac* voltage can be maintained at a given frequency in the circuit which is shown in Fig.12.3.

Carrier Current $I_0(t)$ and External Current $I_e(t)$

- The total field across the diode is the sum of the dc and ac fields. This total field causes break down at the $n^+ - p$ junction during the positive half of the ac voltage cycle.
- If the field is above the breakdown voltage, and the ***carrier current*** $I_0(t)$ generated at the $n^+ - p$ junction by the avalanche multiplication grows exponentially with time while the field is above the critical value.
- During the negative half cycle, when the field is below the breakdown voltage, the carrier current $I_0(t)$ decays ***exponentially*** to a small steady – state value.
- The carrier current $I_0(t)$ reaches its maximum in the middle of the *ac* voltage cycle. Under the influence of the electric field the generated holes are injected into the space – charge region toward the negative terminal.
- The injected holes traverse the drift space, they induce a current $I_e(t)$ in the external circuit.

$$I_e(t) = \frac{Q}{\tau} = \frac{v_d Q}{L} \quad \ldots (3)$$

where, Q – Total charge of the moving holes, and

v_d – Hole drift velocity, and

L – Length of the drift i region.

- When the pulse of hole current $I_0(t)$ is suddenly generated in the n^+-p junction, a constant current $I_e(t)$ starts flowing in the external circuit and continues to flow during the time τ in which the holes are moving across the space –charge region.

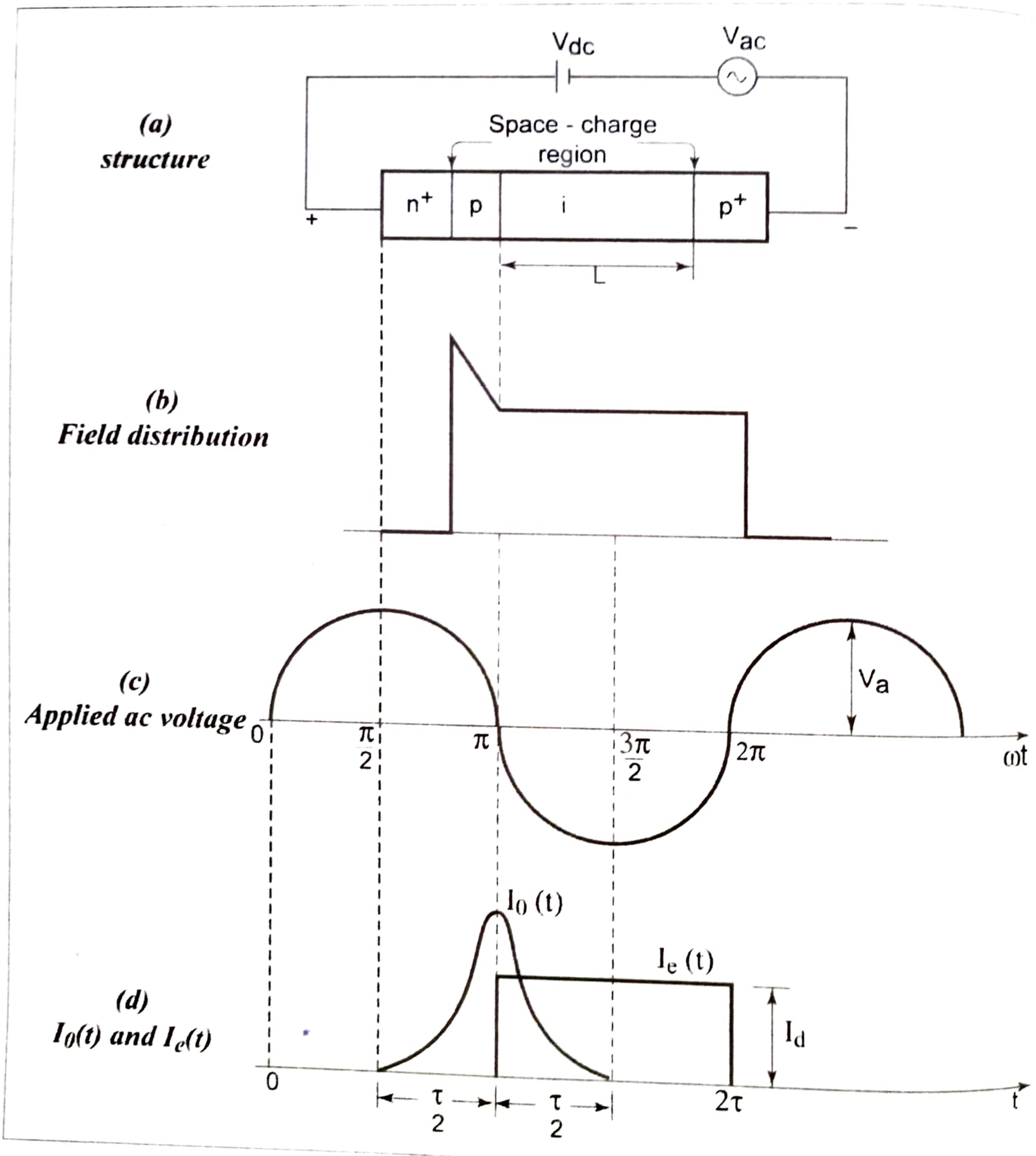


Fig.12.3. Field, voltage and currents in IMPATT diode

12.2.4. NEGATIVE RESISTANCE

- These diodes exhibit a differential negative resistance by two effects:
  (i) The ***impact ionization avalanche effect,*** which causes the carrier current $I_0(t)$ and the *ac* voltage to be out of phase by 90°.
  (ii) The ***transit – time effect,*** which further delays the external current $I_e(t)$ relative to the *ac* voltage by 90°.

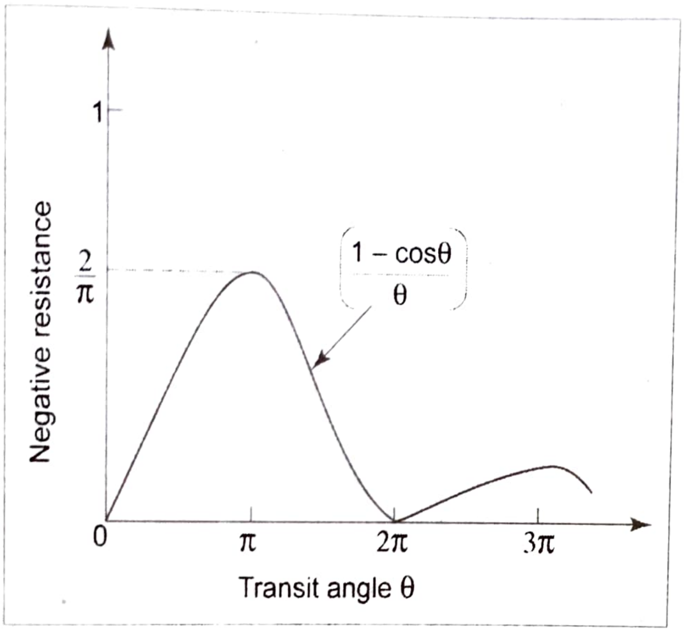


Fig.12.4. Negative resistance versus transit angle

θ is the ***transit angle*** and it is given as,

$$\theta = \omega\tau = \omega\frac{L}{v_d} \qquad \dots (4)$$

- The peak value of the negative resistance occurs near θ = π. For transit angles larger than π, the negative resistance of the diode decreases rapidly.

Resonant Frequency:

The resonant frequency of the cavity is given by

$$f = \frac{1}{2\tau} = \frac{v_d}{2L} \qquad \dots (5)$$

- If the resonator is tuned to this frequency, IMPATT diodes provide a high-power CW and pulsed microwave source.

12.2.5. POWER OUTPUT AND EFFICIENCY

- For an ***uniform avalanche,*** the maximum voltage is applied to the diode,

$$V_m = E_m L \qquad \dots (6)$$

where, L – Length of the ***depletion region,*** and

E_m – Maximum electric field.

This maximum applied voltage is ***limited*** by the ***breakdown voltage.***

- The maximum current that can be carried by the diode is also limited by the avalanche breakdown process, for the current in the space – charge region causes an increase in the electric field.

$$\text{Maximum current, } I_m = J_m A$$

$$= \sigma E_m A$$

$$= \frac{\varepsilon_s}{\tau} E_m A$$

$$= \frac{v_d \varepsilon_s E_m A}{L} \qquad \dots (7)$$

- The upper limit of the power input,

$$P_m = I_m V_m \qquad \dots (8)$$

By substituting equations (6) and (7) in equation (8) we get

$$= \frac{v_d \varepsilon_s E_m A}{L} \times E_m L$$

$$= E_m^2 \varepsilon_s v_d A \qquad \dots (9)$$

- The ***capacitance*** across the space – charge region is defined as,

$$C = \frac{\varepsilon_s A}{L} \qquad \dots (10)$$

Efficiency:

- The ***efficiency*** of the IMPATT diode is given by,

$$\eta = \frac{P_{ac}}{P_{dc}} = \frac{\text{RF power output}}{\text{dc input power}}$$

$$= \left(\frac{V_a}{V_d}\right)\left(\frac{I_a}{I_d}\right) \qquad \text{... (11)}$$

where, V_a & I_a - *ac* voltage and current.

V_d & I_d - *dc* voltage and current.

$$\text{Output power} = P_{out} = P_{ac} = \eta\, P_{dc} \qquad \text{... (12)}$$

Performance Characteristics:

Theoretically, η = 30% (<30% in practical), and

15% for Si;

23% for GaAs.

- ***GaAs*** IMPATTs have ***higher power*** and ***efficiency*** in the ***40 – to –60GHz*** region whereas ***Si*** IMPATTs are produced with ***higher reliability*** and yield in the same frequency region.

12.2.6. IMPATT DIODE POWER AMPLIFIER

- The IMPATT diode can be used as an amplifier with the same basic circuit arrangement as oscillator, provided $R_L > |R_d|$, where R_L is the load resistance and R_d is the diode negative resistance.
- The circulator is incorporated with IMPATT diode as shown in Fig.12.5. The negative resistance is used to terminate one port of the circulator and actual load is connected to the other port.
- The input RF power is fed from the remaining port. The negative resistance results in voltage reflection coefficient at the port which is greater than unity.

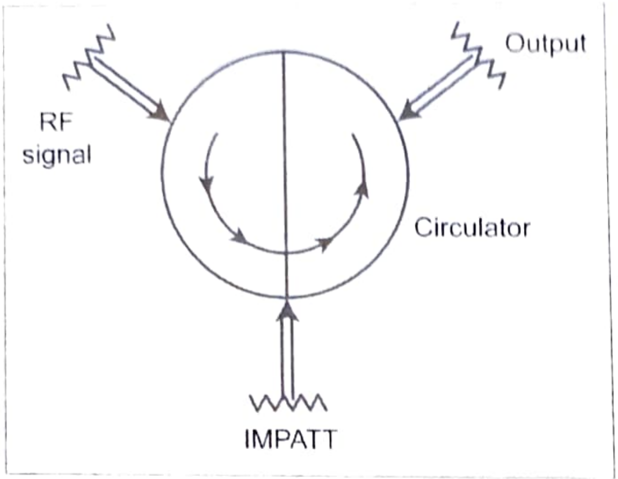


Fig.12.5. IMPATT circulator type amplifier

- Thus the average power from the source P_{av} circulates to the negative resistance port and the reflected power is greater than the incident power. The reflected power is delivered to the load.

Power Gain:

$$G_p = \frac{|\Gamma|^2 P_{av}}{P_{av}} = |\Gamma|^2 > 1$$

$$= \left|\frac{-|R_d| - R_L}{-|R_d| + R_L}\right|^2$$

Equivalent Circuit of IMPATT diode:

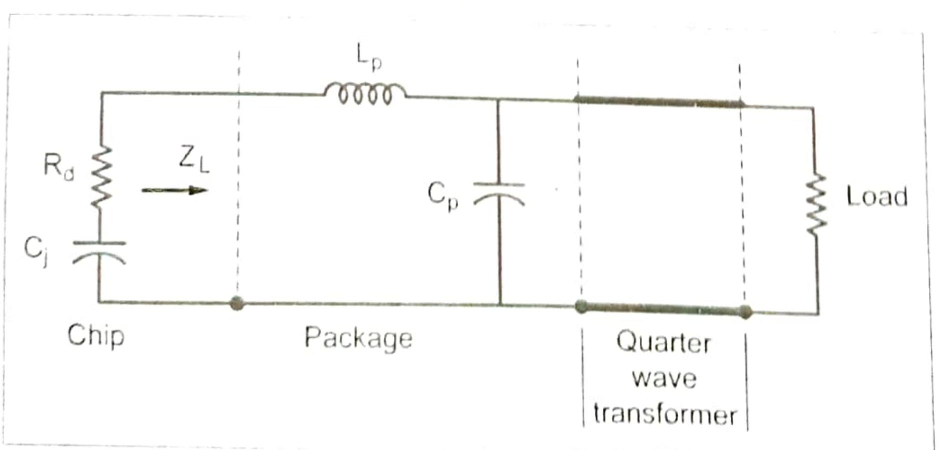


Fig.12.6. Equivalent Circuit of IMPATT Diode

- Here, R_d is the diode negative resistance and L_P, C_P are the package lead inductance and capacitance, respectively.

12.2.7. ADVANTAGES, DISADVANTAGES AND APPLICATIONS

Advantages:

IMPATT diodes provide ***potentially reliable, compact, inexpensive,*** and ***moderately efficiency*** microwave power sources.

Disadvantages:

The major disadvantages of the IMPATT diodes are,

(i) IMPATT diodes have ***low efficiency***.

(ii) It tends to be noisy due to the ***avalanche process*** and requires the high level of operating current.

(iii) A typical noise figure is 30dB which is worse than that of Gunn diodes.

Applications:

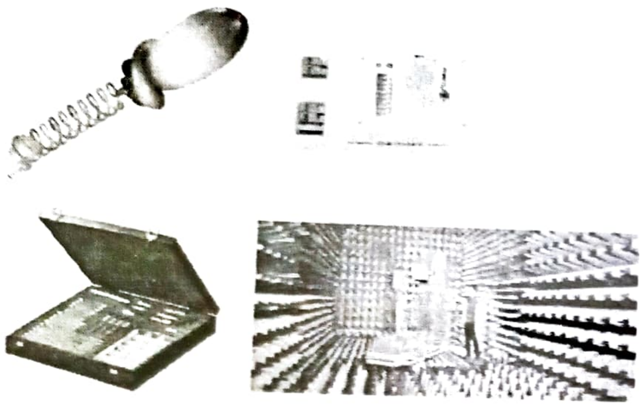
(i) Used in microwave generators.

(ii) Used in modulated output oscillators.

(iii) Used in receiver local oscillators.

(iv) Used in parametric amplifier as pumps.

(v) IMPATT diodes are also suitable for ***negative resistance amplification***.



14

TRAVELING – WAVE TUBE (TWT) AMPLIFIER

14.1. INTRODUCTION

14.1.1. MICROWAVE RESONATORS

- *Microwave resonators are **tunable circuits** used in microwave oscillators, amplifiers, wave meters and filters.*
- At the tuned frequency the circuit resonates where the average energies stored in the electric field (or in the capacitor) W_e and magnetic field (or in the inductor) W_m are equal and the circuit impedance becomes purely real.
- The ***total energy*** is therefore ***twice*** the ***electric or magnetic energy*** stored in the resonator.

Resonant Frequency:

*Resonant frequency f_r at which the **energy** in the **cavity attains** maximum value* $= 2W_e$ **or** $2W_m$.

