Exercise #10 - Turing
MB&B361/562
Spring 2024

Due: before class on Tuesday, April 16, 2024
Please upload it to the Canvas Box (title: 'LastnameFirstname_ExerciseXX').

Please read sections 1 and 4 of Turing (1952), which is uploaded to Canvas.

Question 1 (2 points)
The bicoid protein (look it up in Wiki) is a transcription factor. What are two properties of transcription factors that make them morphogens according to Turing's (rather vague) definition?

Question 2 (4 points)
At the end of Section 4 Turing considers when a reaction diffusion system becomes unstable. Let's take the example of two cells from the class:

$$\dot{\Delta}_x = (5 - 4D_x)\Delta_x - 6\Delta_y$$
$$\dot{\Delta}_y = 6\Delta_x - (7 + 4D_y)\Delta_y$$

Consider the case where $D_x = 0.25$ is fixed and suppose that the diffusion coefficient of the inhibitor, D_y, is a control parameter.
(i) Calculate the trace and determinant of the Jacobian with $D_x = 0.25$.
(ii) Is the system stable when $D_y = 0$?
(ii) Is the system stable when $D_y = 1$?
(iii) As D_y increases, what is the critical value when the system becomes unstable?
(iv) Does the ratio of this critical value of D_y to D_x conform to the general principle of how activators and inhibitors create patterns?

Question 3 (2 points)
In Section 4, Turing points out that statistical fluctuations in number of molecules leads to inhomogeneities:

> larly shaped mass of tissue, it will continue indefinitely to be homogeneous. In practice, however, the presence of irregularities, including statistical fluctuations in the numbers of molecules undergoing the various reactions, will, if the system has an appropriate kind of instability, result in this homogeneity disappearing.

Consider a cell that is 10 μm x 10 μm x 10 μm. Assume that a morphogen has a concentration of 1 nM.
(i) Approximately how many molecules are there in the cell (round to a power of 10)? Assume that Avogadro's number is $10 \times 10^{23} = 10^{24}$
(ii) (Graduate students) How large do you expect the number fluctuations to be (i.e. what is the expected standard deviation of the number)?