MB&B 562: Exercise 2

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Exercise 1

Live script submitted as PDF. Numerical solutions for in-script questions follow.

Q2) setting $\beta = 0$,

$$\dot{x} = r - \alpha x$$

$$\int \frac{dx}{r - \alpha x} = \int dt$$

$$\frac{-1}{\alpha} \ln(r - \alpha x) = t + c$$

At $t = 0$, $x = x_0 = 0$

$$c = \frac{-1}{\alpha} \ln(r)$$

Using value of c,

$$\frac{-1}{\alpha} \left(\ln(r - \alpha x) + \ln(r) \right) = t$$

$$r / (r - \alpha x) = e^{-\alpha t}$$

when $t \to \infty$,

$$r = \alpha x_{ss} \quad \text{OR} \quad r = 0$$

Using $r = 0.1$, $\alpha = 0.05$

$$\boxed{x_{ss} \approx 20}$$

Q3) Setting feedback term to β,

$$\dot{x} = r - \alpha x + \beta$$

Solving $|||^{ly}$ to Q2,

$$\frac{-1}{\alpha} \ln (r + \beta - \alpha x) = t + c$$

At $t = 0$, $x = x_0 = 100$

$$c = \frac{-1}{\alpha} \ln (r + \beta - 100\alpha)$$

Using the value of c

$$\frac{-1}{\alpha} \left(\ln \left((r + \beta - \alpha x)(r + \beta - 100\alpha \right) \right) = t$$

$$(r + \beta - \alpha x)(r + \beta - 100\alpha) = e^{-\alpha t}$$

When $t \to \infty$,

$$r + \beta - \alpha x = 0$$

$$x_{ss} = \frac{r + \beta}{\alpha}$$

Using $r = 0.1$, $\beta = 3$, $\alpha = 0.05$

$$x_{ss} = \frac{3.1}{0.05}$$

$$\boxed{x = 62}$$

Exercise 2

2) a) $ax^2 + bx + c = 0$

(where $a \neq 0$ for quadratic form

$ax^2 + bx = -c$

To complete the square on LHS,

& comparing with :

$(m+n)^2 = m^2 + 2mn + n^2$

Multipying by $4a$,

$4a^2x^2 + 4abx = -4ac$

$(2a)^2x^2 + 2(2a)(b)x = -4ac$

Now, adding b^2 on either side,

$(2a)^2x^2 + 2(2a)(b)x + b^2 = b^2 - 4ac$

$(2ax + b)^2 = b^2 - 4ac$

$2ax + b = \pm\sqrt{b^2 - 4ac}$

$$\boxed{x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}}$$

is the general soln.

b) Sum of roots (Σx_i)

from ⓐ,

$$\Sigma x_i = \frac{-b + \cancel{\sqrt{b^2 - 4ac}}}{2a} + \frac{-b - \cancel{\sqrt{b^2 - 4ac}}}{2a}$$

$$\boxed{\Sigma x_i = \frac{-b}{a}}$$

Product of roots (Πx_i)

from ⓑ,

$$\Pi x_i = \left(\frac{-b + \sqrt{b^2 - 4ac}}{2a}\right)\left(\frac{-b - \sqrt{b^2 - 4ac}}{2a}\right)$$

$$= \frac{\cancel{b^2} - (\cancel{b^2} - 4ac)}{4a^2}$$

$$\boxed{\Pi x_i = \frac{c}{a}}$$

Exercise 3

My programming ability: **moderate**.