- Quality factor 'Q' which is a measure of the frequency selectivity of a cavity.

$$\boxed{Q = \frac{2\pi \times \text{maximum energy stored}}{\text{Energy dissipated per cycle}}}$$

Drawbacks of Klystrons

- The klystrons are having the following drawbacks:
  - (i) Klystrons are essentially ***narrow band*** devices as they utilize cavity resonators to velocity modulate the electron beam over a narrow gap.
  - (ii) In klystrons and magnetrons, the microwave circuit consists of a ***resonant structure*** which ***limits*** the ***bandwidth*** (or the ***operating frequency range***) of the tube.

Major Differences Between the TWT and Klystron

- The major differences between the TWT and klystron are,
  - (i) The interaction between an electron beam and RF field in the TWT is continuous over the entire length of the circuit, but the interaction in the klystron occurs only at the gaps of a few resonant cavities.
  - (ii) The wave in the TWT is a ***propagating wave*** but the wave in the klystron is not propagating.
  - (iii) In the coupled – cavity TWT there is a ***coupling effect*** between the cavities, where as each cavity in the klystron operates independently.

14.2. HELIX TRAVELING–WAVE TUBE (OR) TRAVELLING WAVE TUBE AMPLIFIER (TWTA)

14.2.1. INTRODUCTION

- *A Traveling Wave Tube Amplifier (TWTA) circuit uses a* ***helix slow wave non resonant*** *microwave guiding structure and thus a* ***broad band microwave amplifier***.

- Two main constituents of a TWT are,
  - (i) An ***electron beam,*** and
  - (ii) A structure supporting a slow electromagnetic wave (slow-wave structure).

- In the case of the TWT, the microwave circuit is ***non resonant*** and the wave propagate with the ***same*** speed as the electrons in the beam.
- The initial effect on the beam is a small amount of ***velocity modulation*** caused by the ***weak electric fields*** associated with the traveling wave.
- This ***velocity modulation*** later ***translates*** to ***current modulation***, which then induces an RF current in the circuit, causing an amplification.

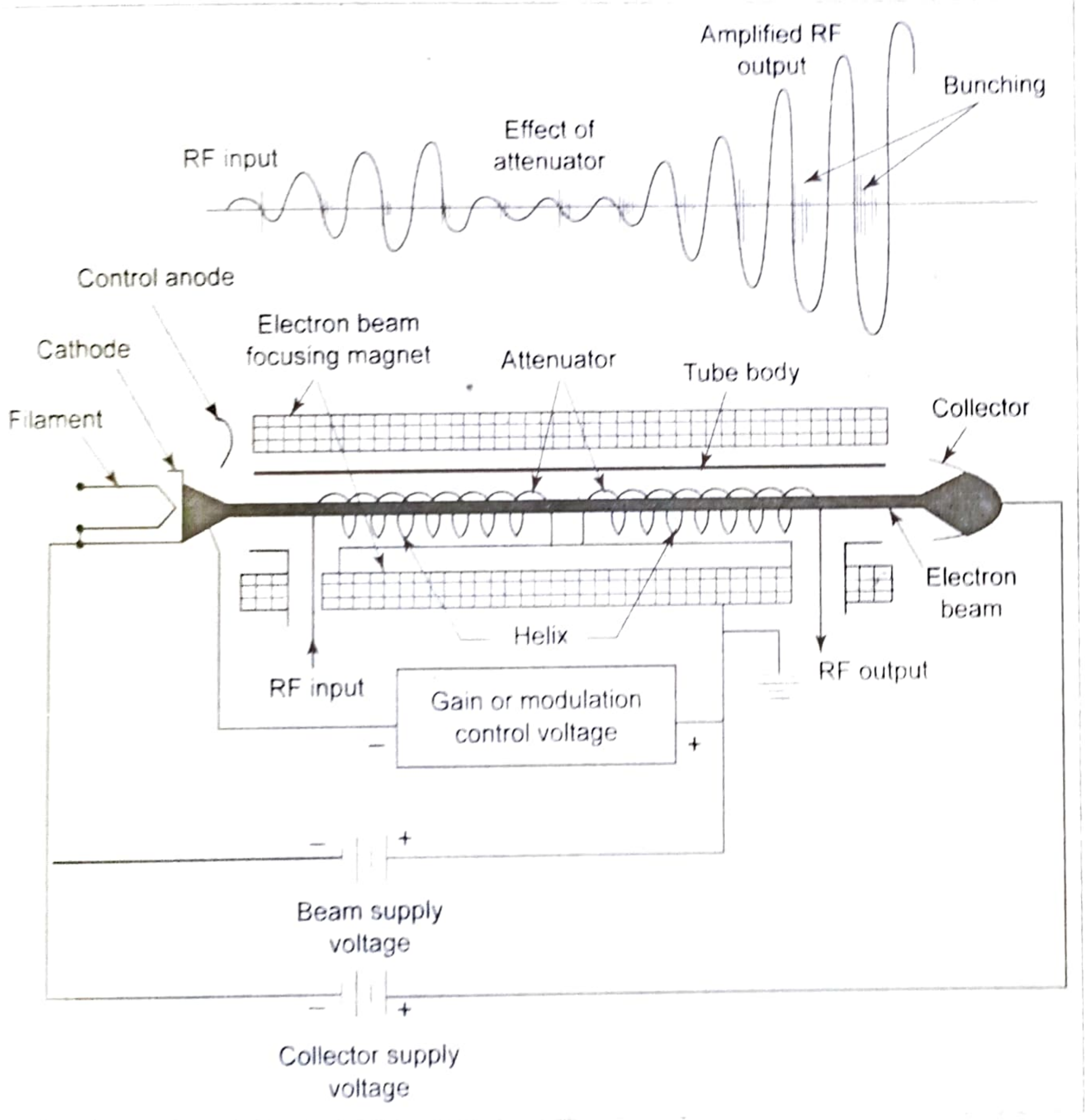


Fig. 14.1. TWT amplifier tube and circuit

14.2.2. OPERATION

- The electron beam is focused axially by a static magnetic field and collected in a collector circuit.
- The microwave input signal is injected on the helix slow-wave circuit surrounding the electron beam, which produces an axial electric field of the signal at the center of the helix and it can interact with the electron beam.
- The *dc* beam voltage is adjusted so that the beam velocity is slightly greater than the axial component of a field on the slow-wave structure.
- During transit along the axis, the electron beam transfers great amount of energy to the traveling signal wave and thus signal field amplitude increases.

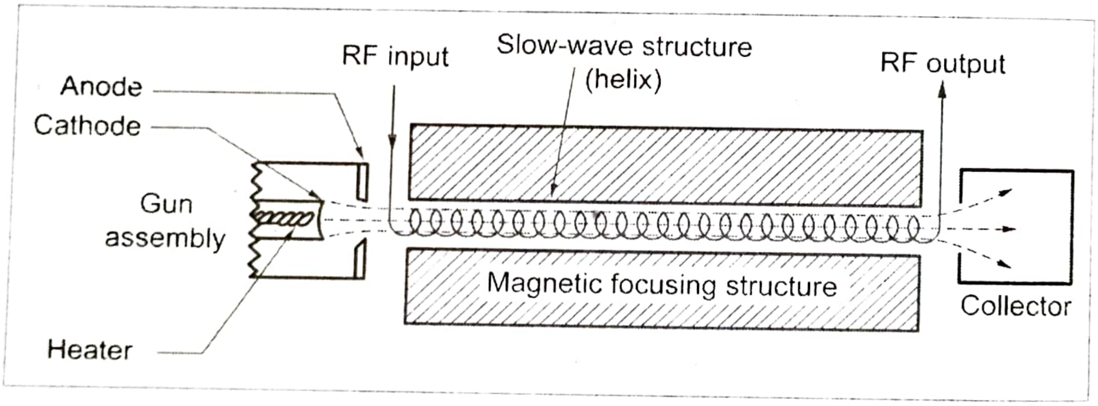


Fig.14.2. Simplified TWTA circuit

Attenuator:

An attenuator is placed over a part of the helix near the output end to attenuate any reflected waves due to impedance mismatch that can be fed back to the input to cause oscillations.

Magnet:

The magnet produces an axial magnetic field to prevent spreading of the electron beam as it travels down the tube.

Need of Slow – Wave Structures (Helix Tube):

Slow – wave structures are special circuits that are used in microwave tubes to reduce the wave velocity in a certain direction so that the electron beam and the signal wave can interact.

14.2.3. CHARACTERISTICS AND APPLICATIONS OF TWTA

Characteristics of TWTA:

Frequency range	:	3GHz and higher
Bandwidth	:	about 0.8GHz
Efficiency	:	20 to 40%
Power output	:	upto 10kW average
Power gain	:	upto 60dB

Applications of TWTA:

The main applications of TWTA are,

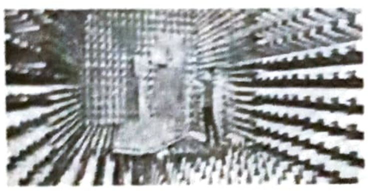
(i) Used in medium power satellite,

(ii) Used in higher power satellite transponder output,

(iii) Used in radar transmitters, and

(iv) Used in broadband microwave amplifier.



15

MICROWAVE CROSSED-FIELD TUBES (M-TYPE)

15.1. INTRODUCTION

- *A crossed-field microwave tube is a device that converts dc into microwave energy using an electronic energy-conversion process.*
- *M – type devices or crossed field tubes in which the* **dc magnetic field** *and* **dc electric field** *are* **perpendicular** *to each other. The principal tube in this type is called* **magnetron.**
- In all crossed – field tubes, the dc magnetic field plays a direct role in the RF interaction process. A magnetron oscillator is used to generate ***high microwave power***.

Classification of Crossed – Field Electron Tubes:

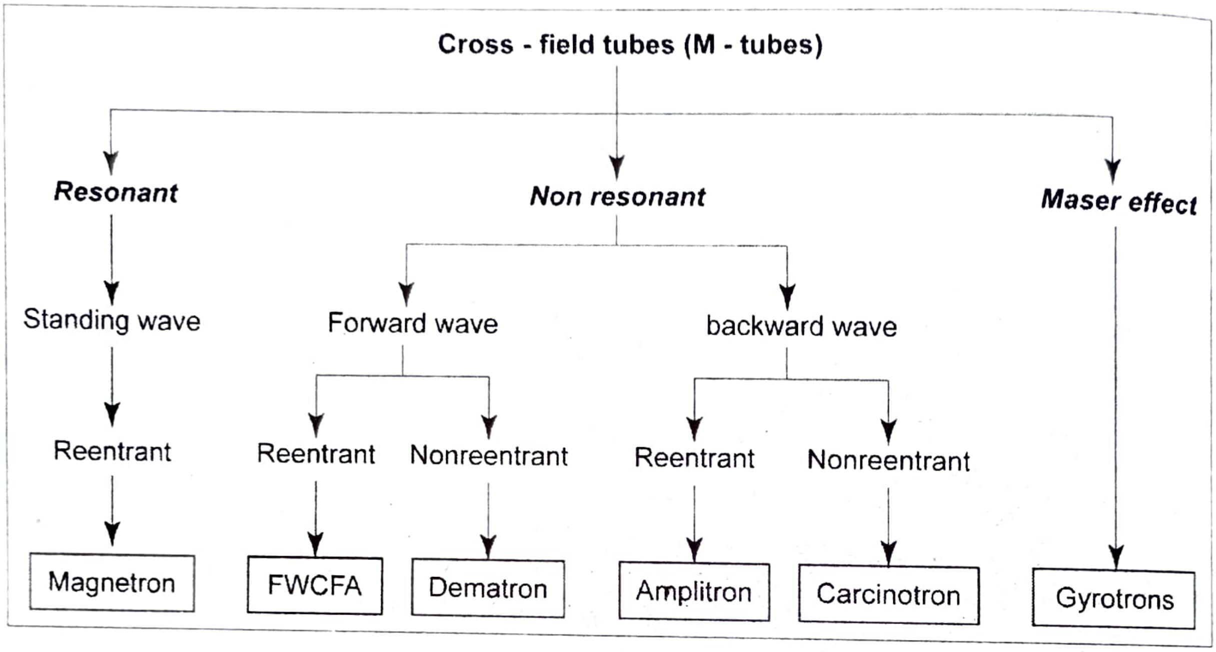


Fig.15.1. Types of cross field tubes

where,

FWCFA – Forward – Wave Crossed – Field Amplifier.
BWCFA – Backward – Wave Crossed – Field Amplifier.
BWCFO – Backward – Wave Crossed – Field oscillator.

15.2. MAGNETRON OSCILLATORS

- Magnetrons provide microwave oscillations of very high peak power. The magnetron was invented by ***Hull*** in ***1921*** and an improved high power magnetron was developed by ***Randall*** and ***Boot*** around ***1939.***
- All magnetrons operated in a dc magnetic field normal to a dc electric field between the cathode and an anode.
- The electrons emitted from the cathode are moved in curved paths due to crossed field between the cathode and anode.
- If the *dc* magnetic field is ***strong*** enough, the electrons will not arrive in the anode but return to the cathode. Consequently, the anode current is cut off.

15.2.1. CLASSIFICATION OF MAGNETRON

- Magnetrons can be classified into three types namely,
  (i) Split – anode magnetron,
  (ii) Cyclotron – frequency magnetrons, and
  (iii) Traveling – wave magnetrons.

Split – Anode (Negative Resistance) Magnetron:

This type of magnetron uses a static ***negative resistance*** between two anode segments but has low efficiency and is useful only at low frequencies (< 500MHz) which is below the microwave region.

Cyclotron – Frequency Magnetrons:

This type operates under an influence of synchronism between an alternating component of electric field and a periodic oscillation of electrons in a direction parallel to the field. These are useful only for frequencies greater than 100MHz.

Traveling – Wave Magnetrons:

This type customarily referred to simply as ***magnetrons.*** These magnetrons depend upon the interaction of electrons with a traveling electromagnetic field of linear velocity.

- ***Cylindrical magnetron, linear (or planar) magnetron, coaxial magnetron, voltage – tunable magnetron, inverted coaxial magnetron***, and ***the frequency agile magnetron*** are the different configurations available in the traveling – wave magnetrons.

15.2.2. POWER OUTPUT AND EFFICIENCY

- A magnetron can deliver a peak power output of up to 40 MW with the dc voltage of 50 kV at 10 GHz. The average power output is 800 kW.
- The magnetron possesses a very high efficiency ranging from 40 to 70%. Magnetrons are commercially available for peak power output from 3kW and higher.

15.2.3. APPLICATIONS

The magnetrons are widely used on,

(i) Radar transmitters with high output power,

(ii) Satellite and missiles for telemetry,

(iii) Industrial heating, and

(iv) Microwave ovens.

15.3. CYLINDRICAL MAGNETRON

15.3.1. INTRODUCTION

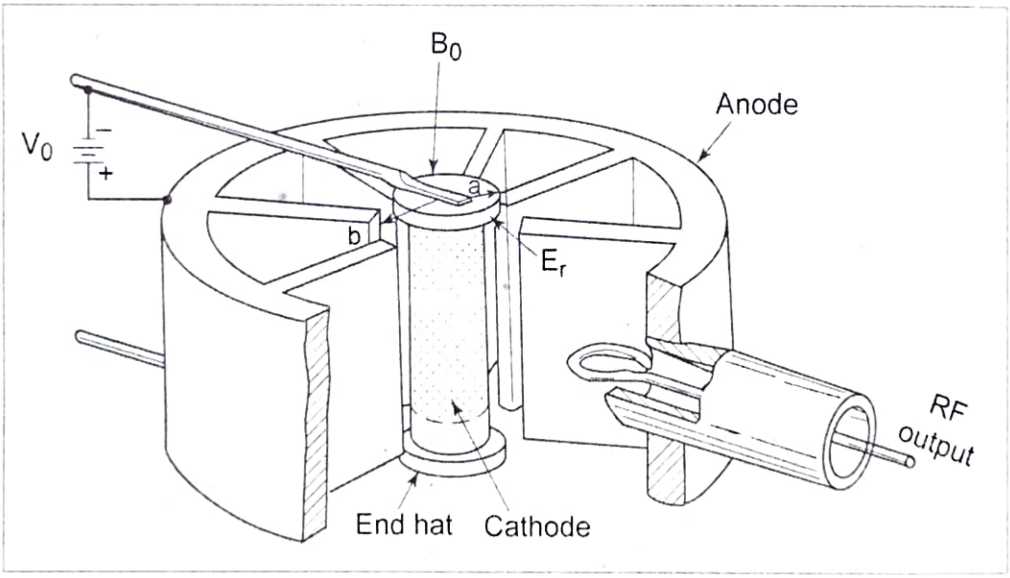


Fig.15.2. Schematic diagram of a cylindrical magnetron

- A schematic diagram of a cylindrical magnetron oscillator is shown in Fig.15.2. A high power microwave oscillator uses traveling- wave cylindrical magnetron tubes.
- This type of magnetron is also called a ***conventional magnetron.*** It consists of a cylindrical cathode of finite length and radius a at the centre surrounded by a cylindrical anode of radius b.

