MB&B 562: Exercise 7

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Q1.1) Capillary : $\frac{d\theta_2}{dt} = \omega_2$

Hair Cell : $\frac{d\theta_1}{dt} = \omega_1 - \epsilon \sin(\theta_1 - \theta_2) + \xi(t)$

Q1.2) $\frac{d}{dt}[\theta_1 - \theta_2] = (\omega_1 - \omega_2) - \epsilon \sin(\Delta\theta) + \xi(t)$

$\frac{d(\Delta\theta)}{dt} = \Delta\omega - \epsilon \sin(\Delta\theta) + \xi(t)$

where, $\Delta\omega = \omega_1 - \omega_2$.

Q1.3) Given, $\omega_1 = 90$ rad/s

$\omega_2 = 60$ rad/s

$\epsilon = \alpha f_0$

Coupling occurs when $\frac{d\theta}{dt} \to 0$

At that point,

$\Delta\omega = \epsilon \sin(\Delta\theta) + \xi(t)$

$= \alpha f_0 \sin(\Delta\theta) + \xi(t)$

Assuming that at coupling onset ($t = 0$), there is no noise,

$\Delta\omega \approx \alpha f_0$

$(90 - 60) \approx \alpha (0.35 \times 10^{-12})$

$\alpha \approx 8.57 \times 10^{13}$ rad $N^{-1} s^{-1}$