# Homework 2 - Due on January 24

## Problem 1

a) Write a function to compute the divided difference: input

$$(x_1, x_2, ..., x_{n+1}) \qquad (f(x_1), f(x_2), ..., f(x_{n+1}))$$

then output

$$f[x_1], f[x_1, x_2], ...., f[x_1, x_2, x_3, ..., x_{n+1}]$$

Please demonstrate correctness of your code.

b) Write a function to compute the polynomial using nested multiplication: input

$$(a_1, a_2, ..., a_{n+1}) \qquad x$$

then output

$$p(x) = a_1 + a_2(x - x_1) + a_3(x - x_1)(x - x_2) + ... + a_{n+1}(x - x_1)(x - x_2)...(x - x_n)$$

Please demonstrate correctness of your code.

c) Combine a) and b) to write a package for the Lagrange polynomial interpolation. Discuss 1) Use a flow chart to show how you make the combined code in modules to minimize changes when different functions are used for the package; 2) how you test the correctness of your overall implementation.

d) Use your package to obtain the Lagrange polynomial interpolation for the following three functions with $n$ equally spaced points $\in [a, b]$.

1) $f(x) = 3x^3 + 4x^2 + 2x + 1$ with $a = -1, b = 1$.

2) $f(x) = \sin(x)$ with $a = 0, b = 2\pi$

3) $f(x) = 1/(1 + 25x^2)$ with $a = -1, b = 1$.

Discuss how the errors between the interpolation function and $f(x)$ depend on $n$.

## Problem 2

Demonstrate the the order of accuracy computationally for the cubic spline using Matlab. Construct at least two examples (at least two different non-trival functions) to show the order of accuracy. Please try two different boundary conditions implemented in Matlab.