- The anode is a slow wave structure consisting of several reentrant cavities equi-spaced around the circumstance and coupled together through the anode cathode space by means of slots.
- The *dc* voltage V_0 is applied between the cathode and the anode and a dc magnetic flux density B_0 is maintained in the positive z direction by means of a permanent magnet or an electromagnet.
- When the *dc* voltage and the magnetic flux are adjusted properly, the electrons emitted from the cathode try to travel to anode, but with the influence of crossed fields E and H in the space between anode and cathode, the electrons takes a curved path.
- The accelerated electrons in the curved trajectory, when retarded by the RF field, the transfer energy from the electron to the cavities to grow RF oscillations till the system RF losses balances the RF *oscillations* for stability.

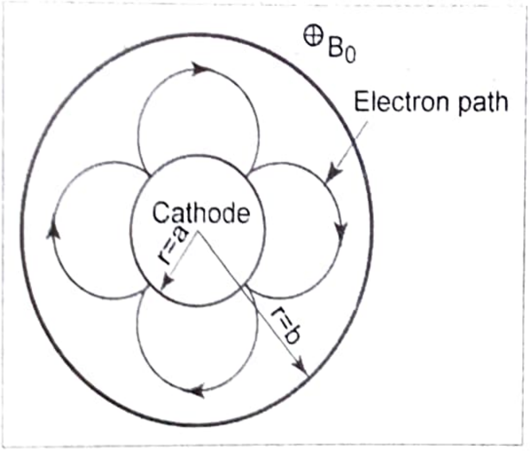


Fig.15.3. Electron path in a cylindrical magnetron

15.3.2. EQUATIONS OF ELECTRON MOTION (OR) HULL CUT OFF VOLTAGE

- The equations of motion for an electron in a cylindrical magnetron can be written as,

$$\frac{d^2r}{dt^2} - r\left(\frac{d\phi}{dt}\right)^2 = \frac{e}{m}E_r - \frac{e}{m}rB_0\frac{d\phi}{dt} \qquad \ldots (1)$$

$$\frac{1}{r}\frac{d}{dt}\left(r^2\frac{d\phi}{dt}\right) = \frac{e}{m}B_z\frac{dr}{dt} \qquad \dots (2)$$

where, $\frac{e}{m}$ = ***Charge to mass ratio*** of electron

$$= 1.759 \times 10^{11} \text{ C/kg}$$

- $\boldsymbol{B_0 = B_z}$ is assumed in the positive z direction and rearrange the equation (2) as,

$$\frac{d}{dt}\left(r^2\frac{d\phi}{dt}\right) = \frac{e}{m}B_z\frac{r\,dr}{dt} \qquad \left[\because \frac{dr^n}{dt} = n\,r^{n-1}\frac{dr}{dt};\ \frac{dr^2}{dt} = 2r\frac{dr}{dt};\ r\frac{dr}{dt} = \frac{1}{2}\frac{dr^2}{dt}\right]$$

$$= \frac{1}{2}\omega_c\frac{d}{dt}\left(r^2\right) \qquad \dots (3)$$

where, $\omega_c = \frac{e}{m}B_z$ is the ***cyclotron angular frequency.***

- By integrating equation (3), we get

$$r^2\frac{d\phi}{dt} = \frac{1}{2}\omega_c r^2 + \text{constant} \qquad \dots (4)$$

- At r = a, where 'a' is the radius of the cathode cylinder, and $\frac{d\phi}{dt} = 0$. Then equation (4) becomes,

$$0 = \frac{1}{2}\omega_c r^2 + \text{constant}$$

Therefore, Constant $= -\frac{1}{2}\omega_c a^2$ ... (5)

- By substituting equation (5) in equation (4) we get,

$$r^2 \frac{d\phi}{dt} = \frac{1}{2}\omega_c r^2 - \frac{1}{2}\omega_c a^2$$

$$\frac{d\phi}{dt} = \frac{\omega_c}{2} - \frac{\omega_c a^2}{2r^2}$$

- The ***angular velocity*** of an electrons is,

$$\frac{d\phi}{dt} = \frac{\omega_c}{2}\left[1 - \frac{a^2}{r^2}\right] \quad \ldots (6)$$

- The electrons move in the direction perpendicular to the magnetic field, the kinetic energy of an electron is given by,

$$\frac{1}{2}mv^2 = eV$$

$$v^2 = \frac{2eV}{m}$$

- The electron velocity has r and ϕ components, therefore

$$v_r^2 + v_\phi^2 = \frac{2eV}{m}$$

$$\left(\frac{dr}{dt}\right)^2 + \left(r\frac{d\phi}{dt}\right)^2 = \frac{2eV}{m} \quad \ldots (7)$$

- At $r = b$, where 'b' is the radius from the center of the cathode to the edge of the anode, $V = V_0$ and $\frac{dr}{dt} = 0$, when the electrons just graze the anode. Equations (6) and (7) becomes,

$$\frac{d\phi}{dt} = \frac{\omega_c}{2}\left[1 - \frac{a^2}{b^2}\right] \quad \ldots (8)$$

$$b^2\left(\frac{d\phi}{dt}\right)^2 = \frac{2eV_0}{m} \quad \ldots (9)$$

- By substituting equation (8) in equation (9) we get,

$$\left(\frac{d\phi}{dt}\right)^2 b^2 = \frac{2eV_0}{m}$$

$$\frac{2eV_0}{m} = b^2\left[\frac{\omega_c}{2}\left(1-\frac{a^2}{b^2}\right)\right]^2$$

- The electron will acquire a tangential as well as a radial velocity. Whether the electron will just graze the anode and return toward the cathode depends on the relative magnitudes of V_0 and B_0.

$$\frac{2eV_0}{m} = \frac{b^2\omega_c^2}{4}\left[1-\frac{a^2}{b^2}\right]^2$$

$$\frac{2eV_0}{m} = \frac{b^2 e^2 B_{0C}^2}{4m^2}\left[1-\frac{a^2}{b^2}\right]^2 \qquad \left(\because \omega_c = \frac{e}{m}B_{0C}\right)$$

$$8V_0\frac{e}{m} = b^2\frac{e^2}{m^2}B_{0C}^2\left[1-\frac{a^2}{b^2}\right]^2$$

($\because B_{0C}$ = ***Cut-off magnetic flux density***)

$$8\,V_0 = b^2\frac{e}{m}B_{0C}^2\left[1-\frac{a^2}{b^2}\right]^2$$

$$8\,V_0\frac{m}{e} = b^2 B_{0C}^2\left[1-\frac{a^2}{b^2}\right]^2$$

$$B_{0C}^2 = \frac{8\,V_0\dfrac{m}{e}}{b^2\left[1-\dfrac{a^2}{b^2}\right]^2}$$

Hull Cutoff Magnetic Equation

$$\boxed{B_{0C} = \frac{\left(8V_0 \dfrac{m}{e}\right)^{\frac{1}{2}}}{b\left(1 - \dfrac{a^2}{b^2}\right)}} \quad \ldots (10)$$

- The above equation is called as ***Hull cutoff magnetic equation***. *The magnetic field required to return electrons back to cathode just grazing the surface of the anode is called the **cut-off magnetic field** (or) **cut-off magnetic flux density.***
- If $B_0 > B_{0C}$ for a given V_0, the electrons will not reach the anode.

Hull Cutoff Voltage Equation

$$\boxed{V_{0C} = \frac{e}{8m} B_0^2 b^2 \left(1 - \frac{a^2}{b^2}\right)^2} \quad \ldots (11)$$

- The above equation (9) is called as ***Hull cutoff voltage equation***. If $V_0 < V_{0C}$ for a given B_0, the electrons will not reach the anode.

15.3.3. CYCLOTRON ANGULAR FREQUENCY

- The magnetic field is normal to the motion of electrons that travel in a ***cycloidal path,*** the outward ***centrifugal force*** is equal to the pulling force.

$$\frac{mv^2}{R} = evB \quad \ldots (12)$$

where, R – Radius of the cycloidal path, and

v – Tangential velocity of the electron.

- The cyclotron angular frequency of the circular motion of the electron is then given by,

$$\omega_C = \frac{v}{R} = \frac{eB}{m} \quad \ldots (13)$$

- The *period* of one complete revolution can be expressed as,

$$T = \frac{2\pi}{\omega} = \frac{2\pi m}{eB} \quad \dots (14)$$

Resonant Modes in a Magnetron:

- If there are N reentrant cavities in the anode structure, there exist N resonant frequencies or modes. The phase shift between two adjacent cavities can be expressed as,

$$\phi_n = \frac{2\pi n}{N} \quad \dots (15)$$

where, mode of oscillation $n = 0, \pm 1, \pm 2, \pm 3, \dots \pm N/2$.

π Mode of Operation of an Eight- Cavity Magnetron:

- Magnetron oscillators are ordinarily operated in the ***π- mode*** where $n = \frac{N}{2}$

$$\phi_n = \pi \quad \dots (16)$$

- In this π mode of operation, the successive cavities in anode have opposite phase, excitation is maximum in the cavities.
- If L is the mean separation between cavities, the phase constant of the fundamental – mode field is expressed as,

$$\beta_0 = \frac{2\pi n}{NL} \quad \dots (17)$$

- The traveling field of the fundamental mode travels around the structure with angular velocity and it is expressed as,

$$\frac{d\phi}{dt} = \frac{\omega}{\beta_0} \quad \dots (18)$$

- When the cyclotron frequency of the electrons is equal to the angular frequency of the field, the interactions between the field and electron occurs and the energy is transferred. The cyclotron angular frequency is expressed as,

$$\omega_C = \beta_0 \frac{d\phi}{dt} \quad \dots (19)$$

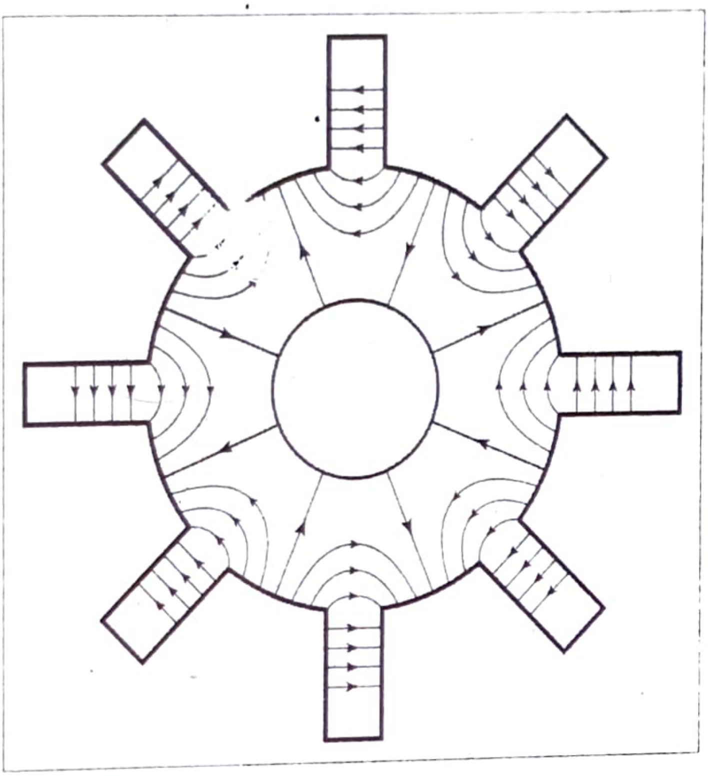


Fig.15.4. Lines of force in π mode of eight cavity magnetron

15.3.4. POWER OUTPUT AND EFFICIENCY

- The efficiency and power output of a magnetron depend on the ***resonant structure*** and the ***dc power supply.***

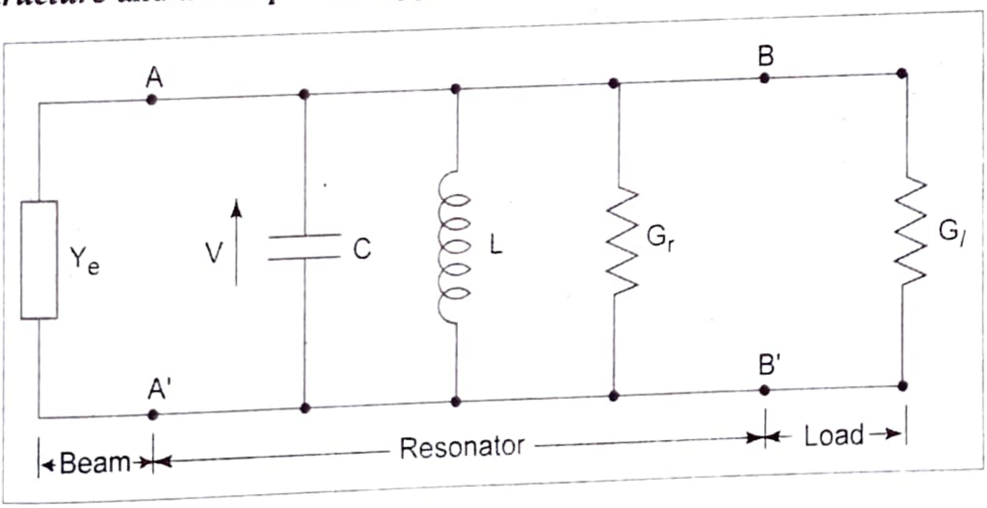


Fig.15.5. Equivalent circuit for one resonator of a magnetron

where, Y_e = Electronic admittance.

V = RF voltage across the vane tips.

C = Capacitance at the vane tips.

L = Inductance of the resonator.

G_r = Conductance of the resonator.

G_l = Load conductance per resonator.

- The ***unloaded quality*** factor of the resonator is expressed as,

$$Q_{un} = \left.\frac{\omega_0 C}{G_r}\right|_{at\,\omega_0} \qquad \dots (1)$$

- Here, angular resonant frequency $\omega_0 = 2\pi f_0$. The unloaded Q is a measure of the quality of the resonant circuit itself.

- Then, the ***external quality*** factor of the load circuit is given as,

$$Q_{ex} = \left.\frac{\omega_0 C}{G_l}\right|_{at\,\omega_0} \qquad \dots (2)$$

- An external Q is a measure of the degree to which the resonant circuit is coupled to the external circuitry. The ***loaded quality*** Q_l of the resonant circuit is expressed as,

$$\frac{1}{Q_l} = \frac{1}{Q_{ex}} + \frac{1}{Q_{un}}$$

$$Q_l = \left.\frac{\omega_0 C}{G_r + G_l}\right|_{at\,\omega_0} \qquad \dots (3)$$

Circuit Efficiency:

- The circuit efficiency of a magnetron is expressed as,

$$\eta_C = \frac{G_l}{G_l + G_r}$$

$$= \frac{1}{1+\dfrac{G_r}{G_l}} = \frac{1}{1+\dfrac{Q_{ex}}{Q_{un}}} \qquad \dots (4)$$

- The maximum circuit efficiency is obtained when the magnetron is heavily loaded, i.e., for $G_l >> G_r$.

