Iterative learning controller

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I. Part-A

A. Task A.1

The system is defined as

$$x_k(t+1) = Ax_k(t) + Bu_k(t)$$

$$y_k(t) = Cx_k(t)$$

Here A = 1, B = 1 , C = 2. y_k is denoted as

$$y_k = Gu_k$$

The matrix is written using the lifted form of G.

$$\begin{bmatrix} y_k(1) \\ y_k(2) \\ . \\ y_k(N) \end{bmatrix} = \begin{bmatrix} CB & & & \\ CAB & CB & & \\ . & . & & \\ CA^{N-1}B & CA^{N-2}B & . \quad . & CB \end{bmatrix} \begin{bmatrix} u_k(0) \\ u_k(1) \\ . \\ u_k(N-1) \end{bmatrix}$$

B. Task A.2

u_{k+1} is governed by the following equation :

$$u_{k+1} = u_k + \gamma_k G^T e_k$$

e_{k+1} is reference signal subtracted by Gu_{k+1}. Substituting and simplifying we would arrive at the following equation.

$$e_{k+1} = (I - G\gamma_k G^T)e_k$$

$$G.G^T = \begin{bmatrix} (CB)^2 & & & \\ (CAB)^2 & (CB)^2 & & \\ . & . & & \\ (CA^{N-1}B)^2 & (CA^{N-2}B)^2 & . \quad . & (CB)^2 \end{bmatrix}$$

The cost of the gradient algorithm is given by

$$(\|e_{k+1}\|)^2 + \omega(\gamma_k)^2$$

Applying matrix properties we get

$$e_{k+1}(e_{k+1})^T + \omega(\gamma_k)^2$$

To minimise it we differentiate it wrt γ_k and set it to zero. Substituting the values of e_{k+1} we get the following :

$$(I - \gamma_k GG^T)^2 e_k + \omega(\gamma_k)^2$$

differentiating it setting it equal to zero we get:

$$\gamma_k = \frac{e_k^T GG^T e_k}{e_k(GG^T)^2 e_k + \omega}$$

C. Task A.3

To prove that $(\|e_{k+1}\|) < (\|e_k\|)$, we first prove $(\|e_{k+1}\|)^2 < (\|e_k\|)^2$ and simplifying further we get $e_{k+1}(e_{k+1})^T < e_k(e_k)^T$. Simpplifying we get down to the equation :

$$(I - \gamma_k GG^T)(I - \gamma_k GG^T)^T < 1$$

The values of the LHS can be calculated and are shown below :

$$\begin{bmatrix} (1-\gamma_k(CB)^2)^2 & & & \\ (-\gamma_k(CAB)^2)^2 & (1-\gamma_k(CB)^2)^2 & & \\ . & & & \\ . & & & \\ (-\gamma_k(CA^{N-1}B)^2)^2 & . & . & (1-\gamma_k(CB)^2)^2 \end{bmatrix}$$

Substituting the values of C,A,B and γ_k, we can see that value all the elements would be less than 1.

II. Part-B

A. Task B.1

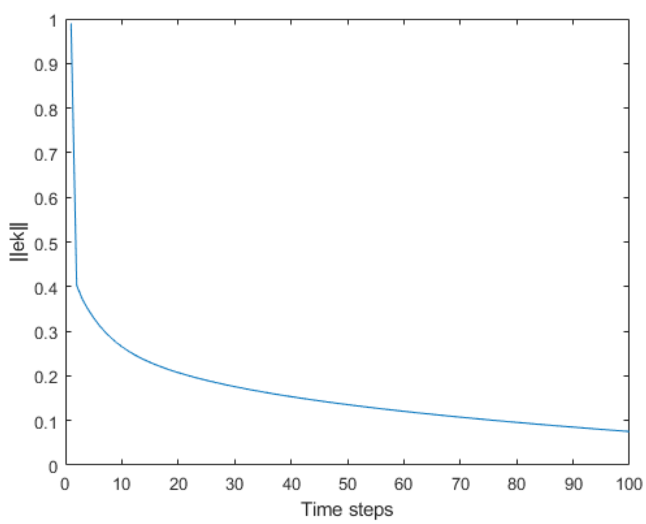

Fig. 1. Tracking error norm for gamma 100.

The above figure illustrates the error norm iterations done for 100 time steps. We can see that the error norm converges to 0 when γ_k value is set to 100 . It posses the same properties as discussed in task A.3 where the tracking error norm doesn't exceed 1.The curve converges monotonically for a high value of γ_k .

