

#### Exercise #4

MB&B 361/562.

Due: before class on Tuesday, February 13, 2024

Please upload it to the Canvas Box (title: 'LastnameFirstname\_Exercise2'). You can scan handwritten parts.

1. Consider the Collins switch from the Class 5 notes:

$$\dot{u} = f(u, v) = \frac{2}{1+v^4} - u$$

$$\dot{v} = g(u, v) = \frac{2}{1+u^4} - v$$

(i) Let  $u^* = \varepsilon$  be the small- $u$  fixed point (actually  $\varepsilon \cong 0.2$  reading off the graph in the lecture notes). Calculate the approximate value of the associated value of  $v^*$ ?

(ii)\* Calculate the Jacobian (approximately) and show that  $(u^*, v^*)$  is a stable fixed point.

2. Let  $A = \begin{pmatrix} -1 & 0.5 \\ 0.5 & -1 \end{pmatrix}$

(i) Find the eigenvalues of  $A$ . Let  $\lambda_1$  be the larger one and  $\lambda_2$  be the smaller one.

(ii) Show that  $A \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \lambda_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix}$  and  $A \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \lambda_2 \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ . The vectors  $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$  and  $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$  are called the eigenvectors associated with the respective eigenvalues.

(iii) Consider the  $\dot{x} = Ax$  with initial value  $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ . Describe the trajectory of  $x$ .

\*Bonus points for undergraduates