Electronic Efficiency:

- The electronic efficiency of a magnetron is expressed as,

$$\eta_e = \frac{P_{gen}}{P_{dc}} = \frac{V_0 I_0 - P_{lost}}{V_0 I_0} \qquad \dots (5)$$

where,

P_{gen} - RF power induced into the anode circuit.

P_{dc} - $V_0 I_0$ power from the dc power supply.

V_0 - Anode voltage.

I_0 - Anode current.

P_{lost} - Power lost in the anode circuit.

- The ***RF power generated*** by the electrons is expressed as,

$$P_{gen} = V_0 I_0 - P_{lost}$$

$$= V_0 I_0 - I_0 \frac{m\,\omega_0^2}{2e\,\beta^2} + \frac{E_{max}^2}{B_z^2}$$

$$= \frac{1}{2} N |V|^2 \frac{\omega_0 C}{Q_l} \qquad \dots (6)$$

where,

N = Total number of resonators.

V = RF voltage across the resonator gap.

$E_{max} = \frac{M_1 |V|}{L}$ is the maximum electric field.

$$M_1 = \frac{\sin\left(\beta_n \frac{\delta}{2}\right)}{\left(\beta_n \frac{\delta}{2}\right)} = 1$$ for small δ is the gap factor for the π-mode operation.

β = Phase constant.

β_z = Magnetic flux density.

L = Center – to – center spacing of the vane tips.

- The power generated by the electrons may be simplified to,

$$P_{gen} = \frac{N L^2 \omega_0 C}{2 M_1^2 Q_l} E_{max}^2 \quad \ldots (7)$$

- The ***electronic efficiency*** may be rewritten as,

$$\eta_e = \frac{P_{gen}}{V_0 I_0}$$

$$\boxed{\eta_e = \frac{1 - \dfrac{m\omega_0^2}{2eV_0 \beta^2}}{1 + \dfrac{I_0 m M_1^2 Q_l}{B_z e N L^2 \omega_0 C}}} \quad \ldots (8)$$

10.6. SCHOTTKY BARRIER DIODE (SBD)

- *SBD is a simple metal semiconductor barrier diode that exhibiting a non-linear impedance and it is basically an extension of the point contact diode.*

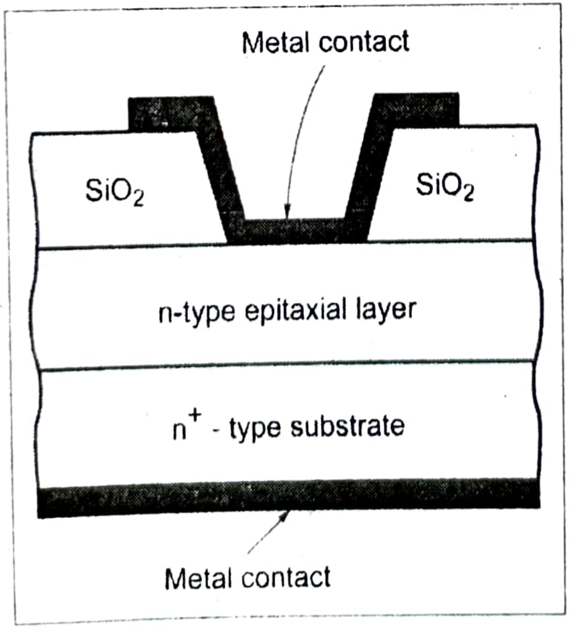


Fig.10.12. Schottky diode

Construction

- The diode is constructed on a thin silicon (n^+- type) substrate by growing epitaxially on n-type active layer of about 2-micron thickness. A thin SiO_2 layer is grown thermally over this active layer.
- Metal-semiconductor junction is formed by depositing metal over SiO_2. Schottky diodes also exhibit a square-law characteristic and have a *higher burn out rating, lower 1/f noise* and *better reliability* than point contact diodes.

Operation

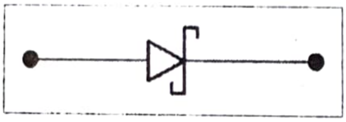


Fig.10.13. Symbol of SBD

- When the device is forward biased, the barrier height gets reduced. The major carriers (electrons) can be easily injected from the highly doped n- semiconductor material into the metal with an approximately exponential V-I characteristic.
- When it is reverse-biased, the barrier height becomes too high for the electrons to cross and thus no conduction takes place.

Equivalent Circuit

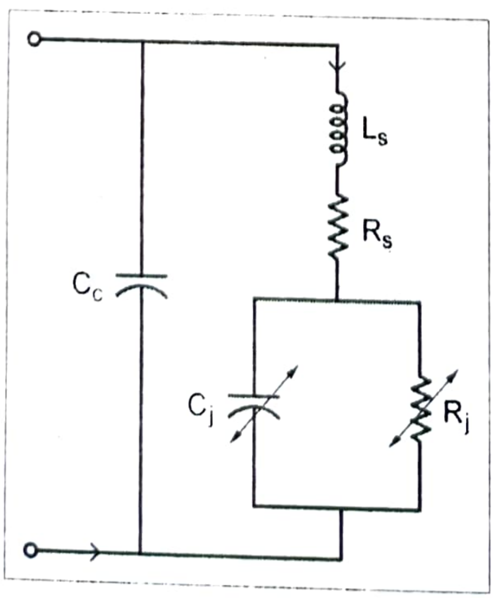


Fig.10.14. Equivalent of SBD

where, R_j – Resistance of metallic junction.

C_j – Barrier capacitance (0.3-0.5 pF).

R_s – Bulk resistance of heavily doped Si substrate (4-6 ohm).

L_s – Bond wire inductance $(0.4 - 0.9\,nH)$.

C_c – Case capacitance.

Applications:

SBD's are used as,

(i) Low noise mixer.

(ii) Balanced mixer in a CW Radar.

(iii) Microwave detectors.

10.9. PIN DIODE

- A PIN diode consist of a high-resistivity intrinsic *(i)* semiconductor layer between two highly doped p^+ and n^+ Si layers as shown in Fig.10.19. The device acts as electrically variable resistor which is related to the '*i*' layer thickness.

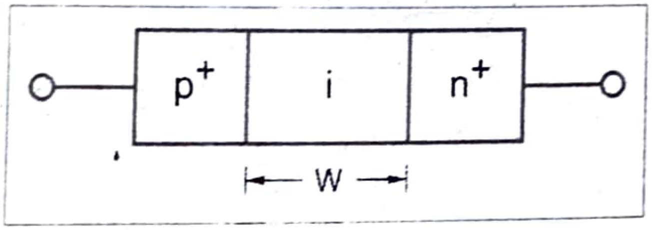


Fig.10.19. PIN diode

- The intrinsic layer has a very large resistance in reverse bias and it decreases in forward bias. When mobile carriers from *p* and *n* regions are injected into *i* layer, carriers take time such that the diode ceases to act a rectifier at microwave frequency and appears as a linear resistance.

- This property makes it suitable as a variable attenuator at microwave frequencies.

Equivalent Circuit

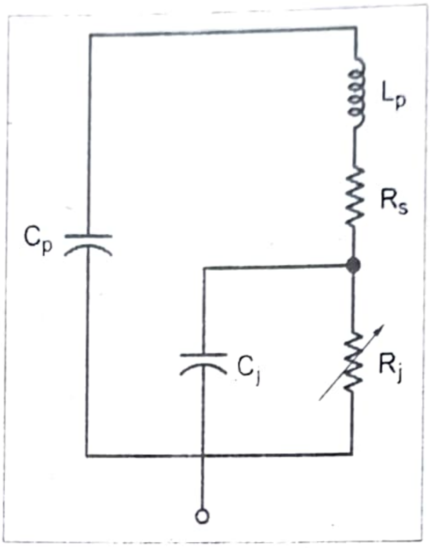


Fig.10.20. Equivalent circuit

where,

R_j, C_j – Junction resistance, capacitance of i layer.

R_s – Bulk semiconductor (p^+ and n^+) layer and contact resistance.

L_p, C_p – Package inductance, capacitance.

10.9.1. OPERATION OF PIN DIODE

- The operation can be explained by considering zero bias, reverse and forward bias conditions shown by Fig.10.21.

Zero bias:

- At zero bias, the diffusion of the holes and electrons across the junction causes space charge(density) region of thickness which is inversely proportional to the impurity concentration.

- An ideal 'i' layer has no depletion region *i.e.*, p layer has a fixed negative charge and n layer has a fixed positive charge under zero bias.

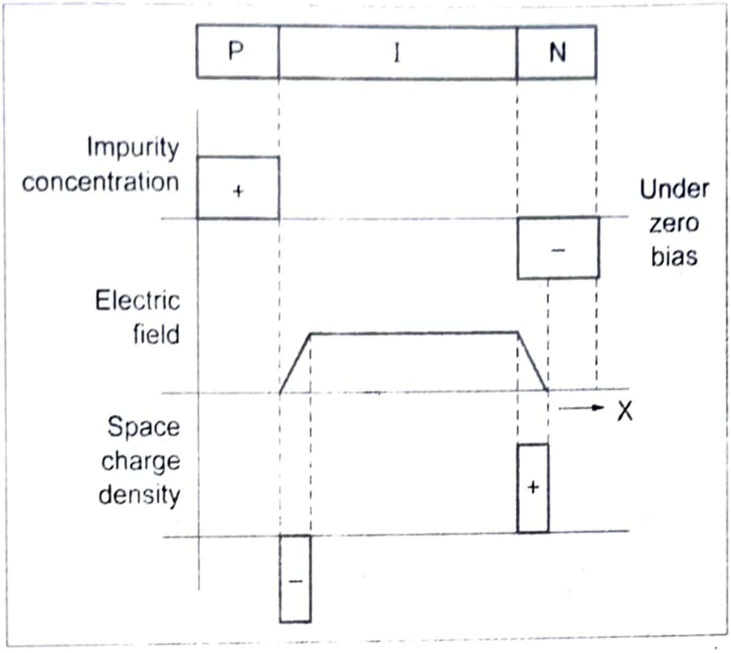


Fig.10.21. Operation of PIN diode

Reverse bias:

- As reverse bias is applied, the space charge regions in the *p* and *n* layers will become thicker. The reverse resistance will be very high and almost constant.

Forward bias:

- With forward bias, carriers will be injected into the *i* layer and the *p* and *n* space charge regions will become thinner *i.e.,* electrons and holes are injected into the '*i*' layer from *p* and *n* layers respectively.
- This results in the carrier concentration in the '*i*' layer becoming raised above the equilibrium levels and the resistivity drops as the forward bias is increased. Thus, the low resistance is offered in the forward direction.

10.9.2. APPLICATIONS OF PIN DIODES

1. PIN diode as a switch:

- It can be used as either in series or in shunt. In the first case, when the diode is reverse biased, switch is "off" or open and when it is forward biased it is closed. The bias is changed by a suitable control system as shown in Fig.10.22.

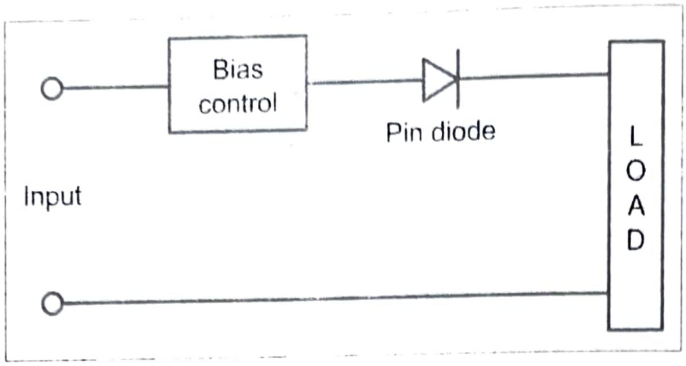


Fig.10.22. PIN diode as a switch.

(a) Single PIN switch:

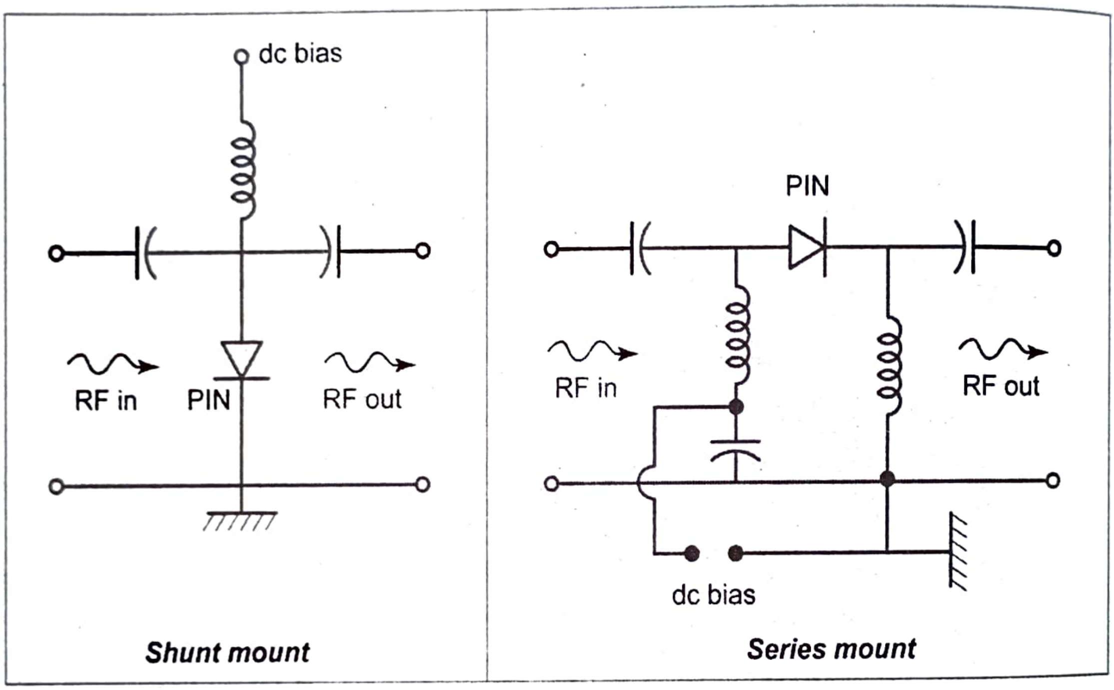


Fig.10.23. Single PIN switch

- Fig.10.23 shows the schematic circuits of a single PIN switch in shunt and series mounting configurations. *AC* blocking inductor is realised from a high impedance strip line section and *dc* blocking capacitor is realised from a gap in the line.

- For series configuration, transmission is "ON" for forward bias and "OFF" for reverse bias. For shunt configuration, reverse biasing produces transmission "ON" due to high impedance shunt and forward biasing produces transmission "OFF" due to the low impedance shunt.

(b) Double switch:

- The PIN diode double switch circuit uses two diodes called as Single-Pole Double-Throw (SPDT) is shown in Fig.10.24.

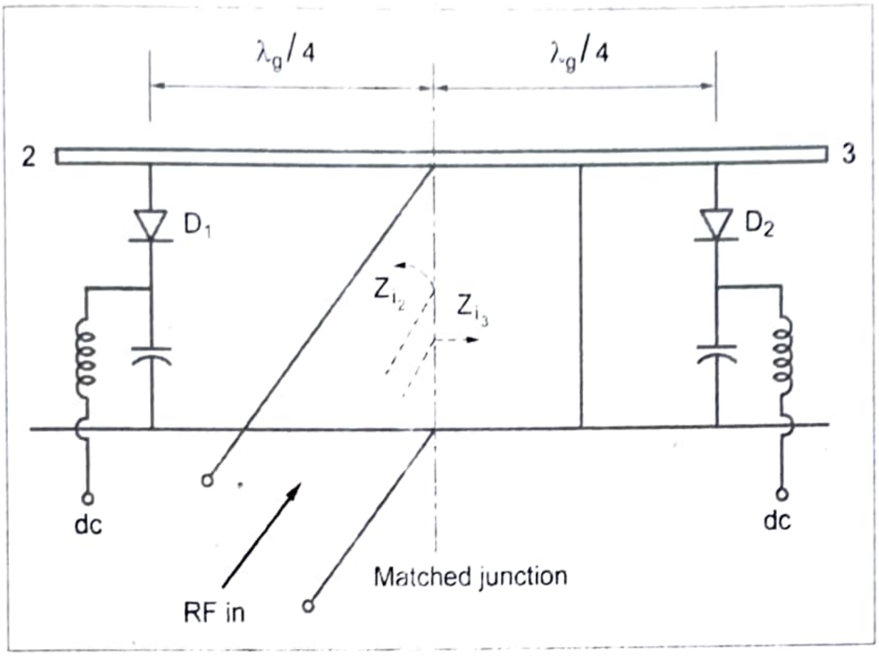


Fig.10.24. SPDT double switch

- Diodes are biased through RF chokes to isolate *ac* component from *dc* source. The impedance matching between RF feeder line and switch is obtained from the quarterwave lines which provide zero impedance when the switch is OFF (open circuit) and infinite impedance when the switch is ON (short circuit).

2. PIN Phase Shifter

- Electronic phase shifters designed using PIN diodes are extensively used in phased array. The phase shift is obtained by perturbing parameters of the transmission line.

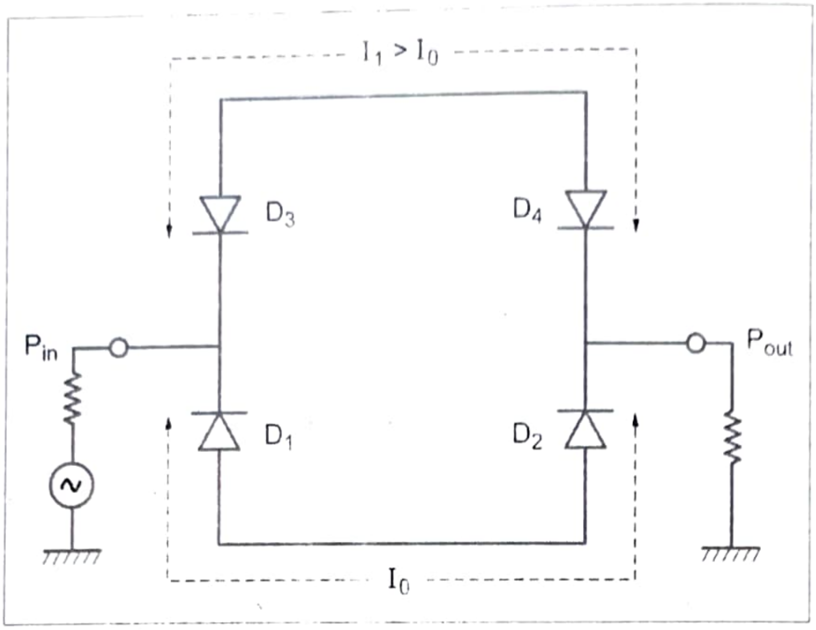


Fig.10.25. PIN phase shifter

- Here D_1, D_2, D_3 and D_4 are identical PIN diodes. Operation is based on switching the diodes from one type (FB) biasing to another (RB) so that a differential phase change occurs at the output.

3. PIN Attenuator

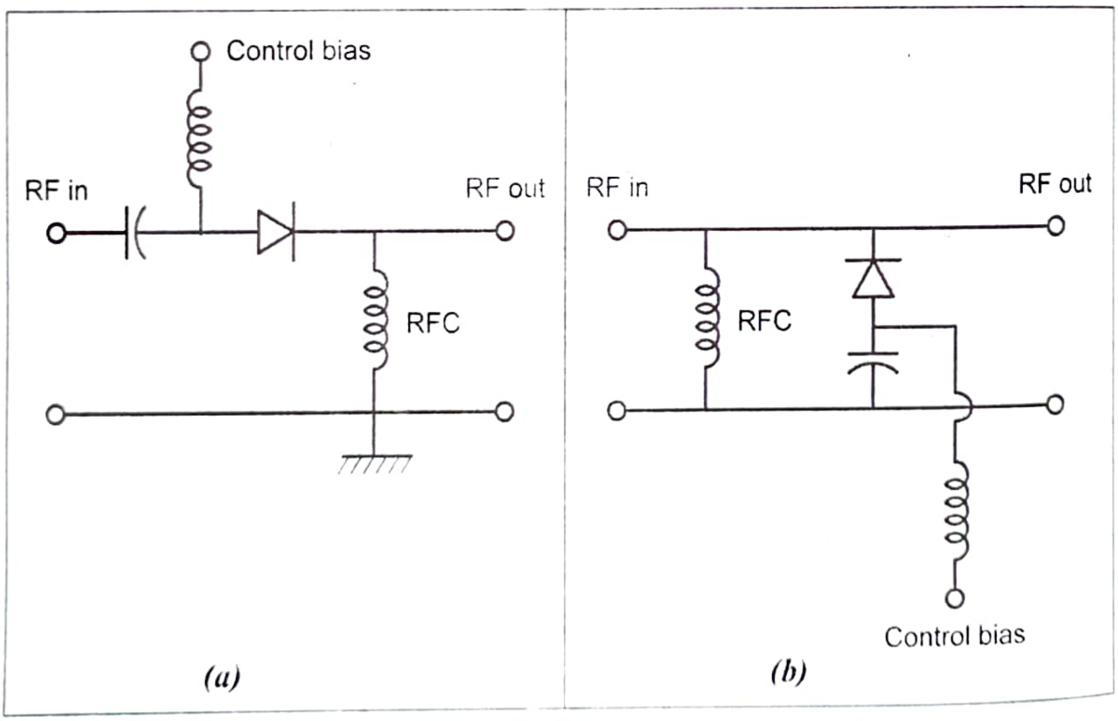


Fig.10.26. PIN attenuator (a) Series (b) Shunt

- The resistance of diode decreases with an increase in the forward bias, the diode acts as a variable attenuator when put in series or in shunt on a line.
- In series circuit, the attenuation decreases with an increase in the biasing current and at a consequent decrease of RF resistance.
- In shunt circuit attenuation increases with biasing current because most of the RF energy is absorbed in the diode.

Klystrons:

- A klystron is a vacuum tube that can be used either as a ***generator*** or as an ***amplifier*** of power at microwave frequencies operated by the principles of ***velocity*** and ***current modulation.***

- There are ***two basic configurations*** of klystron tubes,

  (i) **Reflex Klystron** – It is used as low power ***microwave oscillator,*** and

  (ii) **Two cavity** (or) **Multi cavity Klystron** – It is used as low power ***microwave amplifier.***

13.3. TWO CAVITY KLYSTRON AMPLIFIER

13.3.1. INTRODUCTION

- A two-cavity klystron amplifier is a ***velocity modulated tube*** in which the velocity modulation process produces ***density modulated stream of electrons.*** It consists of two cavities namely, ***buncher (input) cavity*** and ***catcher(output) cavity.***

Drift Space:

The separation between buncher and catcher grids is called as ***drift space.***

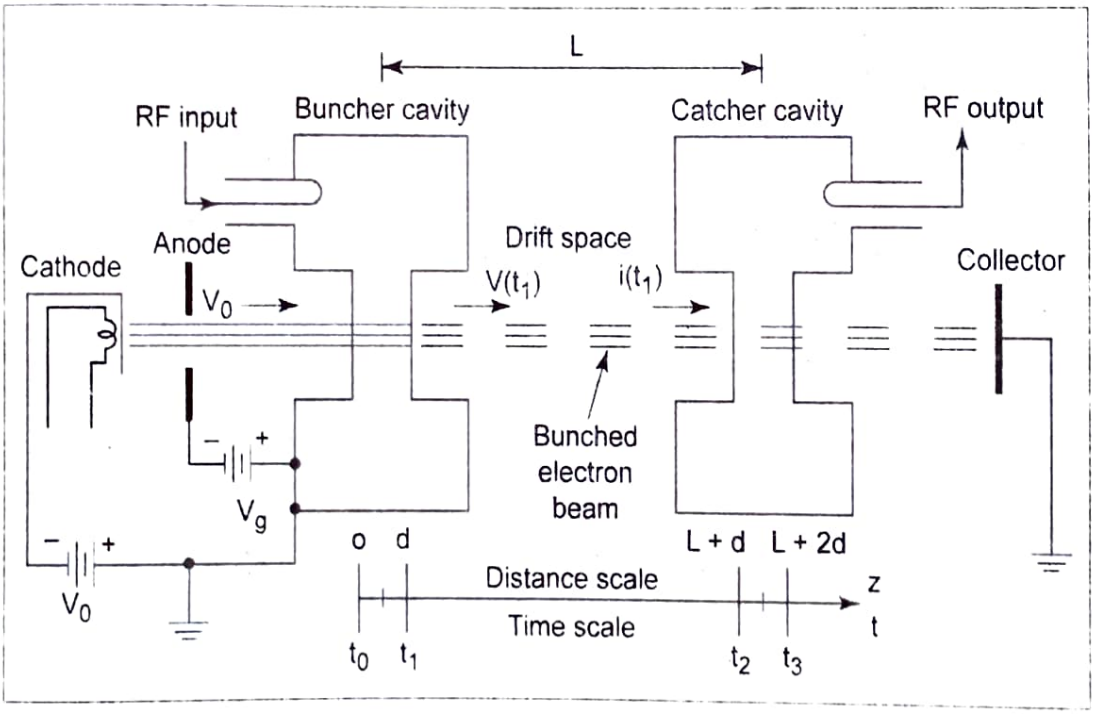


Fig. 13.5. Two – cavity klystron amplifier

13.3.2. OPERATION

- Cathode emit an electrons beam. This electrons beam first reaches the anode. The accelerating anode produces a high velocity electrons beam.
- The input RF signal to be amplified excites the buncher cavity with a coupling loop.

Bunching:

The electrons beam passing the buncher cavity gap at zeros of the gap voltage V_g (voltage between buncher grids) passes through with an unchanged velocity.

*The **electrons beam** passing through the **positive half cycles** of the gap **voltage** undergoes an **increase** in **velocity,** those passing through the **negative swings** of the gap voltage undergoes a **decrease** in **velocity.** As a result of these actions, the electrons **gradually bunch** together as they travel down the **drift space**. This is called bunching.*

- The ***first cavity*** acts as the ***buncher*** and ***velocity – modulates*** the beam. Thus the electron beam is velocity modulated to form bunches or under goes density modulation in accordance with the input RF signal cycle.

Velocity – Modulation:

*The **variation in electron velocity** in the **drift space** is known as **velocity modulation***

- When this density modulated electron beam passing through the catcher cavity grid, it induces RF current (ac current) and thereby excite the RF field in the output cavity at an input signal cycle.
- The *ac* current on the beam is such that the level of excitation of the second cavity is much greater than that in the buncher cavity and hence the amplification takes place.
- If desired, a portion of the amplified output can be fed back to the buncher cavity in a regenerative manner to obtain a ***self – sustained oscillations.***
- The maximum bunching should occur approximately at a midway between the second cavity grids during its retarding phase, thus the ***kinetic energy*** is ***transferred*** from the ***electrons*** to the ***field*** of the ***second cavity***.
- The electrons then emerge from the second cavity with reduced velocity and terminates at the collector.

Catcher Cavity:

The ***output cavity catches energy*** *from the* ***bunched electron beam****. Therefore, it is also called as* ***catcher cavity****.*

13.3.3. ANALYSIS OF TWO-CAVITY KLYSTRON AMPLIFIER

- The analysis for RF amplification by a two-cavity klystron amplifier is based on the following assumptions:
  (i) The transit time in the cavity gap is very small compared to the period of the input RF signal cycle.
  (ii) The input RF signal amplitude V_1 is very small compared to the dc beam voltage V_0 (anode potential with respect to the cathode potential)
  (iii) The cathode, anode, cavity grids and collector are parallel and the cavity grids do not intercept any electron while passing.
  (iv) No space charge or debunching takes place at the bunch point.
  (v) The RF fields are totally confined in the cavity gaps so that the field is zero in the drift space L.
  (vi) The electrons leave the cathode with a zero initial velocity.

13.3.4. VELOCITY-MODULATION PROCESS

- When electrons are ***first accelerated*** by the high *dc* beam voltage V_0 before entering the buncher grids, their velocity (v_o) is uniform.

$$v_0 = \sqrt{\frac{2eV_0}{m}} = 0.593 \times 10^6 \sqrt{V_0} \ m/s \qquad \ldots (1)$$

Assume that electrons leave the cathode with zero velocity.

- When a microwave signal is applied to the input terminal of the buncher cavity, the gap voltage between the buncher grids can be written as,

$$V_s = V_1 \sin(\omega t) \qquad \ldots (2)$$

where, V_1 is the amplitude of the signal and assume ($V_1 << V_0$).

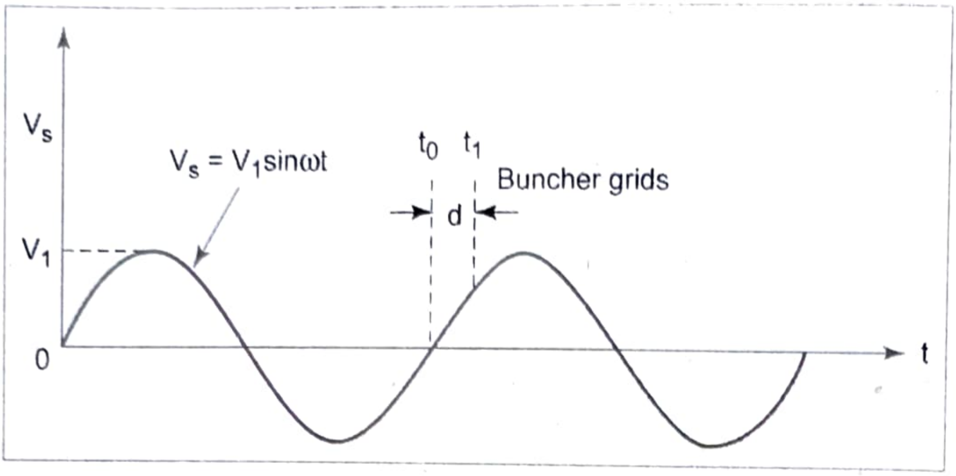


Fig.13.6. Signal voltage in buncher gap

- ***Average transit time*** through the buncher cavity grids gap distance 'd' is,

$$\tau \approx \frac{d}{v_0} = t_1 - t_0 \qquad \ldots (3)$$

- The ***average gap transit angle*** is,

$$\theta_g = \omega\tau = \omega(t_1 - t_0) = \frac{\omega d}{v_0} \qquad \ldots (4)$$

- The ***average microwave voltage*** in the buncher gap can be written as,

$$\langle V_s \rangle = \frac{1}{\tau}\int_{t_0}^{t_1} V_1 \sin(\omega t)dt$$

$$= -\frac{V_1}{\omega\tau}\left[\cos(\omega t_1) - \cos(\omega t_0)\right] \qquad \ldots (5)$$

From equation (4),

$$\omega(t_1 - t_0) = \frac{\omega d}{v_0}$$

$$\omega t_1 = \frac{\omega d}{v_0} + \omega t_0 \qquad \ldots (6)$$

- By substituting equation (6) in equation (5) we get,

$$= \frac{V_1}{\omega\tau}\left[\cos(\omega t_0) - \cos\left(\omega t_0 + \frac{\omega d}{v_0}\right)\right] \qquad \ldots (7)$$

Let $\omega t_0 + \dfrac{\omega d}{2 v_0} = \omega t_0 + \dfrac{\theta_g}{2} = A$ and

$$\frac{\omega d}{2 v_0} = \frac{\theta_g}{2} = B$$

$$(A - B) = \omega t_0 \qquad \boxed{\theta_g = \omega\tau = \frac{\omega d}{v_0}}$$

$$(A + B) = \omega t_0 + \frac{\theta_g}{2} + \frac{\theta_g}{2}$$

$$= \omega t_0 + \theta_g$$

$$\boxed{(A + B) = \omega t_0 + \frac{\omega d}{v_0}}$$

- By using trigonometric relation in equation(7) we get,

$$\cos(A-B) - \cos(A+B) = 2\sin A \sin B$$

$$\langle V_s \rangle = \frac{V_1}{\omega\tau} 2\sin\left[\frac{\omega d}{2v_0}\right] \sin\left(\omega t_0 + \frac{\omega d}{2 v_0}\right)$$

Substitute $\tau = \dfrac{d}{v_0}$

$$= \frac{V_1 \sin\left(\dfrac{\omega d}{2v_0}\right)}{\dfrac{\omega d}{2v_0}} \sin\left(\omega t_0 + \frac{\omega d}{2 v_0}\right)$$

$$\boxed{\langle V_s \rangle = V_1 \frac{\sin\left(\frac{\theta_g}{2}\right)}{\frac{\theta_g}{2}} \sin\left(\omega t_0 + \frac{\theta_g}{2}\right)} \quad \dots (8a)$$

$$\langle V_s \rangle = V_1 \beta_i \sin\left(\omega t_0 + \frac{\theta_g}{2}\right) \quad \dots (8b)$$

- β_i is the ***beam – coupling coefficient*** of the input cavity gap and it is defined as,

$$\frac{\sin\left(\frac{\omega d}{2v_0}\right)}{\frac{\omega d}{2v_0}} = \frac{\sin\left(\frac{\theta_g}{2}\right)}{\frac{\theta_g}{2}} = \beta_i \quad \dots (9)$$

- From equation (9) it is clear that ***increasing*** the gap transit angle θ_g ***decreases the coupling between the electron beam*** and the ***buncher cavity,*** i.e., the velocity modulation of the beam for a given microwave signal is decreased.

