NAME:____

Math 227A, Computational Biology Homework 5

Fall 2025, German Enciso
Due: Friday, November 7th, at the beginning of class

1. Consider the system $x' = -x + yx^3$, $y' = y - y^3x$.
   a) Find the steady states of this system
   b) Linearize around each steady state to determine the qualitative local behavior around it.
   c) Use Hartman Grobman's Theorem to justify whether this information can be transferred to the original system, and draw an estimated phase portrait of the original nonlinear system.
   d) Now consider the system $x' = -x + yx$, $y' = y - yx$, which is identical to the first one but with different exponents. Linearize around the positive steady state $(1, 1)$. What can you conclude about the original system around the equilibrium based on the linearization alone?

2. Consider a transcription factor that promotes its own growth. Let x be the messenger RNA, and y the protein itself. Assume that $x' = \sigma(y) - x$, $y' = x - y$, where $\sigma(x) = 2x^2/(1 + x^2)$. Notice this system is only defined for $x \leq 0$, $y \geq 0$.
   a) Find the steady states of the system.
   b) Linearize around each steady state to determine the qualitative local behavior around it.
   c) Use Hartman Grobman's Theorem to justify whether this information can be transferred to the original system, and draw an estimated phase portrait of the original nonlinear system.

3. Consider the system $x' = y - 2x$, $y' = \mu + x^2 - y$.
   a) Sketch the nullclines
   b) Find and classify the bifurcations that occur as μ varies.
   c) Sketch the phase portrait as a function of μ.

4. Carry out the following problem from Strogatz:

8.1.10 (Budworms vs. the forest) Ludwig et al. (1978) proposed a model for the effects of spruce budworm on the balsam fir forest. In Section 3.7, we considered the dynamics of the budworm population; now we turn to the dynamics of the forest. The condition of the forest is assumed to be characterized by $S(t)$, the average size of the trees, and $E(t)$, the "energy reserve" (a generalized measure of the forest's health). In the presence of a constant budworm population B, the forest dynamics are given by

$$\dot{S} = r_S S\left(1 - \frac{S}{K_S}\frac{K_E}{E}\right), \qquad \dot{E} = r_E E\left(1 - \frac{E}{K_E}\right) - P\frac{B}{S},$$

where $r_S, r_E, K_S, K_E, P > 0$ are parameters.

a) Interpret the terms in the model biologically.
b) Nondimensionalize the system.
c) Sketch the nullclines. Show that there are two fixed points if B is small, and none if B is large. What type of bifurcation occurs at the critical value of B?
d) Sketch the phase portrait for both large and small values of B.

Figure 1: