NAME:_____

Math 227A, Computational Biology Homework 2

Fall 2025, German Enciso
Due: Friday, October 17, at the beginning of class

1. Study the steady states of the following ODE $y' = f(y)$, and determine their stability. Draw on a separate diagram the graphs of two sample solutions for each system.

   a) $y' = 2y - 2y^3$

   b) $y' = e^{-y}\sin(y)$

2. Working backwards, find an equation for a system $y' = f(y)$, with exactly four steady states at $y = 0, 1, 2, 3$, and which are stable, unstable, stable, and unstable, respectively. Is there more than one system with the same steady states and stability? Explain.

3. Consider the ODE $y' = y^2 - 1 + r$.

   a) Draw a phase plot for two different values of the bifurcation parameter.

   b) Draw the bifurcation diagram for this system, and determine what type of bifurcation it is.

   c) Determine the value of the bifurcation point $\bar{r}$. Hint: how should the phase plot look like at the bifurcation point itself?

4. Consider the ODE $y' = -r\ln y + y - 1$.

   a) In order to study the steady states for different values of r, set the ODE equal to zero, that is, $r\ln y = y - 1$. Plot both functions (right and left hand side) on the same graph, and show how this plot can be used to graphically display the steady states for every value of r. Show that $y = 1$ is a steady state for every value of r.

   b) Show that $\bar{r} = 1$ is a bifurcation point for this system, with the associated critical point $\bar{y} = 1$. What information do you need to know to show this?

   c) Sketch a phase plot for the system when $0 < r < 1$ and $r > 1$. How many steady states are there in each case, and what is their stability?

   d) Sketch the bifurcation diagram for this system and determine the type of bifurcation (eg saddle-node, transcritical).

5. Consider the ODE $y' = ay(y-b)(1-y/K)$ encountered in ecological dynamics. Draw a bifurcation diagram for this system using the parameter K. What type of bifurcation is present?

6. Consider the system $y' = y^2(5-y) + 1 - ry$.
   a) Carry out a mathematical analysis to show that this system presents hysteresis.
   b) Use Matlab to numerically plot the bifurcation graph of this system. You can use a Matlab function to find the roots of algebraic equations.