- After ***velocity modulation,*** the exit velocity from the buncher gap is given by,

$$v(t_1) = \sqrt{\frac{2e}{m}(V_0 + V_S)}$$

$$v(t_1) = \sqrt{\frac{2e}{m}\left[V_0 + \beta_i V_1 \sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right]}$$

$$= \sqrt{\frac{2e}{m} V_0 \left[1 + \frac{\beta_i V_1}{V_0} \sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right]} \quad \dots (10)$$

- Thus, the electrons in the beam are velocity modulated by the input RF signal with a ***depth of velocity modulation*** $= \frac{\beta_i V_1}{V_0}$. Since $\beta_i V_1 << V_0$, the binomial expansion of equation (10) gives,

$$\boxed{v(t_1) = v_0\left[1+\frac{\beta_i V_1}{2V_0}\sin\left(\omega t_0+\frac{\theta_g}{2}\right)\right]} \quad \ldots (11)$$

- Equation (11) is the ***equation of velocity modulation.*** Alternatively, the equation of velocity modulation can be given by,

$$\boxed{v(t_1) = v_0\left[1+\frac{\beta_i V_1}{2V_0}\sin\left(\omega t_1-\frac{\theta_g}{2}\right)\right]} \quad \ldots (12)$$

13.3.5. BUNCHING PROCESS

- The ***effect*** of ***velocity modulation*** produces bunching of the ***electron beam*** or ***current modulation***. The electrons that pass the buncher at $V_s = 0$ travel through with unchanged velocity v_0.
- The electrons that pass the buncher cavity during the ***positive half cycles*** of microwave input voltage V_s ***travel faster*** than the electrons that passed the gap when $V_s = 0$.
- The electron beams that pass the buncher cavity during the ***negative half cycles*** of the voltage V_s travel ***slower than*** the electrons that passed the gap when $V_s = 0$.
- The distance from the buncher grid to the location of the dense electron bunching for the electron at t_b is

$$\Delta L = v_0 (t_d - t_b) \quad \ldots (1)$$

where,

$$t_c = t_b + \frac{\pi}{2\omega}$$

$$t_b = t_a + \frac{\pi}{2\omega}$$

$$t_a = t_b - \frac{\pi}{2\omega}$$

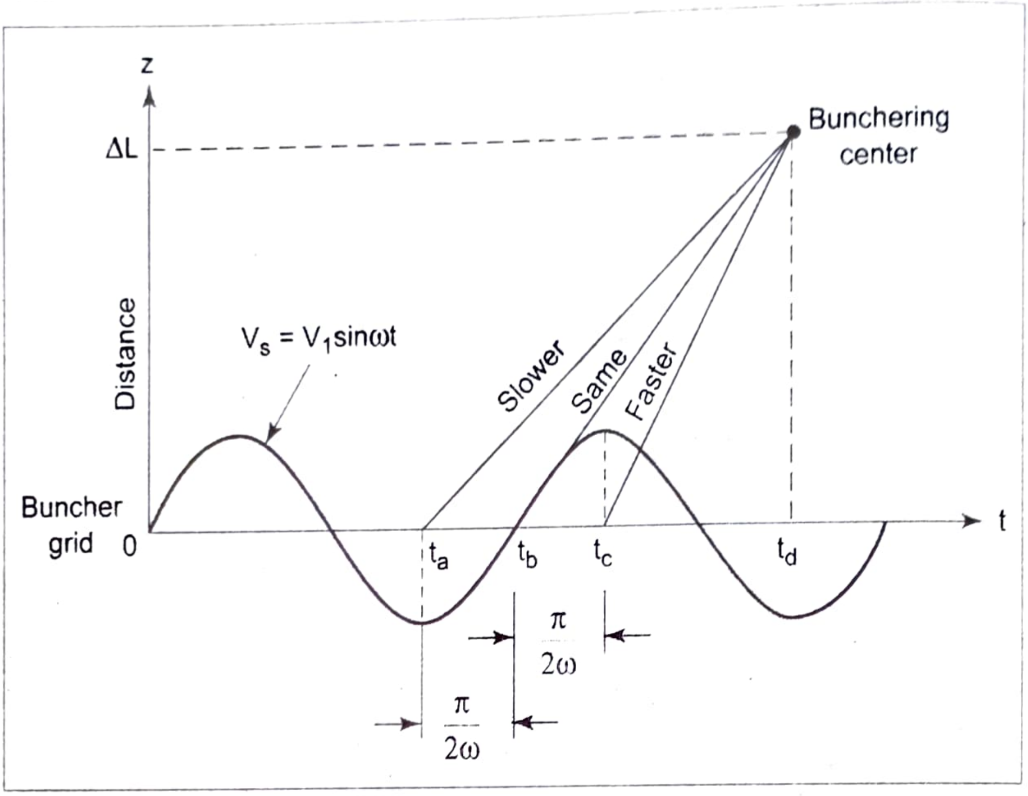


Fig.13.7. Bunching distance

- Similarly, the distances for the electrons at t_a and t_c are

$$\Delta L = v_{min}\left(t_d - t_a\right)$$

$$= v_{min}\left(t_d - t_b + \frac{\pi}{2\omega}\right) \quad \dots (2)$$

$$\Delta L = v_{max}\left(t_d - t_c\right)$$

$$= v_{max}\left(t_d - t_b - \frac{\pi}{2\omega}\right) \quad \dots (3)$$

- From the velocity modulation process the ***maximum*** and ***minimum*** velocities are

$$v_{min} = v_0\left(1 - \frac{\beta_i V_1}{2 V_0}\right) \quad \dots (4)$$

$$v_{max} = v_0\left(1 + \frac{\beta_i V_1}{2 V_0}\right) \quad \dots (5)$$

- By substituting equation (4) in equation (2),

$$\Delta L = \Delta L = v_{min}\left((t_d - t_b) + \frac{\pi}{2\omega}\right)$$

$$= v_0\left(1 - \frac{\beta_i V_1}{2V_0}\right)\left((t_d - t_b) + \frac{\pi}{2\omega}\right)$$

$$= v_0(t_d - t_b) + v_0\frac{\pi}{2\omega} - \beta_i V_1\frac{v_0}{2V_0}(t_d - t_b) - \left(\frac{v_0\beta_i V_1}{2V_0}\right)\left(\frac{\pi}{2\omega}\right)$$

$$\Delta L = v_0(t_d - t_b) + \left[v_0\frac{\pi}{2\omega} - \frac{v_0\beta_i V_1}{2V_0}(t_d - t_b) - \frac{v_0\beta_i V_1}{2V_0}\frac{\pi}{2\omega}\right] \quad \ldots (6)$$

- By substituting equation (5) in equation (3) we get,

$$\Delta L = v_{max}\left((t_d - t_b) - \frac{\pi}{2\omega}\right)$$

$$= v_0\left(1 + \frac{\beta_i V_1}{2V_0}\right)\left((t_d - t_b) - \frac{\pi}{2\omega}\right)$$

$$= \left(v_0 + \frac{v_0\beta_i V_1}{2V_0}\right)\left((t_d - t_b) - \frac{\pi}{2\omega}\right)$$

$$= v_0(t_d - t_b) - \frac{v_0\pi}{2\omega} + \frac{v_0\beta_i V_1}{2V_0}(t_d - t_b) - \frac{v_0\beta_i V_1}{2V_0}\frac{\pi}{2\omega}$$

$$\Delta L = v_0(t_d - t_b) + \left[-\frac{v_0\pi}{2\omega} + \frac{v_0\beta_i V_1}{2V_0}(t_d - t_b) - \frac{v_0\beta_i V_1}{2V_0}\frac{\pi}{2\omega}\right] \quad \ldots (7)$$

Applegate Diagram:

- The applegate diagram represents the internal operation of two cavity klystron by ***distance–time plot*** which includes velocity modulation process, bunching process, energy transfer etc.,

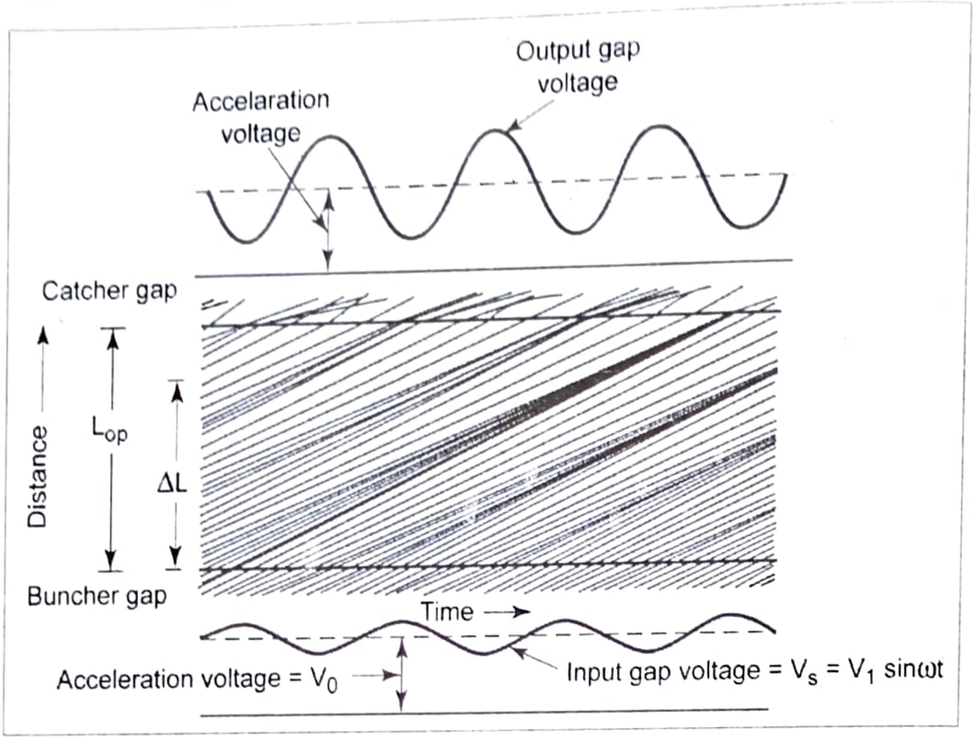


Fig.13.8. Applegate diagram

- From equation (6) and (7), the ***necessary condition*** for those electrons at t_a, t_b and t_c to meet at the same distance ΔL is,

$$\frac{v_0 \pi}{2\omega} - \frac{v_0 \beta_i V_1}{2 V_0}\left(t_d - t_b\right) - \frac{v_0 \beta_i V_1}{2 V_0}\frac{\pi}{2\omega} = 0 \qquad \ldots (8)$$

$$-\frac{v_0 \pi}{2\omega} + \frac{v_0 \beta_i V_1}{2 V_0}\left(t_d - t_b\right) - \frac{v_0 \beta_i V_1}{2 V_0}\frac{\pi}{2\omega} = 0 \qquad \ldots (9)$$

- By equating equations (8) and (9) we get,

$$\frac{v_0 \pi}{2\omega} - \frac{v_0 \beta_i V_1}{2 V_0}\left(t_d - t_b\right) - \frac{v_0 \beta_i V_1}{2 V_0}\frac{\pi}{2\omega} = -\frac{v_0 \pi}{2\omega} + \frac{v_0 \beta_i V_1}{2 V_0}\left(t_d - t_b\right) - \frac{v_0 \beta_i V_1}{2 V_0}\frac{\pi}{2\omega}$$

$$\frac{v_0 \pi}{2\omega} + \frac{v_0 \pi}{2\omega} = \frac{v_0 \beta_i V_1}{2 V_0}\left(t_d - t_b\right) + \frac{v_0 \beta_i V_1}{2 V_0}\left(t_d - t_b\right)$$

$$\frac{v_0 \pi}{\omega} = \frac{v_0 \beta_i V_1}{V_0}(t_d - t_b)$$

$$\boxed{t_d - t_b = \frac{\pi V_0}{\omega \beta_i V_1}} \quad \dots (10)$$

- By substituting equation (10) in equation (1), we get the expression for minimum distance at which maximum bunching occur,

$$\Delta L = v_0 (t_d - t_b)$$

$$\boxed{\Delta L = v_0 \frac{\pi V_0}{\omega \beta_i V_1}} \quad \dots (11)$$

13.3.5.1. MAXIMUM BUNCHING

- Now, we have to find out the ***spacing between the buncher and catcher cavities*** in order to achieve a ***maximum degree of bunching.***
- The transit time for an electron to travel a distance of L is,

$$T = t_2 - t_1 = \frac{L}{v(t)} \quad \dots (12)$$

- By substituting equation (10) for $v(t)$ from velocity modulation process in equation (12), and use the ***binomial expansion*** as,

$$(1 + x)^{-1} = (1 - x) \text{ for } |x| << 1$$

$$= \frac{L}{v_0 \left[1 + \frac{\beta_i V_1}{2 V_0} \sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right]}$$

$$T = T_0 \left[1 - \frac{\beta_i V_1}{2 V_0} \sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right] \quad \dots (13)$$

where $T_0 = \frac{L}{v_0}$ is the ***dc transit time.***

Bunching Parameter and DC Transit Angle:

- In terms of radians, the equation (13) becomes,

$$\omega T = \omega(t_2 - t_1)$$

$$= \left[\omega T_0 - \left(\frac{\omega T_0 \beta_i V_1}{2 V_0}\right)\sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right]$$

$$= \theta_0 - X\sin\left(\omega t_1 - \frac{\theta_g}{2}\right) \quad \dots (14)$$

dc transit angle between cavities $\theta_0 = \dfrac{\omega L}{v_0} = 2\pi N$ ... (15)

where, *N* is the ***number of electron transit cycles*** in the drift space.