When the value of γ_k is fixed to a lower value of 10 as shown in figure 2. We see that the tracking error norm converges to

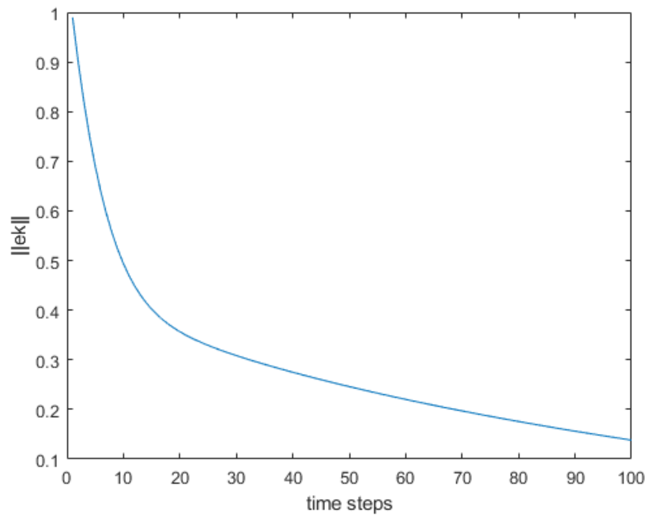

Fig. 2. Tracking error norm for gamma 10.

0 in an asymptotic manner taking upto 20 iterations to reach the value of 0.4.

B. Task B.2

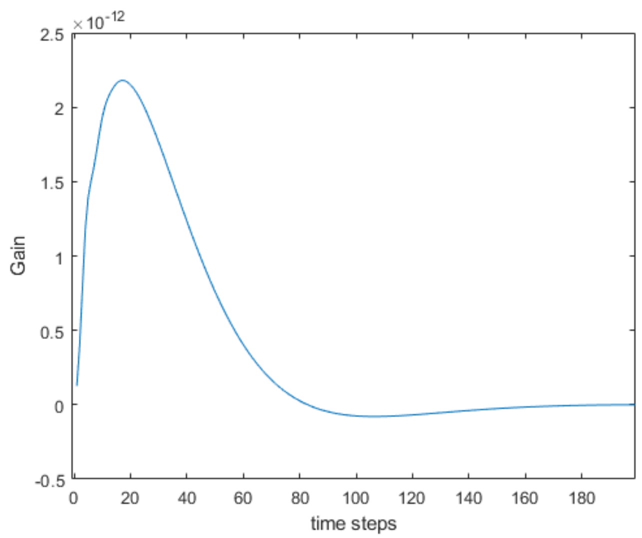

Fig. 3. variation of gamma with iteration.

The above figure illustrates the variation of γ_k as the time steps increase. It is supposed to increase as the rate of change of gradient of the tracking norm increases. It goes stable as the gradient of the tracking norm approaches zero.

C. Task B.3

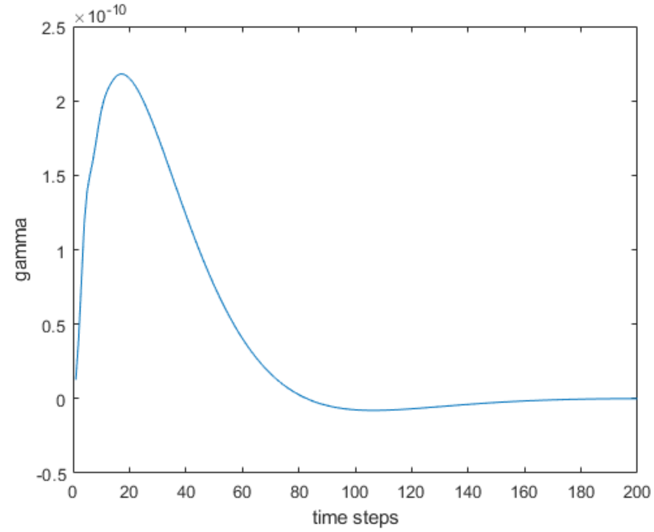

Fig. 4. variation of gamma with changes in omega.

When omega increases , the denominator of the equation in γ_k increases. However the changes brought by ω is very insignificant and is evident in the graph of γ_k . When we increase the value ofω, it restricts the range of omega. As we can see when omega increases , the range has got restricted but by a very small fraction .