- Then, the ***bunching parameter*** of a klystron is expressed as,

$$X = \frac{\beta_i V_1}{2 V_0}\theta_0 \quad \dots (16)$$

Beam Current in Catcher Cavity (Current Modulation):

- The bunched ***beam current*** at the catcher cavity is a periodic waveform of period $\dfrac{2\pi}{\omega}$ about ***dc current*** and it is expressed as,

$$\boxed{i_2 = I_0 + \sum_{n=1}^{\infty} 2 I_0 J_n(nX)\cos\left[n\omega\left(t_2 - \tau - T_0\right)\right]} \quad \dots (17)$$

where, I_0 = ***dc beam current in buncher cavity.***

- Equation (17) is the generalized expression of a bunched beam current consisting of a ***dc component t*** (I_0) plus the **fundamental ac component**. The klystron is generally tuned to fundamental ac component of current and it is given by,

$$I_f = 2 I_0 J_1(X)\cos(\omega t_2 - \tau - T_0) \quad \dots (18)$$

- The ***fundamental ac component*** of the beam current at the catcher cavity has a ***magnitude*** as,

$$I_f = 2\, I_0\, J_1(X) \qquad \text{... (19)}$$

- This fundamental ac component I_f can be ***maximum*** when $J_1(X) = 0.582$ at $X = 1.841$ by adjusting *dc* beam voltage V_0.

- The optimum distance L at which the maximum fundamental *ac* component of current occurs from equation (16) as,

$$X = \frac{\beta_i V_1}{2V_0}\theta_0$$

$$X = \frac{\beta_i V_1 \omega L}{2 V_0 v_0}$$

$$L = \frac{2 X V_0 v_0}{\omega \beta_i V_1} \qquad \text{... (20)}$$

- In equation (20), replace $L = L_{opt}$ when $X = 1.841$ to get the maximum bunching is,

$$L_{opt} = \frac{2 X V_0 v_0}{\omega \beta_i V_1}$$

$$= \frac{2 \times 1.841 \times V_0 v_0}{\omega \beta_i V_1}$$

$$\boxed{L_{opt} = \frac{3.682\, V_0 v_0}{\omega \beta_i V_1}} \qquad \text{... (21)}$$

13.3.6. OUTPUT POWER

- The maximum bunching should occur approximately midway between the catcher grids.
- When the ***electrons*** emerge from the ***catcher grids,*** they have ***reduced velocity*** and are finally ***collected*** by the ***collector.***

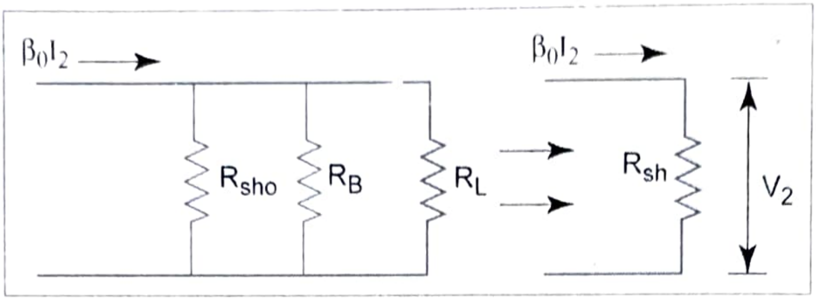


Fig.13.9.Equivalent circuit of output cavity

- The output cavity can be represented by an equivalent circuit as shown in Fig.13.9.

where, **R_{sho}** - Wall resistance of catcher cavity,

R_B - Beam loading resistance,

R_L - External load resistance, and

R_{sh} - Total equivalent shunt resistance of the catcher circuit, including the load.

Induced Current (i_{2ind}) in the Catcher Cavity:

- The ***current induced*** by the electron beam in the walls of the catcher cavity is ***directly proportional*** to the ***amplitude*** of the ***microwave input voltage V_1.***
- The ***fundamental component*** of RF beam current passing through the catcher cavity gap induces a current in the catcher cavity and it is expressed as,

$$i_{2\,ind} = \beta_0 i_2 = \beta_0 2I_0 J_1(X)\cos[\omega(t_2 - \tau - T_0)] \qquad \text{... (22)}$$

- Here, β_0 - ***beam coupling coefficient*** of the catcher gap. $\beta_0 = \beta_i$ when both buncher and catcher cavities are identical. The ***magnitude*** of the ***induced current*** in the cavity is given by,

$$I_2 = \beta_0 2I_0 J_1(X) \qquad \text{... (23)}$$

- The ***output power*** delivered to the catcher cavity and the load is given as,

$$P_{out} = \frac{(\beta_0 I_2)^2}{2} R_{sh} \qquad \ldots (24)$$

where,
$$R_{sh} = \frac{V_2}{\beta_0 I_2}$$

V_2 = ***fundamental component of the catcher gap voltage.***

$$= \frac{\beta_0^2 I_2^2}{2} \times \frac{V_2}{\beta_0 I_2}$$

$$\boxed{P_{out} = \frac{\beta_0 I_2 V_2}{2}} \qquad \ldots (25)$$

13.3.7. EFFICIENCY OF KLYSTRON

- The electronic efficiency η of the two-cavity klystron amplifier is defined as the "***ratio of the output power to the input power*** (or) *the* ***ratio of RF output power to the dc beam power***".

$$\eta = \frac{P_{out}}{P_{in}} = \frac{P_{ac}}{P_{dc}} \qquad \ldots (26)$$

The dc power supplied by the beam voltage, $P_{in} = V_0 I_0$ ... (27)

- By substituting equations (25) and (27) in equation (26) we get,

$$\eta = \frac{\beta_0 I_2 V_2}{2 I_0 V_0} \qquad \ldots (28)$$

- In this efficiency, the ***power losses*** to the beam loading and cavity walls are included.

Maximum Efficiency:

- By substituting equation (23) for I_2 in equation (28) we get,

$$= \frac{\beta_0 2 I_0 J_1(X) V_2}{2 I_0 V_0} \qquad \ldots (29)$$

- The ***efficiency*** becomes ***maximum,*** when $\mathbf{J_1(X) = 0.582}$ at $\mathbf{X = 1.841}$ and the output voltage V_2 is equal to V_0 $(V_2 = V_0)$, then equation(29) becomes,

$$= \beta_0 J_1(X)$$

$$= 0.582\,\beta_0$$

If the ***coupling is perfect*** $\beta_0 = 1$, then

$$\boxed{\eta_{max} = 58.2\,\%} \quad \ldots (30)$$

- In practice, the efficiency of a klystron amplifier is in the range from ***15 to 30%.***

13.3.8. VOLTAGE GAIN

- The ***input voltage*** V_1 can be expressed in terms of the bunching parameter X as,

$$V_1 = \frac{2\,V_0}{\beta_0\,\theta_0} X \quad \ldots (31)$$

Already we know that, $$R_{sh} = \frac{V_2}{\beta_0\,I_2}$$

$$V_2 = \beta\,I_2 R_{sh} \quad \ldots (32)$$

- The ***voltage gain*** of a ***klystron amplifier*** is defined as,

$$A_v = \left|\frac{V_2}{V_1}\right| = \frac{\beta_0\,I_2\,R_{sh}}{V_1} \quad \ldots (33)$$

By substituting equation (23) and (31) in equation (42),

$$= \frac{\beta_0\,2\,I_0\,J_1(X)}{\dfrac{2\,V_0\,X}{\beta_0\,\theta_0}} R_{sh}$$

$$= \frac{\beta_0^2\,\theta_0\,I_0\,J_1(X)}{V_0\,X} R_{sh}$$

$$\boxed{A_v = \frac{\beta_0^2\,\theta_0}{R_0}\,\frac{J_1(X)}{X}\,R_{sh}} \qquad \ldots (44)$$

where, $R_0 = \dfrac{V_0}{I_0}$ is the ***dc beam resistance.***

$$A_v = G_m R_{sh} \qquad \ldots (44)$$

13.3.9. CHARACTERISTICS AND APPLICATIONS

Characteristics:

(i) ***Efficiency:*** $\approx$ ***40%.***

(ii) ***Power output:***

(a) Continuous wave average power $\approx$ ***500 kW***

(b) Pulsed ***power 30 MW*** at ***10 GHz.***

(iii) ***Power gain:*** $\approx$ ***30 dB.***

Applications:

(i) Used in Troposphere scatter transmitters.

(ii) Satellite communication ground stations.

(iii) Used in UHF TV transmitters.

(iv) Radar transmitters.

13.4. REFLEX KLYSTRON OSCILLATOR (SINGLE CAVITY KLYSTRON)

13.4.1. INTRODUCTION

- *The reflex klystron is an oscillator with a built in feedback mechanism. It uses the same cavity for both bunching and the output .*
- The repeller electrode is a negative potential and sends the bunched electron beam back to the resonator cavity. This provides a ***positive feedback*** mechanism which supports ***oscillations.***

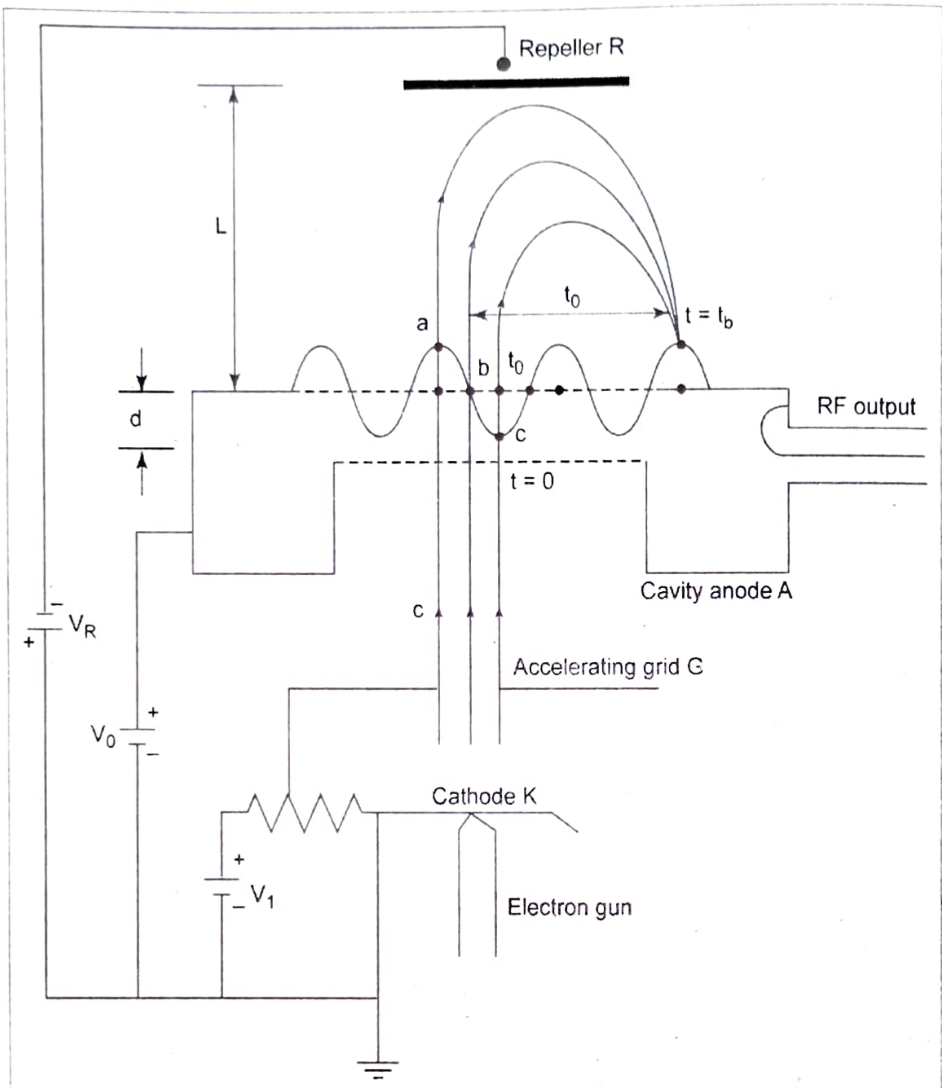


Fig.13.10. Schematic diagram of a reflex klystron

- Due to *dc* voltage (V_0) in the cavity circuit, RF noise is generated in the cavity. This electromagnetic noise field in the cavity act as cavity resonant frequency.
- When the oscillation frequency is varied, the resonant frequency of cavity and the feedback path phase shift must be readjusted for a positive feedback.

13.4.2. MECHANISM OF OSCILLATION

- The electron beam injected from the cathode is first ***velocity – modulated*** by cavity – gap voltage.
- The electrons which encountered the ***positive half cycle*** of the RF field, where in the cavity gap velocity will be ***accelerated.***

- The electrons which encountered ***zero RF field*** will pass with ***unchanged original velocity***, and the electrons which encountered the ***negative half cycle*** velocity will be ***decelerated.***
- All these velocity modulated electrons will be repelled back to the cavity by the repeller due to its negative potential.
- The repeller distance L and the voltages (beam& repeller voltage) can be adjusted to receive all the velocity modulated electrons at a same time on the positive peak of the cavity RF field.
- The velocity modulated electrons are bunched together and lose their kinetic energy when they encounter the positive cycle of the RF field. This loss of energy is transferred to the cavity to conserve the total power.
- If the ***power delivered*** by the bunched electrons to the cavity is ***greater than*** the ***power loss*** in the cavity, the electromagnetic field amplitude at the resonant frequency of the cavity will increase to produce microwave oscillations.
- The electrons are finally collected by the walls of the cavity or other grounded metal parts of the tube.

13.4.3. APPLEGATE DIAGRAM

- *The electrons passing through the buncher grids are accelerated / retarded / passed through with an unchanged initial dc velocity depending upon whether they encounter the RF signal field at the buncher cavity gap at positive / negative / zero crossing phase of the cycle, respectively as shown by distance – time plot. This is called the* ***applegate diagram.***

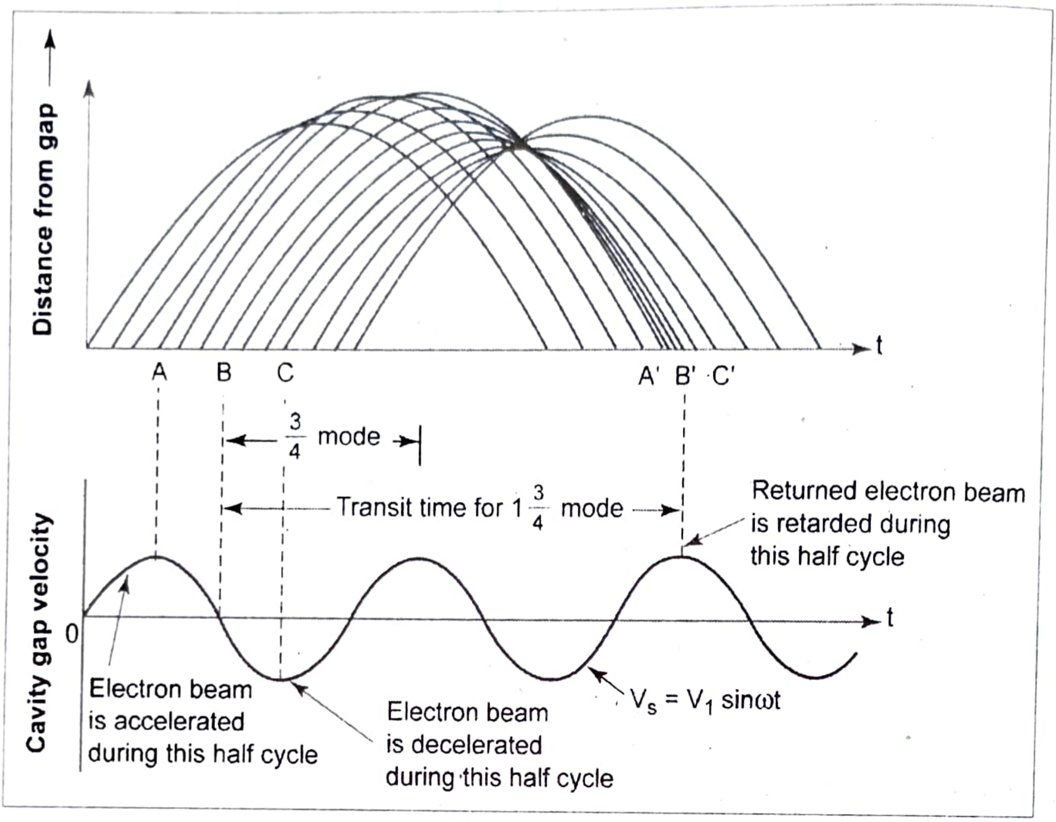


Fig. 13.11. Applegate diagram with gap voltage for a reflex klystron.

Explanation:

- When the gap voltage is at ***positive peak,*** electron passing at this moment is called ***early electron***. This electron is accelerated towards repeller and travels at a distance, which is longer comparatively.
- The electron at neutral zero of gap voltage is called ***reference electron***. When the gap voltage is at ***negative peak*** the corresponding electron is called ***late electron***. This electron decelerated and travels at a less distance.
- These electrons have different velocities cover different distances forms bunch at cavity gap.

13.4.4. MODES OF OSCILLATION

- The condition for oscillation is,

$$\boxed{t_0 = \left(n + \frac{3}{4}\right)T}$$

$$= NT$$

where, $N = n + \frac{3}{4}$ and

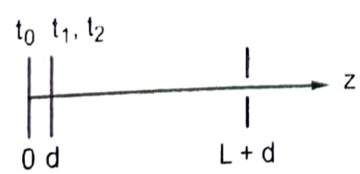
Mode of oscillation, $n = 0, 1, 2, 3, \ldots$

T is the ***time period at the resonant frequency.***

t_0 is the ***time taken*** by the ***reference electron*** to travel in the repeller space.

13.4.5. VELOCITY MODULATION

- For calculation of RF power, it is assumed that,
  - (i) Cavity grids and repeller are plane parallel and very large in extent.
  - (ii) No RF field is excited in the repeller space.
  - (iii) Electrons are not intercepted by the cavity anode grid.
  - (iv) No debunching takes place in the repeller space.
  - (v) The cavity RF gap voltage amplitude V_1 is small compared to the dc beam voltage V_0 : $V_1 << V_0$



t_0 = time for electron entering cavity gap at z = 0

t_1 = time for same electron leaving cavity gap at z = d

t_2 = time for same electron returned by retarding field z = d and collected on walls of cavity

Fig.13.12.

- The ***electrons enter*** into the ***cavity gap*** from the cathode at z = 0 and time t_0 is assumed to have ***uniform velocity.***

$$v_0 = 0.593\times10^6\sqrt{V_0} \quad \cdots (1)$$

- The same electron ***leaves*** the ***cavity gap*** at z = d at time t_1 with velocity (***exit velocity***) as,

$$v(t_1) = v_0\left[1+\frac{\beta_i V_1}{2V_0}\sin\left(\omega t_1-\frac{\theta_g}{2}\right)\right] \quad \cdots (2)$$

- The same electron is forced back to the cavity $z = d$ and time t_2 by the retarding electric field E,

$$E = \frac{V_r+V_0+V_1\sin(\omega t)}{L} \quad \cdots (3)$$

- This retarding field E is assumed to be constant in the z direction. The force equation for one electron in the repeller region is,

$$E = \frac{V_r+V_0}{L} \text{ for } |V_1\sin\omega t| << (V_r+V_0) \quad \cdots (4)$$

$$\text{Force of electron} = -eE = -e\left(\frac{V_r+V_0}{L}\right) \quad \cdots (5)$$

- Force of electrons = mass × acceleration = $\frac{md^2z}{dt^2}$ (z-distance)

Therefore,

$$m\frac{d^2z}{dt^2} = -eE$$

$$= -e\frac{V_r+V_0}{L}$$

$$\frac{d^2z}{dt^2} = -e\left[\frac{V_r+V_0}{mL}\right] \quad \cdots (6)$$

where, **E** = – ∇ **V** is used in the z direction only.

V_r is the **magnitude** of the **repeller voltage.**

- By integrating equation (6) twice with respect to "t" and "t_1" we get,

$$\frac{dz}{dt} = -e\frac{(V_r + V_0)}{mL}\int_{t_1}^{t} dt$$

$$= -e\frac{(V_r + V_0)}{mL}(t - t_1) + K_1 \qquad \text{... (7)}$$

At $t = t_1$, $\quad \frac{dz}{dt} = v(t_1) = K_1$; then

$$z = -e\left[\frac{V_r + V_0}{mL}\right]\int_{t_1}^{t}(t - t_1)dt + v(t_1)\int_{t_1}^{t} dt$$

$$= -e\left[\frac{V_r + V_0}{2mL}\right](t - t_1)^2 + v(t_1)(t - t_1) + K_2 \qquad \text{... (8)}$$

At $t = t_1$, $\quad z = d = K_2$; then

$$z = \frac{-e(V_r + V_0)}{2mL}(t - t_1)^2 + v(t_1)(t - t_1) + d \qquad \text{... (9)}$$

- The electron leaves the cavity gap at $z = d$ and time t_1 with a velocity of $v(t_1)$ and due to ***repeller negative potential*** returns to the gap at $z = d$ and time t_2, then at $t = t_2$, $z = d$.

$$0 = \frac{-e(V_r + V_0)}{2mL}(t_2 - t_1)^2 + v(t_1)(t_2 - t_1) \qquad \text{... (10)}$$

Transit Time:

- The ***round – trip transit time*** in the repeller region is given by,

$$T' = \frac{2\,\text{Velocity}}{\text{Acceleraton}}$$

The factor 2 in the numerator arises because of the to and fro journey of electrons,

$$T' = \frac{2v(t_1)}{\frac{d^2z}{dt^2}} \qquad \text{... (11)}$$

- By substituting equation (6) for $\frac{d^2z}{dt^2}$ in equation (11) we get

$$T' = t_2 - t_1 = \frac{2mL}{e(V_r + V_0)} v(t_1) \qquad \text{... (12)}$$

- Now, the ***negative sign*** is ***not*** taken as the electron bunch travels in the ***reverse direction***. By substituting equation (2) in equation (12) we get,

$$= T_0'\left[1 + \frac{\beta_i V_1}{2V_0} \sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right] \qquad \text{... (13)}$$

- The ***round – trip dc transit time*** of the center of the bunch electron is expressed as,

$$T_0' = \frac{2mLv_0}{e(V_r + V_0)} \qquad \text{... (14)}$$

Multiply the equation (13) by a ***radian frequency***

$$\omega(t_2 - t_1) = \omega T_0' + \omega T_0' \frac{\beta_i V_1}{2V_0} \sin\left(\omega t_1 - \frac{\theta_g}{2}\right)$$

$$= \theta_0' + X' \sin\left(\omega t_1 - \frac{\theta_g}{2}\right) \qquad \text{... (15)}$$

where, ***Round – trip dc transit*** angle of the center of the bunch electron $\theta_0' = \omega T_0'$, and

Bunching parameter of the reflex klystron oscillator, $X' = \frac{\beta_i V_1}{2V_0} \theta_0'$.

13.4.6. POWER OUTPUT

- A maximum amount of kinetic energy can be transferred from the returning electrons to the cavity walls.
- For a ***maximum energy transfer***, the round – trip transit angle (center of the bunch) must be given by,

$$= \omega(t_2 - t_1) = \omega T_0' = \left(n - \frac{1}{4}\right)2\pi \quad \ldots (16)$$

$$= N2\pi = 2\pi n - \frac{\pi}{2} \quad \ldots (17)$$

where, assume $V_1 << V_0$

n = any positive integer for cycle number, and

$N = n - \frac{1}{4}$ is the number of modes.

- The beam current injected into the cavity gap from the repeller region flows in the ***negative z direction***. The beam current of a reflex klystron oscillator can be written as,

$$i_{2t} = -I_0 - \sum_{n=1}^{\infty} 2 I_0 J_n (nX') \cos\left[n\left(\omega t_2 - \theta_0' - \theta_g\right)\right] \quad \ldots (18)$$

where, I_0 = ***dc beam current.***

- The fundamental component of the current induced in the cavity by the modulated electron beam is given by $(\theta_g << \theta_0)$.

$$i_2 = -\beta_i I_2$$

$$= 2 I_0 \beta_i J_1 (X') \cos(\omega t_2 - \theta_0') \quad \ldots (19)$$

- The ***magnitude*** of the fundamental component is,

$$I_2 = 2 I_0 \beta_i J_1 (X') \quad \ldots (20)$$

- The ***dc power*** supplied by the beam voltage V_0 is,

$$P_{dc} = V_0 I_0 \quad \dots (21)$$

- The ***ac power*** delivered to the load is,

$$P_{ac} = \frac{V_1 I_2}{2}$$

$$= V_1 I_0 \beta_i J_1(X') \quad \dots (22)$$

where, X' is the ***bunching parameter*** of the reflex klystron

$$X' = \beta_i V_1 \frac{\theta'_0}{2 V_0} \quad \dots (23a)$$

$$\theta'_0 = \omega T'_0 = 2\pi n - \frac{\pi}{2} \quad \dots (23b)$$

- From equations (22a) and (22b) we get

$$2 V_0 X' = \beta_i V_1 \left(2\pi n - \frac{\pi}{2}\right)$$

$$\frac{V_1}{V_0} = \frac{2X'}{\beta_i \left(2\pi n - \frac{\pi}{2}\right)}$$

$$V_1 = \frac{2X' V_0}{\beta_i \left(2\pi n - \frac{\pi}{2}\right)} \quad \dots (24)$$

- By substituting equation (24) in equation (22) we obtain,

$$P_{ac} = \frac{2X' V_0 I_0 \beta_i J_1(X')}{\beta_i \left(2\pi n - \frac{\pi}{2}\right)}$$

$$\boxed{P_{ac} = \frac{2 V_0 I_0 X' J_1(X')}{2\pi n - \frac{\pi}{2}}} \quad \dots (25)$$

13.4.7. EFFICIENCY (η)

- The ***electronic efficiency*** of a reflex klystron oscillator is,

$$\text{Efficiency} = \frac{P_{ac}}{P_{dc}} \quad \text{... (26)}$$

- By substituting equations (21) and (25) in equation (26) we get,

$$= \frac{2 V_0 I_0 X' J_1(X')}{\left(2\pi n - \frac{\pi}{2}\right) V_0 I_0}$$

$$\boxed{\eta = \frac{2X' J_1(X')}{2\pi n - \frac{\pi}{2}}} \quad \text{... (27)}$$

Maximum Efficiency:

- The factor $X' J_1(X')$ reaches a maximum value of 1.25 at,

$$\boldsymbol{X' = 2.408} \text{ and } \boldsymbol{J_1(X') = 0.52}$$

- The ***maximum efficiency*** is obtained when n = 2 or $1\frac{3}{4}$ mode, then maximum ***theoretical efficiency*** is,

$$\eta_{max} = \frac{2(2.408) J_1(2.408)}{2\pi(2) - \frac{\pi}{2}}$$

$$= \frac{4.816 \times 0.52}{4 \times 3.14 - \frac{3.14}{2}}$$

$$= \frac{2.5}{12.56 - 1.57} = \frac{2.5}{10.99}$$

$$\boxed{\eta_{max} = 22.78\%} \quad \text{... (28)}$$

- The maximum theoretical efficiency of a reflex klystron oscillator ranges from ***20 to 30%.***

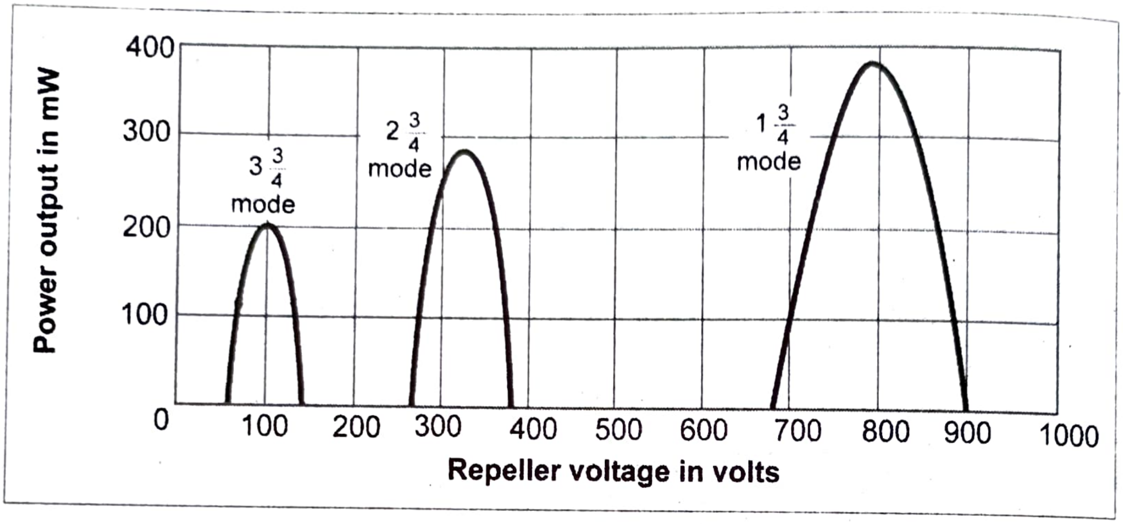


Fig.13.11. Power output and frequency characteristics of a reflex klystron.

Power Output in terms of Repeller Voltage V_R

- For a given beam voltage V_0, the relationship between the repeller voltage and cycle number n required for oscillation is given by,

$$\frac{V_0}{(V_r + V_0)^2} = \frac{\left(2\pi n - \frac{\pi}{2}\right)^2}{8\omega^2 L^2} \frac{e}{m} \qquad \dots (29)$$

- The power output can be expressed in terms of the repeller voltage V_r is given by,

$$\boxed{P_{ac} = \frac{V_0 I_0 X' J_1(X')(V_r + V_0)}{\omega L} \sqrt{\frac{e}{2m V_0}}} \qquad \dots (30)$$

Equivalent Circuit of a Reflex Klystron

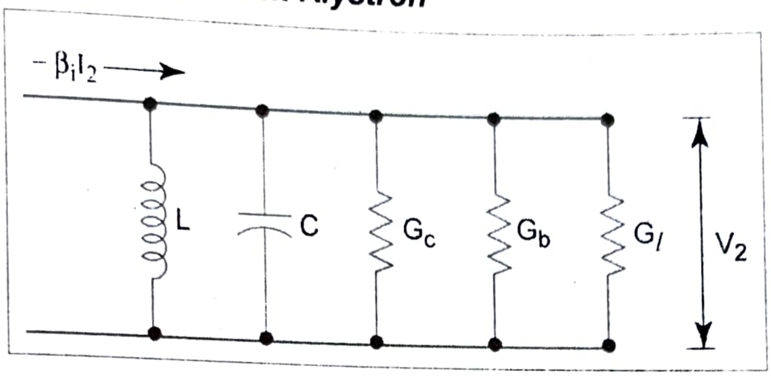


Fig.13.12. Equivalent circuit of a reflex klystron

- In this circuit, ***L*** and ***C*** are the ***energy storage elements*** of the cavity.

  G_c – Copper losses of the cavity,

  G_b – Beam loading conductance, and

  G_l – Load conductance.

13.4.8. CHARACTERISTICS AND APPLICATIONS OF REFLEX KLYSTRON

Characteristics :

Frequency range: 1 to 25GHz.

Power output: It is a low – power generator of 10 to 500mW

Efficiency: About 20 to 30%.

Applications:

(i) This type is widely used in the laboratory for microwave measurements.

(ii) In microwave receivers, as local oscillators in commercial and military applications.

(iii) Also plays a role in airborne Doppler radars as well as missiles.